FTC, MVT ∫, Avg Value, FTC #2

Thursday, February 1, 2018 9:16 AM

aka 4.4

A. FTC : $\int_a^b f(x)\,dx = F(b) - F(a)$

where $F(x)$ is an antiderivative of $f(x)$. ✓

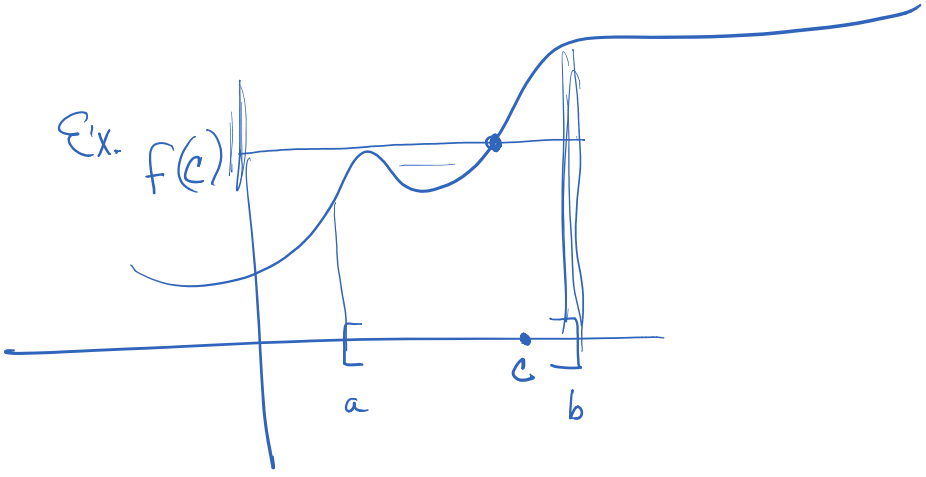
B. MVT for integrals

f cont on $[a,b]$

then

$\int_a^b f(x)\,dx = f(c)\cdot(b-a)$ ← mean value

for some c in $[a,b]$.



C. Avg. Value.

$\frac{1}{b-a}\int_a^b$ ← average value

$$\frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$$ value

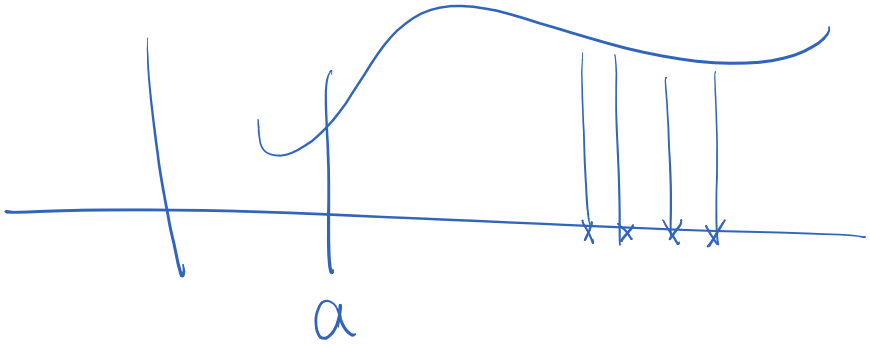
Duh!

D. FTC #2

f continuous.

$$\frac{d}{dx}\int_a^{x} f(t)\,dt = f(x).$$

$$\frac{d}{dx}\int_a^{u} f(t)\,dt = f(u)\frac{du}{dx}$$



what is the rate of change of the area as x moves along the x axis?

$$\frac{d}{dx}\int_a^x 3t^2\,dt = \boxed{3x^2}$$

$$\frac{d}{dx}\left[\frac{3}{3}t^3\Big|_a^x\right] = \frac{d}{dx}\left[x^3 - a^3\right]$$

$$= \boxed{3x^2}$$

$$\frac{d}{dx}\int_a^{3x^4} 5t^2\,dt = \underline{\text{Rule}} \quad 5\left(3x^4\right)^2 \cdot 12x^3$$

$$\frac{d}{dx}\left[\frac{5}{3}t^3\Big|_a^{3x^4}\right]$$

1 2

$$\frac{d}{dx}\left[\frac{5}{3}\left(3x^4\right)^3 - \frac{5}{3}a^3\right]$$

$$5\left(3x^4\right)^2 12x^3$$