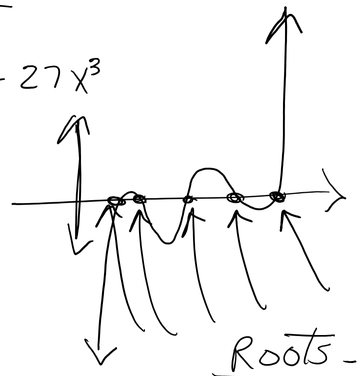
Roots

Wednesday, January 10, 2018 11:28 AM

Goal.

$p(x) = 3x^5 + 18x^4 + 27x^3$

graph.



Roots.

A.)

Roots -

- Zeros
- x-intercepts.

} all 3 same

B.) Factors $\Longrightarrow$ Roots.

Zero product property.

$$(x-3)(x+1)(x) = 0$$

ZPP

$x - 3 = 0$ $\quad$ $x + 1 = 0$ $\quad$ $x = 0$

$x = 3$ $\quad$ $x = -1$

$$\boxed{\text{Roots: } 3, -1, 0}$$

where the polynomial crosses the x axis.

Ex.) Solve by factoring:

$$3x^5 + 18x^4 + 27x^3 = 0$$

1.) Factor: $3x^3(x^2 + 6x + 9) = 0$

$3x^3(x+3)(x+3) = 0$

Set each factor $= 0$

$3x^3 = 0 \Longrightarrow x = 0$

$x + 3 = 0 \Longrightarrow x = -3$

$x+3=0 \implies x=-3$

$\boxed{\text{Roots aro } 0 \text{ and } -3}$

$\boxed{x=0 \quad x=-3 \quad \text{are solution}}$

Solve: $x^4-13x^2=-36$

$x^4-13x^2+36=0$ Quadratic Type.

$(x^2-9)(x^2-4)=0$

$(x+3)(x-3)(x+2)(x-2)=0$

$\boxed{x=-3,\ 3,\ -2,\ 2}$

Multiplicity

Sometimes a factor appears more than once.

Ex. $(x-3)^2=0$

$(x-3)(x-3)=0$

3 is a root w/ multiplicity 2.

State the muliplicity of each root.

Ex: $(x-2)(x-1)^3(x+4)^2=0$

2 Root w/ muliplicity 1

1 Root w/ multiplicity 3

-4 Root w/ multiplicity 2.

Rational Root Theorem:

$\underbrace{Qx^n+\cdots\cdots+P}_{\text{Standard form}}$

All Rational Roots have form

$\pm\dfrac{P \text{ factors}}{Q \text{ factors}}$

Standard form

$\pm \frac{P}{Q}$ factors / factors.

Ex: $\overset{Q}{5}x^3 + 3x^2 + 11x + \overset{P}{8}$

$\frac{8}{1 \cdot 8,\ 2 \cdot 4}$ $\quad$ $\frac{5}{1 \cdot 5}$

$$\pm \frac{P}{Q} = \pm \frac{8}{5}$$

Potential list: $\pm \left\{ \frac{1}{1}, \frac{1}{5}, \frac{8}{1}, \frac{8}{5}, \frac{2}{1}, \frac{2}{5}, \frac{4}{1}, \frac{4}{5} \right\}$

Solve: $x^2 = 5$

$x = \pm\sqrt{5}$

Solve: $(x-1)^2 = 6$

$x - 1 = \pm\sqrt{6}$

$x = 1 \pm \sqrt{6}$

$$\underbrace{(1+\sqrt{6}) \qquad (1-\sqrt{6})}_{\text{Conjugate pair.}}$$

IRRational Root thm:

$\boxed{\text{IRRational Roots occur in conjugate pairs.}}$

A

I know 2 and $1-\sqrt{3}$ are Roots.

$\Rightarrow$ $\quad$ $1+\sqrt{3}$ also a Root.

$\boxed{\text{Root} \longleftrightarrow \text{Factor} \longleftrightarrow \text{Polynomial.}}$

Roots $\quad$ Factors $\quad$ Polynomial

-1 and -6 $\leftarrow (x+1)(x+6) \leftarrow x^2+7x+6$

Old

Roots	Factors	New
$3, -7$ $\longrightarrow$	$(x-3)(x+7)$ $\longrightarrow$	$x^2+4x-21$

$\sqrt{7} \Longrightarrow (x-\sqrt{7})(x+\sqrt{7}) \Longrightarrow x^2 + \sqrt{7}x - \sqrt{7}x - 7$

$\searrow -\sqrt{7}$ also

$(-\sqrt{7})(+\sqrt{7})$

$-\sqrt{49} = -7$

x^2-7

Complex #'s

$\sqrt{x^2} = \sqrt{-7}$

$x = \sqrt{-1}\sqrt{7}$

imaginary

$\sqrt{7}i$

complex number

Complex Roots occur in conjugate pairs.

Root	Factor	polynomial
$2-i$	$(x-(2-i))(x-(2+i)) \Longrightarrow$	leave Factored

$\searrow$ $2+i$ also Root

Q P

$4x^4-21x^3+18x^2+19x-6=0$

List of all possible Rational Roots:

$$\pm \frac{6}{4} \qquad \frac{6}{\substack{1\cdot 6\\2\cdot 3}} \qquad \frac{4}{\substack{1\cdot 4\\2}}$$

$$\pm\left\{\frac{1}{1}, \frac{1}{4}, \frac{1}{2}, \frac{6}{1}, \frac{6}{4}, \frac{6}{2}, \frac{2}{1}, \frac{2}{4}, \frac{2}{2}, \frac{3}{1}, \frac{3}{4}, \frac{3}{2}\right\}$$

Gift: 2 and $-\frac{3}{4}$

2	4	−21	18	19	−6
	↓	8	−26	−16	6
	4	−13	−8	3	0

$-\frac{3}{4}$	4	−13	−8	3
	↓	−3	12	−3
	4	−16	4	0

$$\frac{4x^2 - 16x + 4 = 0}{4}$$

$$x^2 - 4x + 1 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad 4 \pm \sqrt{16-4}$$

$\sqrt{12}$

$\sqrt{4}\cdot\sqrt{3}$

$2\sqrt{3}$

$\frac{4 \pm \sqrt{12}}{2}$

$\frac{4 \pm 2\sqrt{3}}{2}$

$2 \pm \sqrt{3}$

$\boxed{\text{Roots: } 2,\ -\frac{3}{4},\ 2+\sqrt{3},\ 2-\sqrt{3}}$

6.5 : 1-21 odd . 29, 30, 33

6.6 : 7-9; 11-13; 15, 24-26, 28-30

48-51

desmos

wolfram