Student Handbook

Extra Practice

Chapter 1 ■ Skills Practice

Lesson 1-1

Order the given numbers from least to greatest. Then classify each number by the subsets of the real numbers to which it belongs.

1. $2.3, \frac{5}{2}, \sqrt{10}, 2.\overline{4}, 2\sqrt{3}$ **2.** $-3, -\sqrt{12}, \frac{2}{5}, -2\pi, -\pi$ **3.** $\sqrt{9}, 3.0\overline{2}, 3\frac{1}{16}, 3\frac{2}{30}, \pi$

Use interval notation to represent each set of numbers.

4. $-40 \leq x < -12$ **5.** $-1 < x < 5$ or $x \geq 13$ **6.**

Rewrite each set in the indicated notation.

7. $x \leq 0$ or $4 < x < 8$; set-builder notation **8.** all odd natural numbers; roster notation

Lesson 1-2

Identify the property demonstrated by each equation.

9. $12 + a = a + 12$ **10.** $3 \cdot (9 \cdot 2) = (3 \cdot 9) \cdot 2$ **11.** $2(\sqrt{10} + 4) = 2(\sqrt{10}) + 2(4)$

Use mental math to find each value.

12. a 15% tip on a bill of \$34.60 **13.** a 30% discount on a \$67.80 item

Classify each statement as sometimes, always, or never true. Give examples or properties to support your answer.

14. $a + 4 = b + 4$ **15.** $12b = 6b + 6b$ **16.** $ab = ac$

Lesson 1-3

Estimate to the nearest tenth.

17. $\sqrt{90}$ **18.** $\sqrt{62}$ **19.** $-\sqrt{48}$ **20.** $\sqrt{23}$

Simplify each expression.

21. $\frac{\sqrt{242}}{\sqrt{2}}$ **22.** $\frac{\sqrt{20}}{\sqrt{120}}$ **23.** $2\sqrt{5} + 4\sqrt{20}$ **24.** $2\sqrt{72} - \sqrt{18}$

Lesson 1-4

Write an algebraic expression to represent each situation.

25. the area in square inches of a triangle with base b inches and height 12 inches

26. the amount in dollars remaining from \$55 after spending d dollars

Evaluate each expression for the given values of the variables.

27. $2a^2 + 5a - 3b$ for $a = 4$ and $b = -3$ **28.** $\frac{x + y}{2xy + 2}$ for $x = 3$ and $y = 5$

Simplify each expression.

29. $3n + 5n - 2(n + 2)$ **30.** $3(x - 7) + 4x^2$ **31.** $-4a + 2(12 - 4a)$

Lesson 1-5

Simplify each expression. Assume all variables are nonzero.

32. 4^0 **33.** $3^2 \cdot 3^{-5}$ **34.** $(3x^2y)^4$ **35.** $\frac{(5x)^2}{5y^{-4}}$

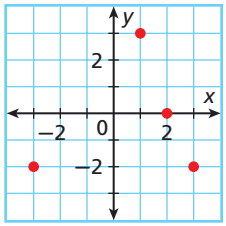
Simplify each expression. Write the answer in scientific notation.

36. $(1.4 \times 10^{12})(2.2 \times 10^3)$ **37.** $\frac{(9.9 \times 10^6)}{(2.2 \times 10^3)}$ **38.** $\frac{24 \times 10^{-5}}{6 \times 10^4}$

Lesson 1-6

Give the domain and range for each relation.

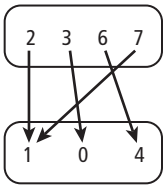
39.



40.

x	y
−5	−10
−2	−2
0	5
4	19

41.



Determine whether each relation is a function.

42.

Average Regular Gasoline Prices August 2005				
Date	8/8	8/15	8/22	8/29
Cost ($/gal)	2.37	2.55	2.61	2.61

43. from a person's age to his or her height

Lesson 1-7

For each function, evaluate $f(0)$, $f(3)$, and $f(-2)$.

44. $f(x) = -4x + 10$

45. $f(x) = \frac{1}{2}x^2$

46. $f(x) = x^2 - 2x + 5$

Graph each function.

47. $g(x) = \frac{1}{2}x - 4$

48.

x	1	2	3	4
y	1	3	5	7

49. $h(x) = -2x + 5$

Lesson 1-8

Perform the given translation on the point $(-3, 4)$. Give the coordinates of the translated point.

50. 3 units right

51. 5 units up

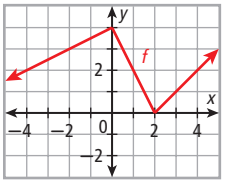
52. 2 units left, 2 units down

Use a table to perform each transformation of $y = f(x)$. Use the same coordinate plane as the original function.

53. reflection across the y-axis

54. translation 2 units up

55. vertical compression by a factor of $\frac{1}{2}$



Lesson 1-9

Identify the parent function for g from its function rule. Then graph g on your calculator and describe what transformation of the parent function it represents.

56. $g(x) = (x + 3)^3$

57. $g(x) = \sqrt{x - 4}$

58. $g(x) = x^2 + 3$

Graph the data from the table. Describe the parent function and the transformation that best approximates the data set.

59.

x	−2	−1	0	1	2
y	−4	−0.5	0	0.5	4

60.

x	1	3	5	7	9
y	16	4	0	4	16

Extra Practice

Chapter 2 ▪ Skills Practice

Lesson 2-1

Solve.

1. $4(x-3)=48$
2. $6x+10=-2x+26$
3. $\frac{1}{2}(10a+12)=a-6$
4. $3z+12=\frac{1}{2}(4z+4)$
5. $11w+4=58-7w$
6. $-5p+32=2(p-2)$

Solve and graph.

7. $3x+7<28$
8. $12y-3\leq 57$
9. $2(4-x)<10$

Lesson 2-2

Solve each proportion.

10. $\frac{3x}{15}=\frac{3}{5}$
11. $\frac{8}{5x}=\frac{2}{11}$
12. $\frac{-4}{5}=\frac{14}{y}$
13. $\frac{2.2}{3}=\frac{n}{5}$
14. $\frac{9.5}{3}=\frac{6+m}{6}$
15. $\frac{-1}{9}=\frac{1.5}{3-x}$

Lesson 2-3

Determine whether each data set could represent a linear function.

16.

x	−2	1	4	7
$f(x)$	−14	−5	4	13

17.

x	−2	−1	0	1
$f(x)$	6	0	−2	0

Graph each line.

18. slope $\frac{2}{3}$; passes through $(3, 4)$
19. slope $-\frac{5}{3}$; passes through $(6, -1)$

Find the intercepts of each line, and graph the lines.

20. $-2y+x=8$
21. $3x+y=6$

Write each function in slope-intercept form. Then graph the function.

22. $3y-2x=3$
23. $4y+3x=20$

Lesson 2-4

Find the slope of each line.

24.

x	−2	1	4	7
$f(x)$	−14	−5	4	13

25. a line through $(-1, 20)$ and $(3, -4)$

Write the equation of each line in slope-intercept form.

26. a line with slope 3 and x-intercept $\frac{4}{3}$
27. a line with slope $-\frac{3}{2}$ passing through $(4, 1)$

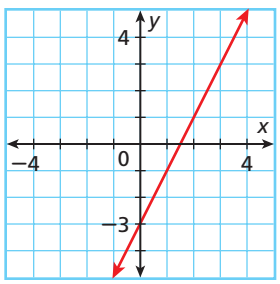
28.

x	−2	0	5	6
$f(x)$	14	15	17.5	18

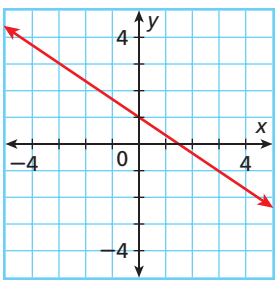
29.

x	7	10	13	16
$f(x)$	−6	−3	0	3

30.



31.



Lesson 2-5

Graph each inequality using intercepts.

32. $y - x > 4$ **33.** $2y - 8x \leq -4$ **34.** $4y + 3x \geq 12$

Solve each inequality for y. Graph the solution.

35. $6x + 6y < 18$ **36.** $12 > 2(x - 3y)$ **37.** $6y \leq 9x - 36$

Lesson 2-6

Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

38. $f(x) = \frac{4}{7}x + 1$; vertical translation 3 units down

39. $f(x) = -4x + 9$; horizontal stretch by a factor of 4

40. linear function defined in the table; reflection across the y-axis

x	−1	0	2	5
y	−17	−11	1	19

Let $g(x)$ be the indicated transformation of $f(x) = x$. Write the rule for $g(x)$.

41. vertical stretch by a factor of 2 followed by a horizontal shift 2 units right

42. horizontal shift 5 units left followed by a reflection across the x-axis

43. vertical stretch by a factor of $\frac{3}{2}$ followed by a vertical shift 8 units down

Lesson 2-7

44. If the points in a scatter plot have a positive correlation, then the r-value is ___?___. If the points have no correlation, then the r-value is ___?___.

45. Make a scatter plot of the data shown in the table.

x	0	2	3	4	6	9
y	18	15	14	10	5	1

46. Find the correlation coefficient and the equation of the line of best fit. Draw the line of best fit on your scatter plot.

Lesson 2-8

Solve each compound inequality. Then graph the solution set.

47. $3 - x > 4$ or $2x + 7 \geq 17$ **48.** $8x \leq 0$ and $4x + 6 \geq -10$

Solve each equation.

49. $|x - 9| = 1$ **50.** $|5x + 5| = 20$ **51.** $4|-2x| = 48$

Solve each inequality. Then graph the solution set.

52. $|x + 7| > 4$ **53.** $|2x - 5| < 21$ **54.** $3\left|\frac{1}{2}x + 1\right| \geq 12$

Lesson 2-9

Let $g(x)$ be the indicated transformation of $f(x) = |x|$. Write the rule for $g(x)$.

55. 7 units up **56.** 3 units right

Translate $f(x) = |x|$ so that the vertex is at the given point. Then graph.

57. $(1, 4)$ **58.** $(-3, 2)$ **59.** $(1.5, -2.5)$

Perform each transformation. Then graph.

60. Stretch $f(x) = |x - 1|$ vertically by a factor of 2.

61. Reflect $f(x) = |x + 4| - 1$ across the y-axis.

Chapter 3 ▪ Skills Practice

Lesson 3-1

Use substitution to determine if the given ordered pair is an element of the solution set for the system of equations.

1. $(2, 8)$
$\begin{cases} y - 2x = 4 \\ 2y + x = -8 \end{cases}$

2. $(4, 13)$
$\begin{cases} y - 3x = 1 \\ 4x - y = 3 \end{cases}$

3. $(3, 2.5)$
$\begin{cases} 2y - x = 2 \\ 3x - 2y = 4 \end{cases}$

4. $(5, 4)$
$\begin{cases} x - y = 1 \\ x - 2y = 8 \end{cases}$

Use a graph and a table to solve each system. Check your answer.

5. $\begin{cases} 4y - x = 12 \\ 3x - 4y = -16 \end{cases}$

6. $\begin{cases} 3x + 3y = 6 \\ 2x - y = 4 \end{cases}$

7. $\begin{cases} 2y - x = 12 \\ 5x - 2y = -4 \end{cases}$

8. $\begin{cases} y - x = 4 \\ 2y + x = -4 \end{cases}$

Classify each system and determine the number of solutions.

9. $\begin{cases} y - 3x = 2 \\ 2y - 6x = 10 \end{cases}$

10. $\begin{cases} y - x = 3 \\ 3x - 4y = 10 \end{cases}$

11. $\begin{cases} 2y + 3x = 8 \\ 3x + 2y = 8 \end{cases}$

12. $\begin{cases} 4y - x = 6 \\ 2x - 8y = -12 \end{cases}$

Lesson 3-2

Use substitution to solve each system of equations.

13. $\begin{cases} x + y = 22 \\ y = x - 4 \end{cases}$

14. $\begin{cases} y = 2x + 2 \\ 3x + 2y = 18 \end{cases}$

15. $\begin{cases} 4x - y = 3 \\ 3y + 3x = 36 \end{cases}$

16. $\begin{cases} 9x - 3y = 3 \\ 2y - 4x = 16 \end{cases}$

Use elimination to solve each system of equations.

17. $\begin{cases} 3x + y = 5 \\ -2x - y = 1 \end{cases}$

18. $\begin{cases} 3x + y = 11 \\ 3y - 3x = -3 \end{cases}$

19. $\begin{cases} 2y + 5x = 7 \\ 2x - 4y = 10 \end{cases}$

20. $\begin{cases} \frac{1}{3}y + 2x = 11 \\ y - 3x = -12 \end{cases}$

Classify each system and determine the number of solutions.

21. $\begin{cases} 2y + x = 10 \\ -y + 4x = 4 \end{cases}$

22. $\begin{cases} 2y - x = 2 \\ 2x - 4y = 12 \end{cases}$

23. $\begin{cases} 4x - 8y = 16 \\ 12y - 6x = -24 \end{cases}$

24. $\begin{cases} y - \frac{2}{3}x = 3 \\ 2x - 3y = 15 \end{cases}$

Lesson 3-3

Graph each system of inequalities.

25. $\begin{cases} y \le 2x + 3 \\ y \ge x + 4 \end{cases}$

26. $\begin{cases} x + 2y \le 10 \\ -x + 2y > 12 \end{cases}$

27. $\begin{cases} y > 3x + 3 \\ 2y - 3x > 12 \end{cases}$

28. $\begin{cases} 3x + y < 4 \\ 2y - \frac{1}{2}x \ge 8 \end{cases}$

Graph each system of inequalities and classify the figure created by the solution region.

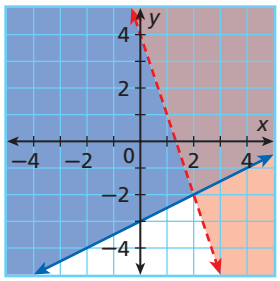
29. $\begin{cases} y \le 2x + 4 \\ y \ge 2x - 1 \\ y \le 4 \\ y \ge -1 \end{cases}$

30. $\begin{cases} y \le 3x \\ y \le -3x + 13 \\ y \ge 0 \end{cases}$

31. $\begin{cases} x \ge -1 \\ x \le 3 \\ y \ge 1 \\ y \le 8 \end{cases}$

32. $\begin{cases} y + x \le 8 \\ y - x \le 1 \\ y \le 3 \\ y \ge -1 \end{cases}$

33. Write a system of inequalities to describe the graph.



Lesson 3-4

Graph each feasible region.

34. $\begin{cases} y \geq 0 \\ x \geq 1 \\ y \leq -x + 8 \\ y \leq 3x \end{cases}$

35. $\begin{cases} y \geq -1 \\ x \geq -2 \\ y \leq -2x + 10 \\ y \leq \frac{1}{2}x + 5 \end{cases}$

36. $\begin{cases} y \geq -8 \\ x \geq -4 \\ y \leq -2x + 1 \\ y \leq -\frac{1}{4}x - 6 \end{cases}$

37. $\begin{cases} y \geq -3 \\ x \leq 1 \\ y \leq x + 8 \\ y \geq -x - 6 \end{cases}$

Maximize or minimize each objective function.

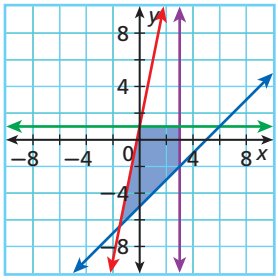
38. Maximize $P = 5x + 3y$ for the constraints from Exercise 34.

39. Maximize $P = 1.2x + 9.5y$ for the constraints from Exercise 35.

40. Minimize $P = 11x - 2.5y$ for the constraints from Exercise 36.

41. Minimize $P = 8x + 24y$ for the constraints from Exercise 37.

42. Maximize $P = 5.5x + 9y$ for the constraints shown on the coordinate grid below.



Lesson 3-5

Graph each point in three-dimensional space.

43. $(0, 4, -2)$ **44.** $(1, 3, 3)$ **45.** $(2, -3, -5)$ **46.** $(-3, -1, 4)$

Graph each linear equation in three-dimensional space.

47. $2x + 2y + z = 10$

48. $3x - 2y + 2z = 6$

49. $6x + 4y + 3z = 12$

50. $\frac{1}{2}x + 4y - z = 4$

Lesson 3-6

Use substitution or elimination to solve each system of equations.

51. $\begin{cases} 3x + y - z = 2 \\ 5x + 3y + 4z = -5 \\ -2x + y + 8z = -12 \end{cases}$

52. $\begin{cases} 2x + 2y - z = 16 \\ 4x - 2y + 2z = 0 \\ -3x - y + 3z = -19 \end{cases}$

53. $\begin{cases} 3x + 4y + 2z = 1 \\ -x + y - 4z = -17 \\ 2x + 8y + 4z = 14 \end{cases}$

54. $\begin{cases} -x + 3y + 3z = 11 \\ -3x + 5y - 7z = 1 \\ 4x - 2y + 3z = 11 \end{cases}$

Classify each system as consistent or inconsistent, and determine the number of solutions.

55. $\begin{cases} 3x + 3y - z = -3 \\ 5x + y + 2z = 14 \\ -4x + 2y + z = -9 \end{cases}$

56. $\begin{cases} 2x + 3y + z = 12 \\ 2x + 3y + z = -8 \\ 4x - y - 4z = 15 \end{cases}$

57. $\begin{cases} 8x - 4y - 16z = 12 \\ -2x + y + 4z = -3 \\ 3x - 2z + 9z = 18 \end{cases}$

Lesson 4-1

Use the following matrices for Exercises 1–4. Add or subtract, if possible.

$$A = \begin{bmatrix} 1 & 3 & 6 \\ 2 & -5 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1.2 & 3.5 & 4 \\ 2.2 & 2.7 & -0.5 \end{bmatrix} \qquad C = \begin{bmatrix} -1 & 3 & 9 \\ 4 & -5 & -2.2 \\ 2 & 1 & 12 \end{bmatrix}$$

1. $A + B$ **2.** $A + C$ **3.** $B - A$ **4.** $C - B$

Use the following matrices for Exercises 5–8. Evaluate, if possible.

$$A = \begin{bmatrix} 4 & 7 & 3 \\ 2 & 12 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} -3 & 10 & -9 \\ 2 & 0 & -6 \end{bmatrix} \qquad C = \begin{bmatrix} 16 & 8 \\ -3 & 2 \\ 21 & 0 \end{bmatrix}$$

5. $4A$ **6.** $-2C$ **7.** $\frac{1}{2}A + B$ **8.** $2C - A$

Lesson 4-2

Tell whether each product is defined. If so, give its dimensions.

9. $A_{2\times 4}$ and $B_{4\times 5}$; AB **10.** $C_{3\times 3}$ and $D_{2\times 3}$; CD **11.** $E_{4\times 7}$ and $F_{7\times 6}$; EF

Use the following matrices for Exercises 12–15. Find each product, if possible.

$$A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 5 \\ 3 & 9 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 7 & 1 \\ 10 & 4 & -2 \end{bmatrix} \quad D = \begin{bmatrix} 12 & 0 \\ -4 & 4 \\ 5 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 5 & 1 & -3 \end{bmatrix}$$

12. AD **13.** BC **14.** ED **15.** CB

Use the following matrices for Exercises 16–19. Evaluate, if possible.

$$A = \begin{bmatrix} 9 & 6 \\ 0 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 5 & 10 \\ -3 & -1 & 6 \\ 2 & 3 & 6 \end{bmatrix} \qquad C = \begin{bmatrix} 12 & 0 & 5 \\ -5 & 7 & 8 \end{bmatrix}$$

16. A^2 **17.** A^3 **18.** B^2 **19.** C^2

Lesson 4-3

Translate the polygon with coordinates $M(3, 0)$, $N(2, 4)$, $O(-1, 3)$, and $P(-2, -1)$ as indicated. Find the coordinates of the vertices of the image, and graph.

20. 3 units right and 2 units down **21.** 1 unit left and 4 units up

Use a matrix to reduce or enlarge the polygon with coordinates $M(3, 0)$, $N(2, 4)$, $O(-1, 3)$, and $P(-2, -1)$ by the given factor. Find the coordinates of the vertices of the image, and graph.

22. Reduce polygon *MNOP* by a factor of 0.25.

23. Enlarge polygon *MNOP* by a factor of 3.

Reflect the figure with coordinates $A(-1, 1)$, $B(1, -3)$, $C(5, -1)$, and $D(2, 4)$ across the given line. Find the coordinates of the vertices of the image, and graph.

24. Reflect *ABCD* across the x-axis.

25. Reflect *ABCD* across the y-axis.

Use each matrix to rotate the figure with coordinates $E(2, 2)$, $F(4, 0)$, $G(-3, -3)$, and $H(-2, 3)$ about the origin. Graph and describe the image.

26. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

27. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Lesson 4-4

Find the determinant of each matrix.

28. $\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$ **29.** $\begin{bmatrix} 0.75 & 3 \\ 1.5 & 4 \end{bmatrix}$ **30.** $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{2}{3} & 8 \end{bmatrix}$ **31.** $\begin{bmatrix} 10 & -5 \\ 12 & \frac{1}{2} \end{bmatrix}$

Use Cramer's rule to solve each system of equations.

32. $\begin{cases} 3x + 2y = 1 \\ -4x + 5y = -32 \end{cases}$ **33.** $\begin{cases} x + 4y = 15 \\ 3x - 10 = 2y \end{cases}$

34. $\begin{cases} 10x + 23 = 7y \\ 2y - 10 = 4x \end{cases}$ **35.** $\begin{cases} \frac{1}{2}x + \frac{3}{2}y = -1 \\ \frac{1}{4}x + 1 + y = 0 \end{cases}$

Find the determinant of each matrix.

36. $\begin{bmatrix} 2 & 3 & 5 \\ -1 & 4 & 4 \\ 5 & 0 & 9 \end{bmatrix}$ **37.** $\begin{bmatrix} 3 & -6 & -1 \\ 2 & 2 & 2 \\ 7 & 1 & -3 \end{bmatrix}$ **38.** $\begin{bmatrix} 9 & 3 & 0 \\ 5 & -5 & 1 \\ 2 & 3 & -2 \end{bmatrix}$

Lesson 4-5

Determine whether the given matrices are inverses.

39. $\begin{bmatrix} 1 & -8 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0.2 & 0.4 \\ -0.1 & 0.05 \end{bmatrix}$ **40.** $\begin{bmatrix} 9 & 3 \\ -6 & -6 \end{bmatrix} \begin{bmatrix} \frac{1}{6} & \frac{1}{12} \\ -\frac{1}{6} & -\frac{1}{4} \end{bmatrix}$

41. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$ **42.** $\begin{bmatrix} 14 & 7 \\ 20 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Find the inverse of the matrix, if it is defined.

43. $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ **44.** $\begin{bmatrix} 3 & 6 \\ -9 & -6 \end{bmatrix}$ **45.** $\begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$ **46.** $\begin{bmatrix} 1 & -4 \\ -\frac{1}{2} & 3 \end{bmatrix}$

Write the matrix equation for the system and solve.

47. $\begin{cases} 4x + 2y = 12 \\ 6x - y = -2 \end{cases}$ **48.** $\begin{cases} \frac{1}{3}x + 2y = -3 \\ y - 4 = -2x \end{cases}$ **49.** $\begin{cases} 3x + 3y = 12 \\ 2x + 9.5 = 5y \end{cases}$

Lesson 4-6

Write the augmented matrix for each system of equations.

50. $\begin{cases} 2x + 8 = 5y \\ 3y - 7 = 12x \end{cases}$ **51.** $\begin{cases} 9 - y = 2x \\ 3y = 18 \end{cases}$ **52.** $\begin{cases} 2x + 9y = 10 \\ 3x - z = 8 \\ 5z + 5 = 13y \end{cases}$ **53.** $\begin{cases} 4 - 5y = 8x \\ 13x + 12 = z \\ 4z - 2y = 0 \end{cases}$

Write the augmented matrix and use row reduction to solve.

54. $\begin{cases} 4x - 2y = 26 \\ x + 6y = -13 \end{cases}$ **55.** $\begin{cases} 8x - \frac{1}{2} = -3y \\ 4y - 8 = 4x \end{cases}$ **56.** $\begin{cases} 6x + \frac{1}{2}y = 6 \\ y + 14x = 12 \end{cases}$ **57.** $\begin{cases} 12x + y = -6 \\ 2y - 2x = 14 \end{cases}$

Chapter 5 ▪ Skills Practice

Lesson 5-1

Graph each function by using a table.

1. $f(x) = \frac{1}{2}x^2 - 4$ **2.** $f(x) = 2x^2 - x + 3$ **3.** $f(x) = -x^2 - 3x$

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

4. $g(x) = (x + 2)^2 + 1$ **5.** $g(x) = -2x^2$ **6.** $g(x) = \frac{1}{4}x^2$

Use the description to write each quadratic function in vertex form.

7. The parent function $f(x) = x^2$ is vertically stretched by a factor of 3 and translated 6 units right to create g.

8. The parent function $f(x) = x^2$ is reflected across the x-axis and translated 12 units down to create g.

Lesson 5-2

Identify the axis of symmetry for the graph of each function.

9. $f(x) = 2x^2 + 1$ **10.** $f(x) = (x + 3)^2 - 5$ **11.** $f(x) = 3(x - 2)^2$

For each function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.

12. $f(x) = 2x^2 - 4x + 5$ **13.** $f(x) = -\frac{1}{2}x^2 - 2x + 3$ **14.** $f(x) = -x^2 - 8x - 6$

Find the minimum or maximum value of each function. Then state the domain and range of the function.

15. $f(x) = 3x^2 + 60x + 294$ **16.** $f(x) = -2x^2 + 28x - 95$ **17.** $f(x) = 2x^2 + 14x + 30$

Lesson 5-3

Find the zeros of each function by using a graph and a table.

18. $f(x) = x^2 + 5x + 6$ **19.** $f(x) = x^2 - 3x - 28$ **20.** $f(x) = -x^2 + 12x - 20$

Find the zeros of each function by factoring.

21. $f(x) = x^2 + 2x - 35$ **22.** $f(x) = x^2 - 8x - 9$ **23.** $f(x) = 2x^2 - 9x$

24. $f(x) = x^2 + 10x + 25$ **25.** $f(x) = x^2 - 49$ **26.** $f(x) = x^2 - 12x + 36$

Write a quadratic function in standard form for each given set of zeros.

27. 5 and 8 **28.** −3 and 1 **29.** 6 and 6 **30.** 12 and 0

Lesson 5-4

Solve each equation.

31. $4x^2 - 10 = 90$ **32.** $x^2 + 8x + 16 = 10$ **33.** $x^2 + 4x + 4 = 8$

Complete the square for each expression. Write the resulting expression as a binomial squared.

34. $x^2 - 16x +$ ▪ **35.** $x^2 + 22x +$ ▪ **36.** $x^2 + 7x +$ ▪

Solve each equation by completing the square.

37. $x^2 + 8x = -10$ **38.** $x^2 - 12x = 13$ **39.** $x^2 + 20 = 10x$

40. $2x^2 + 12x = 14$ **41.** $3x^2 - 18 = 48x$ **42.** $x^2 - 5 = 2x$

Write each function in vertex form, and identify its vertex.

43. $f(x) = x^2 - 2x + 17$ **44.** $f(x) = x^2 + 4x - 8$ **45.** $f(x) = 4x^2 - 24x + 31$

Lesson 5-5

Express each number in terms of i.

46. $2\sqrt{-81}$ **47.** $-\sqrt{-144}$ **48.** $\sqrt{-128}$ **49.** $5\sqrt{-48}$

Solve each equation.

50. $169 + x^2 = 0$ **51.** $2x^2 = -200$ **52.** $x^2 = -90$

Find the zeros of each function.

53. $f(x) = x^2 + 8x + 20$ **54.** $f(x) = x^2 - 14x + 65$ **55.** $f(x) = x^2 - 2x + 46$

Find each complex conjugate.

56. $12i$ **57.** $3 - 6i$ **58.** $10i - 3$ **59.** $2\sqrt{7} - 10i$

Lesson 5-6

Find the zeros of each function by using the Quadratic Formula.

60. $f(x) = x^2 - 10x + 3$ **61.** $f(x) = 2x^2 + 5x + 1$ **62.** $f(x) = -x^2 + 8x - 3$

63. $f(x) = x^2 - 6x + 40$ **64.** $f(x) = x^2 + 7x + 13$ **65.** $f(x) = 2x^2 - 9x + 25$

Find the type and number of solutions for each equation.

66. $x^2 + 8x = -16$ **67.** $x^2 + 3 = 10x$ **68.** $5 + 2x^2 = 12x$ **69.** $4x^2 + 2x = -9$

Lesson 5-7

Graph each inequality.

70. $y \geq (x + 3)^2 + 2$ **71.** $y < 2x^2 - 4x - 1$ **72.** $y < -x^2 + 11x - 24$

Solve each inequality.

73. $x^2 + 13x + 20 < -2$ **74.** $x^2 - 11x \geq -10$ **75.** $x^2 + 6x + 3 > 10$

76. $x^2 - 2x - 20 > 28$ **77.** $2x^2 - 9x \leq 5$ **78.** $3x^2 + 1 \geq 4x$

Lesson 5-8

Determine whether each data set could represent a quadratic function. Explain.

79.

x	3	4	5	6	7
y	−2	−5	−6	−5	−2

80.

x	−2	−1	0	1	2
y	−5	2	3	4	11

81.

x	−6	−5	−4	−3	−2
y	19	10	7	10	19

Write a quadratic function that fits each set of points.

82. $(-2, 0)$, $(1, 6)$, and $(3, -10)$ **83.** $(-4, -25)$, $(0, -9)$, and $(2, 5)$

Lesson 5-9

Graph each complex number.

84. -3 **85.** $2i$ **86.** $2 + 4i$ **87.** $-3 - 3i$

Find each absolute value.

88. $|6 + 9i|$ **89.** $|-3 + 4i|$ **90.** $|-7i|$

Simplify. Write the result in the form $a + bi$.

91. $(3 + 7i) + (-2 + 3i)$ **92.** $(-9 - 4i) + (5 + i)$ **93.** $(10 + 6i) - (3i - 12)$

94. $-3i(9 - 2i)$ **95.** $(2 - i)(4 + 3i)$ **96.** $(6 + 4i)(4 - 5i)$

97. $\dfrac{11 + 3i}{2 + i}$ **98.** $\dfrac{-44 - 40i}{-8 + 2i}$ **99.** $\dfrac{5 + 12i}{3 + 2i}$

Extra Practice

Chapter 6 ■ Skills Practice

Lesson 6-1

Identify the degree of each monomial.

1. $7x^2$ **2.** $-12x$ **3.** $2x^3y^3$ **4.** 8

Rewrite each polynomial in standard form. Then identify the leading coefficient, degree, and number of terms. Name the polynomial.

5. $5x^2 + 6 + 9x - 10x^3$ **6.** $3 - 12x^4 - 6x^2$ **7.** $14x + 15x^5$

Add or subtract. Write your answer in standard form.

8. $(12x^2 + 4x - 9) + (3x^3 - 7x^2 - 1)$ **9.** $(34 + 8x^3 - 9x^2) - (3x^3 + 10x^2 - 4x - 4)$

Graph each polynomial function on a calculator. Describe the graph, and identify the number of real zeros.

10. $f(x) = 5x^3 + 4x - 6$ **11.** $g(x) = 2x^4 - 12x + 3$ **12.** $h(x) = 3x^3 - 4x + 1$

Lesson 6-2

Find each product.

13. $3ab(2a^2 - 5ab + 9b)$ **14.** $-5cd^3(8d + 3c - c^2d)$ **15.** $(x + 3)(2x^2 - x + 6)$

16. $(2x - 1)(-x^2 + 5x + 5)$ **17.** $(2x + 6)^3$ **18.** $(y - 2)^4$

Expand each expression.

19. $(x - y)^5$ **20.** $(y + 4)^4$ **21.** $(2x + y)^5$ **22.** $(x - 2y)^4$

Lesson 6-3

Divide by using long division.

23. $(6x^2 + 7x - 2) \div (x + 4)$ **24.** $(2x^2 - 9x + 10) \div (2x - 1)$

Divide by using synthetic division.

25. $(3x^3 + 4x^2 - 8) \div (x - 2)$ **26.** $(2x^3 + 3x^2 - 6x - 4) \div (x - 1)$

Use synthetic division to evaluate the polynomial for the given value.

27. $P(x) = -2x^3 + 7x^2 - 3x - 9$ for $x = -2$ **28.** $P(x) = 6x^3 - 7x^2 + 10$ for $x = 0.5$

Lesson 6-4

Determine whether the given binomial is a factor of the polynomial $P(x)$.

29. $(x + 2)$; $P(x) = 3x^3 + 11x^2 + 2x - 16$ **30.** $(x - 4)$; $P(x) = 12x^3 + 9x^2 - 2x + 8$

31. $(x + 1)$; $P(x) = x^4 - 3x^3 + 10x + 4$ **32.** $(x - 3)$; $P(x) = x^3 - 3x^2 - 4x + 12$

Factor each expression.

33. $2x^3 + 12x^2 - 4x - 24$ **34.** $2x^3 + 5x^2 - 18x - 45$ **35.** $4x^3 + 12x^2 + 12x + 36$

36. $a^3 + 27$ **37.** $128b - 2b^4$ **38.** $4c^5 + 32c^2$

Lesson 6-5

Solve each polynomial equation by factoring.

39. $2x^3 + 3x^2 - 8x - 12 = 0$ **40.** $-3x^3 + 30x^2 + 5x - 50 = 0$

Identify the roots of each equation. State the multiplicity of each root.

41. $x^3 + 15x^2 + 75x + 125 = 0$ **42.** $x^3 - 2x^2 - 32x + 96 = 0$

43. $8x^3 - 12x^2 + 6x - 1 = 0$ **44.** $4x^3 + 16x^2 - 25x - 100 = 0$

Identify all of the real roots of each equation.

45. $2x^4 - x^3 - 14x^2 - 5x + 6 = 0$ **46.** $6x^3 - 11x^2 - 19x - 6 = 0$

Lesson 6-6

Write the simplest polynomial function with the given zeros.

47. $-1, 1, 4$ **48.** $-3, \frac{1}{2}, \frac{1}{3}$ **49.** $-3, 1, \frac{2}{3}$ **50.** $-5, 1, 2$

Solve each equation by finding all roots.

51. $x^4 - 5x^3 + 15x^2 - 45x + 54 = 0$ **52.** $2x^4 + 5x^3 - 10x^2 + 10x + 8 = 0$

Write the simplest polynomial function with the given zeros.

53. $3, \sqrt{5}$ **54.** $1 + i, 2$ **55.** $-2, 2i$ **56.** $1, \sqrt{2}, i$

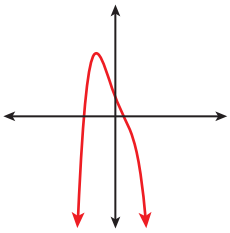
Lesson 6-7

Identify the leading coefficient, degree, and end behavior.

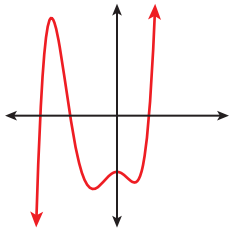
57. $P(x) = 7x^3 - 12x^2 + 9x - 10$ **58.** $Q(x) = -3x^5 + 8x^4 - 16x + 1$

Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient.

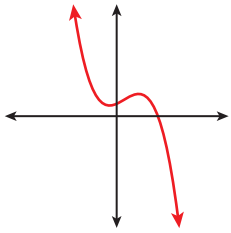
59.



60.



61.



Graph each function.

62. $Q(x) = -4x^3 - 12x^2 + x + 3$ **63.** $R(x) = 2x^4 + x^3 - 19x^2 - 9x + 9$

Graph each function on a calculator, and estimate the local maxima and minima.

64. $S(x) = -2x^4 + x^3 + 5x^2 + 6$ **65.** $T(x) = x^3 + 5x^2 + 3x + 1$

Lesson 6-8

For $f(x) = 2x^3 - 3$, write the rule for each function and sketch the graph.

66. $g(x) = f(x) + 6$ **67.** $h(x) = f(x - 2)$ **68.** $j(x) = f(-x)$ **69.** $k(x) = \frac{1}{2}f(x)$

Let $f(x) = -3x^4 + 2x^2 - 7x + 10$. Write a function g that performs each transformation.

70. Reflect $f(x)$ across the y-axis. **71.** Reflect $f(x)$ across the x-axis.

Let $f(x) = x^3 - 7x^2 + 5$. Graph f and g on the same coordinate plane. Describe g as a transformation of f.

72. $g(x) = f(x + 4)$ **73.** $g(x) = -2f(x)$ **74.** $g(x) = f(-x) - 3$

Write a function that transforms $f(x) = 3x^3 - 5x^2 + x + 1$ in each of the following ways. Support your solution by using a graphing calculator.

75. Stretch vertically by a factor of 3 and move 1 unit to the right.

76. Reflect across the y-axis and move 1 unit down.

Lesson 6-9

Use finite differences to determine the degree of the polynomial that best describes the data.

77.

x	-2	-1	0	1	2	3
y	-24	-4	6	12	20	36

78.

x	-2	-1	0	1	2	3
y	-27	0	9	12	69	288

Extra Practice

Chapter 7 ▪ Skills Practice

Lesson 7-1

Tell whether the function shows growth or decay. Then graph.

1. $f(x) = 12(2.4)^x$
2. $f(x) = 20\left(\frac{4}{5}\right)^x$
3. $f(x) = 0.25(5)^x$

Explain whether each function is exponential.

4. $f(x) = 4x^9$
5. $f(x) = 0.6^x$
6. $f(x) = 10(0)^x$

Lesson 7-2

Graph the relation and connect the points. Then graph the inverse. Identify the domain and range of each relation.

7.

x	1	2	3	4
y	−1	0	2	4

8.

x	−3	−1	2	4
y	−3	−1	−1	−3

Use inverse operations to write the inverse of each function.

9. $f(x) = 15x$
10. $f(x) = x + 9$
11. $f(x) = \frac{x}{7}$
12. $f(x) = 3x + 2$
13. $f(x) = 5 - \frac{3}{4}x$
14. $f(x) = \frac{2x + 1}{5}$

Graph each function. Then write and graph its inverse.

15. $f(x) = 2x + 4$
16. $f(x) = 0.8x + 1$
17. $f(x) = \frac{4x - 5}{3}$

Lesson 7-3

Write each exponential equation in logarithmic form.

18. $3^5 = 243$
19. $51^0 = 1$
20. $16^{1.5} = 64$
21. $7^x = 343$

Write each logarithmic equation in exponential form.

22. $\log_{64}512 = 1.5$
23. $\log_2 0.125 = -3$
24. $\log_4 x = 70$
25. $\log_x 12 = 3$

Evaluate by using mental math.

26. $\log_{10}1000$
27. $\log_5 0.2$
28. $\log_{0.5}0.125$
29. $\log_{1.1}1.21$

Use the given x-values to graph each function. Then graph its inverse. Describe the domain and range of the inverse function.

30. $f(x) = 4^x$; $x = -2, -1, 0, 1, 2$
31. $f(x) = 0.2^x$; $x = -2, -1, 0, 1, 2$

Lesson 7-4

Express as a single logarithm. Simplify, if possible.

32. $\log_2 10 + \log_2 12.8$
33. $\log_4 8 + \log_4 2$
34. $\log_5 1.25 + \log_5 4$
35. $\log_6 144 - \log_6 4$
36. $\log 10{,}000 - \log 100$
37. $\log_8 8 - \log_8 1$

Simplify, if possible.

38. $\log_8 64^4$
39. $\log_7 49^5$
40. $\log_9 1^4$
41. $\log_3 3^{5x+8}$
42. $4^{\log_4 12}$
43. $\log_{1.4} 1.4^5$

Evaluate.

44. $\log_4 256$
45. $\log_4\left(\frac{1}{64}\right)$
46. $\log_3 7$
47. $\log_4 13$

Lesson 7-5

Solve and check.

48. $3^{x+1} = 9^4$ **49.** $32^{x-2} = 8^x$ **50.** $9^x = 12$ **51.** $3.5^{2x-1} = 15$

Solve.

52. $\log_6(4x - 9) = \log_6(x)$ **53.** $\log_7(10x + 13) = 3$ **54.** $\log(20x) - \log 4 = 2$

55. $\log_9 x^3 = 8$ **56.** $\log x + \log(2x - 1) = 1$ **57.** $\log_3\left(\frac{2}{x}\right) + 2 = 0$

Use a table and a graph to solve.

58. $3^{4x-3} = 243$ **59.** $3^x 4^x \geq 1728$ **60.** $\log x^3 = x - 94$ **61.** $3\log x^2 < 6$

Lesson 7-6

Graph.

62. $f(x) = e^x - 1$ **63.** $f(x) = -2e^x + 3$ **64.** $f(x) = 2 - e^{-x}$ **65.** $f(x) = 1.5e^{x+1}$

Simplify.

66. $\ln e^{20}$ **67.** $\ln e^{2x+10}$ **68.** $e^{\ln 5x^2}$ **69.** $e^{2 \ln 2x}$

Lesson 7-7

Make a table of values and graph each function. Describe the asymptote, the domain, and the range. Tell how the graph is transformed from the graph of $f(x) = 4^x$.

70. $g(x) = 4^x - 2$ **71.** $h(x) = 4^{x+2}$ **72.** $j(x) = 4^{x-1} - 4$

Graph each exponential function. Find the y-intercept, the asymptote, the domain, and the range. Describe how the graph is transformed from the graph of its parent function.

73. $g(x) = -\frac{1}{2}(3^x)$ **74.** $h(x) = 3(2^{-x})$ **75.** $j(x) = 5e^{x+1}$

Graph each logarithmic function. Find the asymptote. Then describe how the graph is transformed from the graph of its parent function.

76. $g(x) = -4 \log x$ **77.** $h(x) = 3 \ln(3 - x)$ **78.** $j(x) = \ln(0.5x) - 3$

Write each transformed function by using the given parent function and the indicated transformations.

79. The parent function $f(x) = 6^x$ is horizontally stretched by a factor of 3 and translated 4 units to the left.

80. The parent function $f(x) = \log x$ is vertically compressed by a factor of $\frac{1}{5}$, reflected across the y-axis, and translated 10 units down.

Lesson 7-8

Determine whether f is an exponential function of x. If so, find the constant ratio.

81.

x	−2	−1	1	2	3
y	0.4	2	10	50	250

82.

x	−2	−1	0	1	2
y	−17	−2	13	28	43

83.

x	−2	−1	0	1	2
y	4	2	1	0.5	0.25

84.

x	−2	−1	0	1	2
y	−6	1	12	37	54

Chapter 8 ■ Skills Practice

Lesson 8-1

Given: y varies directly as x. Write and graph each direct variation function.

1. $y = 8$ when $x = 2$ **2.** $y = 21$ when $x = 3$ **3.** $y = 4$ when $x = 2.5$

Given: y varies inversely as x. Write and graph each inverse variation function.

4. $y = 4$ when $x = 2$ **5.** $y = 4$ when $x = \frac{1}{2}$ **6.** $y = \frac{3}{5}$ when $x = 10$

Determine whether each data set represents a direct variation, an inverse variation, or neither.

7.

x	1	3	6
y	2.5	7.5	15

8.

x	2	4	8
y	6	10	18

9.

x	2	8	20
y	5	1.25	0.5

Lesson 8-2

Simplify. Identify any x-values for which the expression is undefined.

10. $\frac{6x^3}{27x^2 + 12x}$ **11.** $\frac{x^2 - x - 2}{3x - 6}$ **12.** $\frac{-x^2 + 16}{-x^2 - 9x - 20}$

Multiply or divide. Assume that all expressions are defined.

13. $\frac{4xy^3}{5x^2} \cdot \frac{20x^3y^2}{-16xy^7}$ **14.** $\frac{x^2 - 9}{2x + 10} \cdot \frac{x + 5}{x - 3}$ **15.** $\frac{x - 4}{2x^2} \cdot \frac{x}{x^2 - x - 12}$

16. $\frac{3x^3}{4x + 4} \div \frac{9x}{x + 1}$ **17.** $\frac{12x^3y^6}{9xy} \div \frac{6y^2}{3x}$ **18.** $\frac{x^2 - 16}{x^2 + 4x + 3} \div \frac{x - 4}{x + 1}$

Lesson 8-3

Find the least common multiple for each pair.

19. $6x^3y$ and $2xy^2$ **20.** $x^2 + 5x$ and $x^2 - 25$ **21.** $x^2 - 3x - 18$ and $x^2 - 5x - 6$

Add or subtract. Identify any x-values for which the expression is undefined.

22. $\frac{x + 9}{2x + 1} + \frac{3x + 6}{2x + 1}$ **23.** $\frac{2}{x + 3} + \frac{4x}{x^2 - 9}$ **24.** $\frac{1}{x^2 + 6x + 8} + \frac{1}{x^2 - 6x - 16}$

25. $\frac{x - 6}{x + 5} - \frac{8x + 7}{x + 5}$ **26.** $\frac{x}{x + 1} - \frac{3}{x + 4}$ **27.** $\frac{7}{x - 9} - \frac{2x - 6}{x^2 - 13x + 36}$

Simplify. Assume that all expressions are defined.

28. $\dfrac{\frac{3x}{3x + 21}}{\frac{9x^2}{x + 7}}$ **29.** $\dfrac{\frac{x}{x - 1}}{\frac{10x^2}{-4x + 4}}$ **30.** $\dfrac{\frac{1}{x - 2}}{\frac{x + 3}{x^2 - 4}}$

Lesson 8-4

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

31. $g(x) = \frac{1}{x - 4}$ **32.** $g(x) = \frac{1}{x} + 6$ **33.** $g(x) = \frac{1}{x + 2} - 5$

Identify the zeros and asymptotes of each function. Then graph.

34. $f(x) = \frac{x^2 - 5x - 24}{2x + 1}$ **35.** $f(x) = \frac{2x^2 - 3x - 2}{x - 4}$ **36.** $f(x) = \frac{-3x^2 + 8x - 4}{x^2 - 25}$

Identify holes in the graph of each function. Then graph.

37. $f(x) = \frac{x^2 - 4x - 21}{x + 3}$ **38.** $f(x) = \frac{x^2 - 4x - 5}{x^2 - 25}$ **39.** $f(x) = \frac{x^2 - 3x}{4x - 12}$

Lesson 8-5

Solve each equation.

40. $12 + \frac{2}{3x} = 6$

41. $x - \frac{1}{x} = \frac{35}{x}$

42. $\frac{x}{x+1} + \frac{x}{4} = \frac{3x}{4x+4}$

43. $\frac{x-1}{x-4} = \frac{x+6}{x}$

44. $\frac{6x}{x+5} = \frac{2x-20}{x+5}$

45. $\frac{4}{x-4} = \frac{-x}{x-4} + \frac{x}{2}$

Solve each inequality by using a graph and a table.

46. $\frac{2x+1}{x} \geq 3$

47. $\frac{4}{x+3} < 2$

48. $\frac{x-4}{2x} \geq 2$

Solve each inequality algebraically.

49. $\frac{3}{x+2} \leq 1$

50. $\frac{10}{x-2} < 2$

51. $\frac{15}{x+3} \leq 1$

Lesson 8-6

Simplify each expression. Assume all variables are positive.

52. $\sqrt[3]{343x^9}$

53. $\sqrt[5]{\frac{x^5}{32}}$

54. $\sqrt[4]{\frac{x^8y^4}{10}}$

Write each expression in radical form, and simplify.

55. $81^{\frac{3}{2}}$

56. $243^{\frac{2}{5}}$

57. $(-8)^{\frac{4}{3}}$

Write each expression using rational exponents.

58. $\sqrt[5]{10^2}$

59. $\sqrt[4]{17^3}$

60. $\left(\sqrt[5]{8}\right)^3$

Simplify each expression.

61. $8^{\frac{1}{2}} \cdot 8^{\frac{5}{2}}$

62. $\frac{4^{\frac{7}{2}}}{4^{\frac{1}{2}}}$

63. $\left(100^{\frac{1}{2}}\right)^3$

Lesson 8-7

Graph each function, and identify its domain and range.

64. $f(x) = \sqrt{x-4} + 1$

65. $f(x) = -\frac{1}{2}\sqrt{x}$

66. $f(x) = 2\sqrt[3]{x+2}$

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

67. $g(x) = \sqrt{x-8}$

68. $g(x) = -6\sqrt{x}$

69. $g(x) = \frac{1}{3}\sqrt{x} + 2$

Graph each inequality.

70. $y \geq \sqrt{x+2} - 3$

71. $y < 2\sqrt{-x}$

72. $y > -4\sqrt[3]{x} + 4$

Lesson 8-8

Solve each equation.

73. $\sqrt{2x+10} = 10$

74. $\sqrt{4x+4} = 2\sqrt{4x-9}$

75. $3\sqrt[3]{x} = \sqrt[3]{7x+40}$

76. $\sqrt{2x+48} = x$

77. $2x + 5 = \sqrt{4x+10}$

78. $x + 6 = \sqrt{4x+21}$

79. $(3x-5)^{\frac{1}{2}} = 4$

80. $(x-4)^{\frac{1}{3}} = -2$

81. $(8x-7)^{\frac{1}{2}} = x$

Solve each inequality.

82. $\sqrt{x-7} < 3$

83. $\sqrt{3x+1} + 2 \leq 6$

84. $\sqrt{2x-3} > 5$

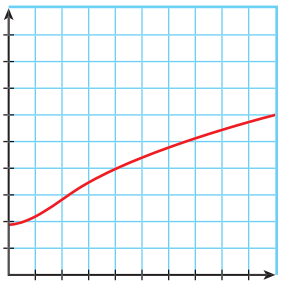
Extra Practice

Chapter 9 ■ Skills Practice

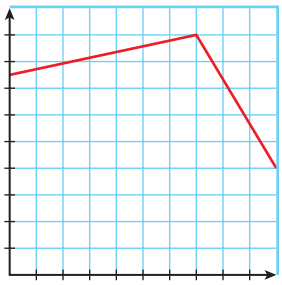
Lesson 9-1

Match each situation to its corresponding graph. Sketch a possible graph of the situation if the situation does not match any of the given graphs.

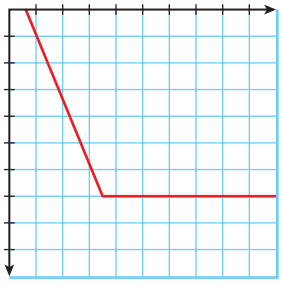
Graph A



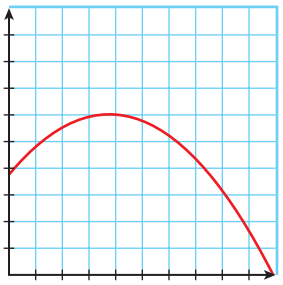
Graph B



Graph C



Graph D

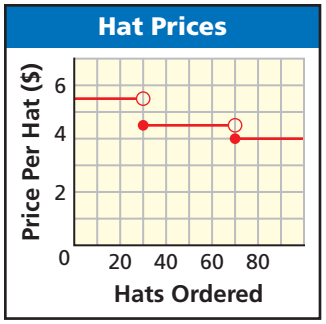


1. A state senator's high approval rating is rising steadily but then drops sharply after a scandal.
2. The value of an antique chair increases steadily.
3. Sales of a valuable stock dip and then recover.
4. A scuba diver descends to 60 ft below sea level and swims around at that depth.

Lesson 9-2

Create a table and a verbal description to represent each graph.

5.



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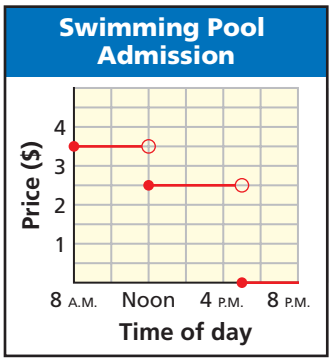


Evaluate each piecewise function for $x = -2$ and $x = 5$.

7. $f(x) = \begin{cases} 10 & \text{if } x \le -4 \\ 7 & \text{if } -4 < x \le 2 \\ 3 & \text{if } x > 2 \end{cases}$

8. $g(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 4 - x & \text{if } x \ge 0 \end{cases}$

9. $h(x) = \begin{cases} x^2 - 3 & \text{if } x \le 2 \\ x + 1 & \text{if } x > 2 \end{cases}$

Graph each function.

10. $f(x) = \begin{cases} 4 & \text{if } x < -1 \\ -1 & \text{if } x \ge -1 \end{cases}$

11. $g(x) = \begin{cases} 2x - 4 & \text{if } x \le 2 \\ -2x + 2 & \text{if } x > 2 \end{cases}$

12. $h(x) = \begin{cases} 2 & \text{if } x < 3 \\ x^2 - 7 & \text{if } x \ge 3 \end{cases}$

Lesson 9-3

Given $f(x) = \begin{cases} 2x - 2 & \text{if } x < 1 \\ -3x & \text{if } x \ge 1 \end{cases}$, write the rule for each function.

13. $g(x)$, a vertical stretch by a factor of 3

14. $h(x)$, a reflection across the y-axis

Identify the x- and y-intercepts of $f(x)$. Without graphing $g(x)$, identify its x- and y-intercepts.

15. $f(x) = -3x + 6$ and $g(x) = f(-2x)$

16. $f(x) = (x - 3)^2$ and $g(x) = -2f(x)$

Lesson 9-4

Given $f(x)$, graph $g(x)$.

17. $f(x) = \frac{1}{2}x - 4$ and $g(x) = f(-x) + 2$

18. $f(x) = |x + 2|$ and $g(x) = \frac{1}{2}f(x - 1) - 4$

Given $f(x) = -2x + 5$ and $g(x) = 4x^2 - 11$, find each function.

19. $(f + g)(x)$

20. $(f - g)(x)$

21. $(g - f)(x)$

Given $f(x) = x - 3$ and $g(x) = x^2 + 3x - 18$, find each function.

22. $(fg)(x)$

23. $\left(\frac{f}{g}\right)(x)$

24. $\left(\frac{g}{f}\right)(x)$

Given $f(x) = \frac{1}{2}x + 5$ and $g(x) = -2x^2$, find each value.

25. $f(g(2))$

26. $g(f(2))$

27. $g(f(-6))$

Given $f(x) = \sqrt{x}$, $g(x) = 2x + 3$, and $h(x) = x^2 + 20$, write each composite function. State the domain of each.

28. $f(g(x))$

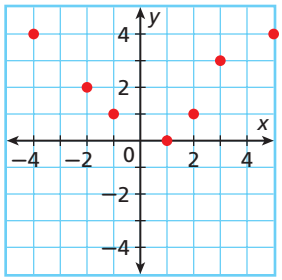
29. $g(f(x))$

30. $g(h(x))$

Lesson 9-5

Use the horizontal-line test to determine whether the inverse of each relation is a function.

31.



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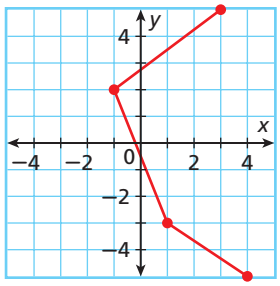

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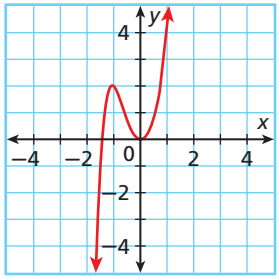


Find the inverse of each function. Determine whether the inverse is a function, and state its domain and range.

34. $f(x) = \frac{1}{2}x - 7$

35. $g(x) = 10 - x^2$

36. $h(x) = \frac{3}{4 + x}$

Determine by composition whether each pair of functions are inverses.

37. $f(x) = \frac{3}{4}x^2$ and $g(x) = \sqrt{\frac{4}{3}x}$ for $x \geq 0$

38. $f(x) = \frac{12x + 1}{5}$ and $g(x) = \frac{5}{12x - 1}$

Lesson 9-6

Use constant differences or ratios to determine which parent function would best model the given data set.

39.

x	y
1	6
3	−12
5	−30
7	−48
9	−66

40.

x	y
2	−3
4	6
6	21
8	42
10	69

41.

x	y
0	0.5
1	2
2	8
3	32
4	128

Lesson 10-1

Graph each equation on a graphing calculator. Identify each conic section. Then describe the center and intercepts.

1. $4x^2 + 16y^2 = 64$ **2.** $x^2 + y^2 = 4$ **3.** $4x^2 + 4y^2 = 100$

Graph each equation on a graphing calculator. Identify each conic section. Then describe the vertices and the direction that the graph opens.

4. $x^2 = y^2 + 16$ **5.** $-10y^2 = x$ **6.** $2y^2 - x^2 = 5$

Find the center and radius of a circle that has a diameter with the given endpoints.

7. $(-2, -1)$ and $(6, 3)$ **8.** $(-4, 0)$ and $(2, 8)$ **9.** $(2, 1)$ and $(8, -1)$

Lesson 10-2

Write the equation of each circle.

10. center $(-4, 3)$ and radius $r = 3$

11. center $(4, 6)$ and radius $r = 9$

12. center $(-3, 3)$ and containing the point $(-3, 0)$

13. center $(2, -5)$ and containing the point $(4, -3)$

Write the equation of the line that is tangent to each circle at the given point.

14. $(x + 2)^2 + (y + 4)^2 = 25; (-5, 0)$ **15.** $(x - 4)^2 + y^2 = 100; (10, 8)$

Lesson 10-3

Find the constant sum of an ellipse with the given foci and point on the ellipse.

16. $F_1(0, 4), F_2(0, -4), P(3, 0)$ **17.** $F_1(6, 0), F_2(-6, 0), P(0, 8)$

Write an equation in standard form for each ellipse with center $(0, 0)$.

18. vertex $(0, 6)$, co-vertex $(5, 0)$ **19.** co-vertex $(0, 5)$, focus $(12, 0)$

Graph each ellipse.

20. $\frac{x^2}{16} + \frac{y^2}{49} = 1$ **21.** $\frac{x^2}{100} + \frac{y^2}{36} = 1$ **22.** $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{64} = 1$

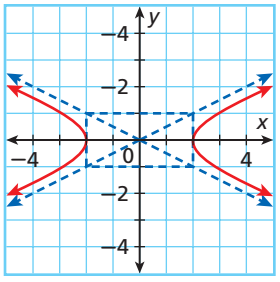
Lesson 10-4

Find the constant difference for a hyperbola with the given foci and point on the hyperbola.

23. $F_1(-15, 0), F_2(15, 0), P(12, 0)$ **24.** $F_1(0, 16), F_2(0, -16), P(0, 10)$

Write an equation in standard form for each hyperbola.

25.



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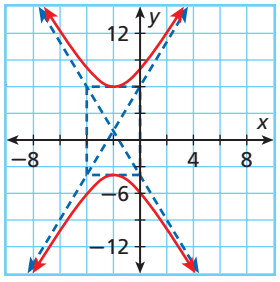


Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

27. $\frac{x^2}{25} - \frac{y^2}{9} = 1$ **28.** $\frac{(x-4)^2}{16} - \frac{(y+2)^2}{4} = 1$ **29.** $\frac{(y-2)^2}{9} - (x-2)^2 = 1$

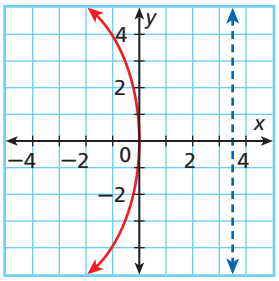
Lesson 10-5

Use the distance formula to find the equation of a parabola with the given focus and directrix.

30. $F(8, 0), x = -8$ **31.** $F(0, 9), y = -9$ **32.** $F(-2, 0), x = 4$

Write the equation in standard form for each parabola.

33.



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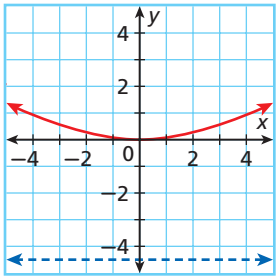

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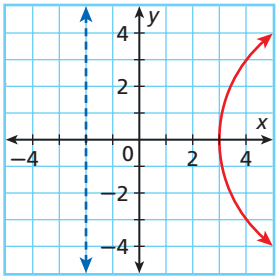


Find the vertex, value of *p*, axis of symmetry, focus, and directrix of each parabola.

36. $y = \frac{1}{20}(x - 1)^2$ **37.** $x = -\frac{1}{12}(y + 4)^2$ **38.** $y - 5 = \frac{1}{26}(x + 3)^2$

Lesson 10-6

Identify the conic section that each equation represents.

39. $\frac{(x + 9)^2}{144} + \frac{(y - 2)}{81} = 1$ **40.** $x^2 + (y - 7)^2 = 81$ **41.** $x + 3 = \frac{1}{6}(y - 6)^2$

42. $4x^2 + 8xy + 5y^2 + 3x + 7 = 0$ **43.** $-2x^2 + 8xy - 8y^2 + 20x = 0$

Find the standard form of each equation by completing the square. Then identify and graph each conic.

44. $4x^2 + 9y^2 - 8x + 72y + 112 = 0$ **45.** $-4x^2 + y^2 + 16x - 4y - 28 = 0$

Lesson 10-7

Solve each system of equations by graphing.

46. $\begin{cases} \frac{(x - 3)^2}{25} + \frac{(y + 2)^2}{64} = 1 \\ 5y + 8x = -26 \end{cases}$ **47.** $\begin{cases} y = -\frac{1}{2}x^2 + 3 \\ y = \frac{1}{2}x - 7 \end{cases}$

48. $\begin{cases} (x + 1)^2 + (y + 4)^2 = 16 \\ x - y = 7 \end{cases}$ **49.** $\begin{cases} 16x^2 + 25y^2 = 400 \\ 20y = 3x^2 \end{cases}$

Solve each system of equations by using the substitution method.

50. $\begin{cases} x^2 + y^2 = 100 \\ y = 3x - 10 \end{cases}$ **51.** $\begin{cases} 4x^2 - 16y^2 = 64 \\ 2y + 12 = 3x \end{cases}$ **52.** $\begin{cases} x^2 + y^2 = 36 \\ 36x^2 + 49y^2 = 1764 \end{cases}$

Solve each system of equations by using the elimination method.

53. $\begin{cases} x^2 + y^2 = 18 \\ x^2 - 5y^2 = -36 \end{cases}$ **54.** $\begin{cases} 3x^2 + 2y^2 = 98 \\ 9x^2 + 4y^2 = 244 \end{cases}$ **55.** $\begin{cases} 4x^2 + 6y^2 = 118 \\ 2y^2 - 2x^2 = -14 \end{cases}$

Lesson 11-1

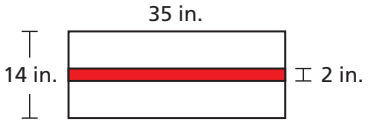
1. When text messaging on a telephone, pressing a 3 types D, E, F, or 3. Pressing a 7 types P, Q, R, S, or 7. How many messages are possible by pressing a 3, a 7, and then a 3?
2. At a company, each employee has an ID that consists of 2 digits followed by a letter. The letters Q and X are not used. How many employee IDs are possible?
3. If there are 8 finalists in a talent show, how many ways can a winner and a runner-up be chosen?
4. Jim's soccer team has 18 members. How many ways can the coach choose a right forward, a center forward, and a left forward?
5. Erin's health club offers 7 types of aerobics classes. She plans to attend 4 classes this week. How many ways can she choose 4 classes that are all different?
6. Francesca can take 4 of her 14 books on a trip. How many ways can she choose them?

Lesson 11-2

Two number cubes are rolled. Find each probability.

7. Both cubes roll the same number.
8. The sum is greater than 8.
9. The sum is 8 or less.
10. Both cubes roll even numbers.
11. What is the probability that a random 2-digit number is a multiple of 7?
12. What is the probability that a randomly selected day in January is after the 20th?
13. A mother is making different lunches for each of her 3 children. If each child grabs a lunch bag at random, what is the probability that all 3 children will get the correct bag?
14. A teacher writes MATHEMATICS on a piece of paper and then cuts out each letter and puts them all in a bag. She will draw two letters at random. What is the probability that she will select an M and an A?

The shaded region is vertically centered in the flag. Find each probability.



15. a random point inside the flag is in the shaded region
16. a random point inside the flag is above the shaded region

A marble is drawn from a bag and then its color is recorded in the table.

17. Find the experimental probability of drawing a blue marble.
18. Find the experimental probability of drawing a pink or a yellow marble.

Marble Drawing Experiment

Color	Times Drawn
Pink	12
Green	10
Blue	16
Yellow	12

Lesson 11-3

Find each probability.

19. rolling a number greater than or equal to 4 on a number cube twice in a row

20. drawing a face card from a deck, replacing it, and drawing a number card

21. Two number cubes are rolled—one blue and one yellow. Find the probability that the yellow cube is even, and the sum is 7. Explain why the events are dependent.

The table shows the results of a schoolwide survey on the homecoming dance. Find each probability.

22. A student who prefers the cafeteria is a girl.

23. A surveyed student is male and prefers the gymnasium.

Homecoming Dance Location Survey

	Girls	Boys
Gymnasium	67	58
Cafeteria	53	37

A bag contains 18 beads—5 blue, 6 yellow, and 7 red. Determine whether the events are independent or dependent. Find the indicated probability.

24. selecting a yellow and then a blue bead when they are chosen with replacement

25. selecting a yellow and then a blue bead when they are chosen without replacement

Lesson 11-4

26. A table was chosen at random in the cafeteria, and there were 2 freshmen, 5 sophomores, 7 juniors, and 2 seniors eating there. A student is chosen at random from the table. What is the probability of choosing a freshman or a senior?

The numbers 1–20 are written on cards and placed in a bag. Find each probability.

27. choosing a number less than 10 or choosing a multiple of 5

28. choosing 20 or choosing an odd number

29. In an apartment building with 50 residents, 16 residents have cats, 28 residents are students, and 9 of the students have cats. What is the probability that a resident is a student or has a cat?

30. There are 8 couples in a dance competition, and each of the 3 judges must pick the couple they believe should win. Suppose the judges picked randomly. What is the probability that at least 2 judges picked the same couple?

Lesson 11-5

Find the mean, median, and mode of each data set.

31. $\{3, 7, 8, 2, 8, 4\}$ **32.** $\{12, 9, 8, 15, 16, 12, 13\}$ **33.** $\{7, 31, 20, 12, 18\}$

34. Find the expected value of the raffle prize.

Raffle Prizes

Value	$0	$5	$20	$200
Probability	0.76	0.16	0.06	0.02

Make a box-and-whisker plot of the data. Find the interquartile range.

35. $\{3, 5, 7, 6, 5, 2, 3\}$ **36.** $\{12, 15, 18, 10, 9, 15, 16\}$

Find the variance and standard deviation.

37. $\{8, 12, 10, 6, 9\}$ **38.** $\{14, 15, 10, 8, 12, 13\}$ **39.** $\{6, 33, 37, 28, 1\}$

Lesson 11-6

Use the Binomial Theorem to expand each binomial.

40. $(x + 4)^5$ **41.** $(2x - 3)^4$ **42.** $(2a + 7b)^3$

43. Patrick takes a multiple-choice quiz that has 4 questions. There are 5 answer choices for each question. What is the probability that he will get at least 2 answers correct by guessing?

Chapter 12 ■ Skills Practice

Lesson 12-1

Find the first 5 terms of each sequence.

1. $a_1 = 16, a_n = 0.25a_{n-1}$
2. $a_1 = -1, a_n = 3a_{n-1} + 1$
3. $a_1 = 2, a_2 = 5, a_n = 2a_{n-1} + a_{n-2}$
4. $a_n = 5(n - 1)$
5. $a_n = 3^n - 4$
6. $a_n = (n + 2)^2$

Write a possible explicit rule for the *n*th term of each sequence.

7. 5, 1, −3, −7, −11, ...
8. 7, 9, 13, 21, 37, ...
9. $\frac{3}{4}, \frac{3}{2}, \frac{9}{4}, 3, \frac{15}{4}, \ldots$
10. 60, 30, 15, $\frac{15}{2}, \ldots$

Lesson 12-2

Write each series in summation notation.

11. $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10}$
12. $0 + 3 + 8 + 15 + 24 + 35$
13. $50 + 41 + 32 + 23 + 14 + 5$
14. $-4 - 2 + 0 + 2 + 4$

Expand each series and evaluate.

15. $\sum_{k=1}^{5}(12k - 7)$
16. $\sum_{k=1}^{4}\frac{(2k)^2}{2}$
17. $\sum_{k=1}^{5}\frac{k-2}{k+1}$

Evaluate each series.

18. $\sum_{k=1}^{55}k$
19. $\sum_{k=15}^{25}12$
20. $\sum_{k=1}^{18}k^2$

Lesson 12-3

Determine whether each sequence could be arithmetic. If so, find the common difference and the next term.

21. 15.5, 28, 40.5, 53, 65.5, ...
22. 9.67, 9.34, 9.01, 8.68, ...
23. $\frac{1}{2}, 2, \frac{9}{2}, 8, \frac{25}{2}, \ldots$
24. 2, 4, 6, 4, 2, ...

Find the 8th term of each arithmetic sequence.

25. 4.5, 6, 7.5, 9, 10.5, ...
26. 74, 68, 62, 56, 50, ...
27. $5, 5\frac{2}{5}, 5\frac{4}{5}, 6\frac{1}{5}, 6\frac{3}{5}, \ldots$

Find the missing terms in each arithmetic sequence.

28. 13, ▢, ▢, 37, ...
29. 9.5, ▢, ▢, ▢, −0.5, ...
30. 10, ▢, ▢, ▢, 26, ...

Find the 9th term of each arithmetic sequence.

31. $a_3 = 29$ and $a_6 = 56$
32. $a_4 = 16$ and $a_7 = -2$
33. $a_{10} = 30.5$ and $a_{14} = 38.5$
34. $a_5 = 3\frac{1}{3}$ and $a_7 = 2\frac{2}{3}$

Find the indicated sum for each arithmetic series.

35. S_{12} for 18, 21, 24, 27, 30, ...
36. S_{15} for 20, 18.5, 17, 15.5, 14, ...
37. $\sum_{k=1}^{9}(5k + 8)$
38. $\sum_{k=1}^{20}(-2.75k + 15)$

Lesson 12-4

Determine whether each sequence could be geometric or arithmetic. If possible, find the common ratio or difference.

39. 7, 14, 28, 56, 112, ...

40. 7, 14, 21, 28, 35, ...

41. $\frac{2}{3}, 1\frac{1}{3}, 2\frac{2}{3}, 5\frac{1}{3}, 10\frac{2}{3}, \ldots$

42. 25.5, 31, 36.5, 42, 47.5, ...

43. −3, 6, 21, 42, 69, ...

44. $4, 1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots$

Find the 7th term of each geometric sequence.

45. 5, 10, 20, 40, 80, ...

46. 200, 100, 50, 25, 12.5, ...

47. −1, 3, −9, 27, −81, ...

48. 7, 70, 700, 7000, ...

Find the 8th term of the geometric sequence with the given terms.

49. $a_4 = 4$, $a_5 = 8$

50. $a_4 = 16$, $a_6 = 256$

51. $a_3 = 125$, $a_5 = 5$

52. $a_4 = 4$, $a_7 = 864$

Find the geometric mean of each pair of numbers.

53. 4 and 36

54. $\frac{1}{4}$ and $\frac{1}{9}$

55. 72 and 288

56. 10 and 20

Find the indicated sum for each geometric series.

57. S_8 for 5, −15, 45, −135, ...

58. S_6 for $\frac{3}{4}$, 3, 12, 48, ...

59. $\sum_{k=1}^{5} 12(2)^{k-1}$

60. $\sum_{k=1}^{7} (-4)^{k-1}$

Lesson 12-5

Determine whether each geometric series converges or diverges.

61. $\frac{5}{2}, \frac{5}{8}, \frac{5}{32}, \frac{5}{128}, \ldots$

62. 0.1, 0.5, 2.5, 12.5, 62.5, ...

63. 1, 1.3, 1.69, 2.197, 2.8561, ...

64. 1, 0.7, 0.49, 0.343, 0.2401, ...

Find the sum of each infinite geometric series, if it exists.

65. 1.1, 1.21, 1.331, 1.4641, ...

66. 1.8, 1.62, 1.458, 1.3122, ...

67. 7, 0.7, 0.07, 0.007, ...

68. $\sum_{k=1}^{\infty} \frac{1}{2}\left(\frac{3}{2}\right)^k$

69. $\sum_{k=1}^{\infty} 8\left(\frac{4}{10}\right)^k$

70. $\sum_{k=1}^{\infty} 100(0.95)^k$

Write each repeating decimal as a fraction in simplest form.

71. $0.\overline{4}$

72. $0.\overline{26}$

73. $0.\overline{892}$

Identify a counterexample to disprove each statement, where the variable is a real number.

74. $4a^3 \geq 8a^2$

75. $5^{2n} \geq 5^n$

76. $x^2 > (x-1)^2$

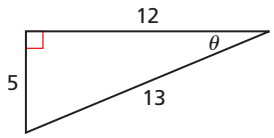
77. $|x+1| \geq |x|$

Extra Practice

Lesson 13-1

Find the value of the sine, cosine, and tangent functions for θ.

1.



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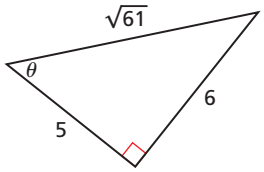

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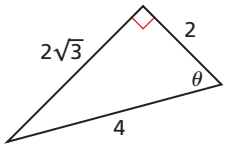
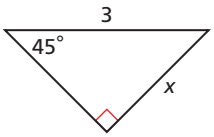
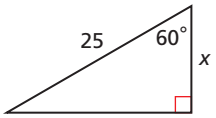


Use a trigonometric function to find the value of x.

4.



5.



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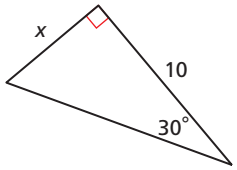
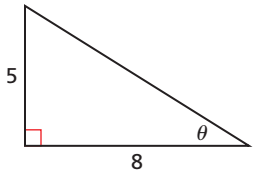


Find the values of the six trigonometric functions for θ.

7.



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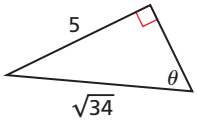

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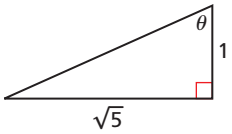


Lesson 13-2

Draw an angle with the given measure in standard position.

10. $-30°$ **11.** $240°$ **12.** $410°$ **13.** $-350°$

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

14. $\theta = 20°$ **15.** $\theta = 400°$ **16.** $\theta = -125°$ **17.** $\theta = -385°$

Find the measure of the reference angle for each given angle.

18. $\theta = -120°$ **19.** $\theta = 175°$ **20.** $\theta = 110°$ **21.** $\theta = 385°$

***P* is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ.**

22. $P(2, 3)$ **23.** $P(-1, 4)$ **24.** $P(-1, -1)$ **25.** $P(2, -8)$

Lesson 13-3

Convert each measure from degrees to radians or from radians to degrees.

26. $60°$ **27.** $-135°$ **28.** $90°$ **29.** $-10°$

30. $-\frac{3\pi}{2}$ **31.** $\frac{\pi}{10}$ **32.** $\frac{\pi}{18}$ **33.** $-\frac{3\pi}{8}$

Use the unit circle to find the exact value of each trigonometric function.

34. $\cos 150°$ **35.** $\tan \frac{7\pi}{4}$ **36.** $\sin \frac{7\pi}{6}$ **37.** $\cos 315°$

38. $\sin \frac{2\pi}{3}$ **39.** $\cos 270°$ **40.** $\csc \frac{4\pi}{3}$ **41.** $\cot 225°$

Use a reference angle to find the exact value of the sine, cosine, and tangent of each angle.

42. $-150°$ **43.** $210°$ **44.** $315°$ **45.** $330°$

46. $\frac{\pi}{4}$ **47.** $-\frac{7\pi}{6}$ **48.** $\frac{5\pi}{4}$ **49.** $\frac{5\pi}{3}$

Lesson 13-4

Find all possible values of each expression.

50. $\tan^{-1}\left(-\sqrt{3}\right)$ **51.** $\cos^{-1}\frac{1}{2}$ **52.** $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.

53. $\mathrm{Tan}^{-1}(-1)$ **54.** $\mathrm{Sin}^{-1}\frac{1}{2}$ **55.** $\mathrm{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Solve each equation to the nearest tenth. Use the given restrictions.

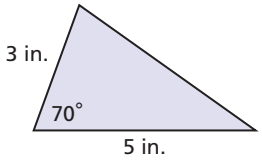
56. $\sin\theta = 0.8$, for $-90° \le \theta \le 90°$ **57.** $\sin\theta = 0.8$, for $90° < \theta < 180°$

58. $\tan\theta = 2.1$, for $-90° < \theta < 90°$ **59.** $\tan\theta = 2.1$, for $180° < \theta < 270°$

Lesson 13-5

Find the area of each triangle. Round to the nearest tenth.

60.



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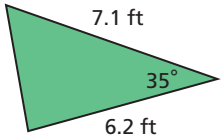

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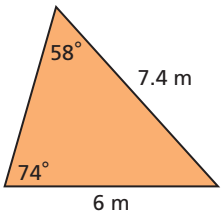
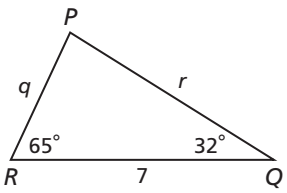


Solve each triangle. Round to the nearest tenth.

63.



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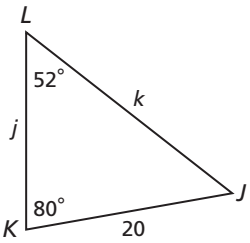

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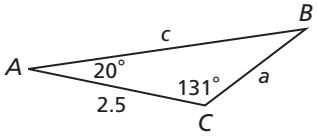

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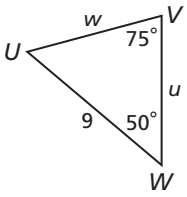

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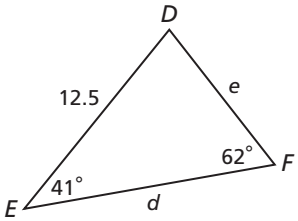

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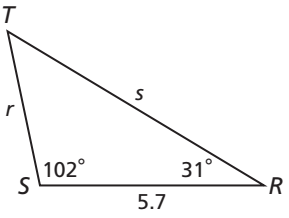
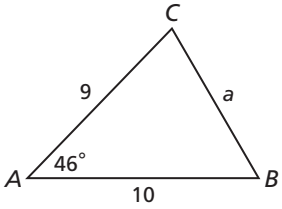


Lesson 13-6

Use the given measurements to solve each triangle. Round to the nearest tenth.

69.



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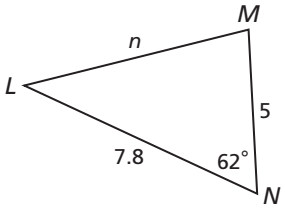

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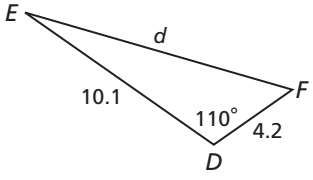

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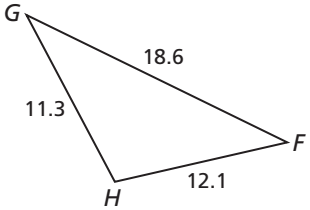

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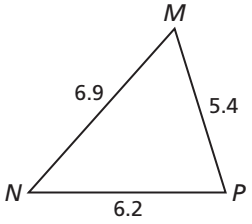

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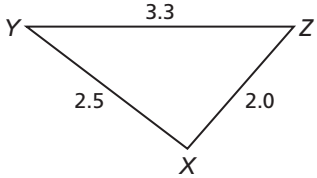
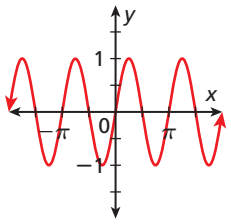
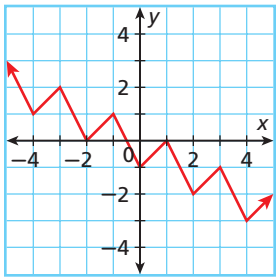


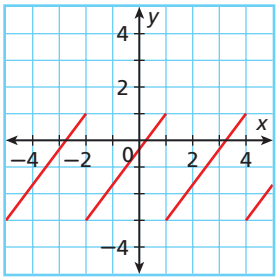
Extra Practice

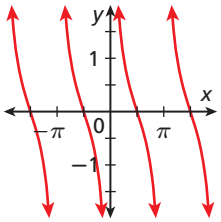
Lesson 14-1

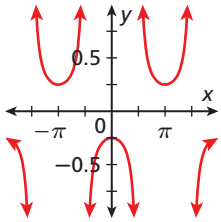
Identify whether each function is periodic. If the function is periodic, give the period.

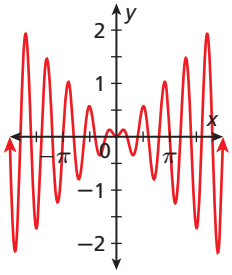
1. 

2. 

3. 

4. 

5. 

6. 

Using $f(x) = \sin x$ or $f(x) = \cos x$ as a guide, graph each function. Identify the amplitude and period.

7. $f(x) = \frac{1}{2}\sin 2x$

8. $g(x) = 3\cos\frac{1}{2}x$

9. $h(x) = 2\cos \pi x$

Using $f(x) = \sin x$ or $f(x) = \cos x$ as a guide, graph each function. Identify the x-intercepts and phase shift.

10. $f(x) = \cos\left(x + \frac{\pi}{2}\right)$

11. $g(x) = \sin(x - \pi)$

12. $h(x) = \sin\left(x + \frac{\pi}{4}\right)$

Lesson 14-2

Using $f(x) = \tan x$ as a guide, graph each function. Identify the period, x-intercepts, and asymptotes.

13. $g(x) = 2\tan 2x$

14. $g(x) = \frac{1}{2}\tan 3x$

15. $h(x) = -\tan \pi x$

Using $f(x) = \cot x$ as a guide, graph each function. Identify the period, x-intercepts, and asymptotes.

16. $g(x) = \frac{1}{2}\cot 2x$

17. $g(x) = -\cot\frac{1}{2}x$

18. $h(x) = \cot 3x$

Using $f(x) = \cos x$ or $f(x) = \sin x$ as a guide, graph each function. Identify the period and asymptotes.

19. $g(x) = \sec\frac{1}{2}x$

20. $g(x) = \csc 2x$

21. $h(x) = \frac{1}{4}\sec x$

Lesson 14-3

Prove each trigonometric identity.

22. $\sec(-\theta) = \sec\theta$

23. $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

24. $\tan^2\theta\left(1-\sin^2\theta\right) = \sin^2\theta$

Rewrite each expression in terms of cos θ and simplify.

25. $\sec\theta\left(1-\sin^2\theta\right)$

26. $\dfrac{\sin^2\theta}{1+\cos\theta}$

27. $\dfrac{\csc\theta-\sin\theta}{\cot\theta}$

Rewrite each expression in terms of sin θ and simplify.

28. $\dfrac{\tan\theta+1}{\sec\theta+\csc\theta}$

29. $\dfrac{\cot\theta}{\csc\theta}$

30. $1-\cot\theta\cos\theta\sin\theta$

Lesson 14-4

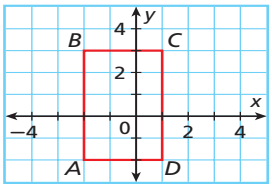
Find each value if $\sin A = \dfrac{12}{13}$ with $0° < A < 90°$ and if $\cos B = -\dfrac{3}{5}$ with $90° < B < 180°$.

31. $\sin(A+B)$

32. $\cos(A-B)$

33. $\tan(A+B)$

Find the coordinates, to the nearest hundredth, of the vertices of figure $ABCD$ with $A(-2,-2)$, $B(-2,3)$, $C(1,3)$, and $D(1,-2)$ after each rotation about the origin.



34. 135°

35. 270°

Lesson 14-5

Find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ for each set of conditions.

36. $\sin\theta = \dfrac{12}{13}$ and $0° < \theta < 90°$

37. $\cos\theta = -\dfrac{3}{5}$ and $180° < \theta < 270°$

38. $\tan\theta = -\dfrac{3}{2}$ and $\dfrac{3\pi}{2} < \theta < 2\pi$

39. $\sin\theta = \dfrac{1}{3}$ and $\dfrac{\pi}{2} < \theta < \pi$

Prove each identity.

40. $\dfrac{\cos 2\theta}{\cos\theta+\sin\theta} = \cos\theta-\sin\theta$

41. $\dfrac{\cos\theta\sin 2\theta}{1+\cos 2\theta} = \sin\theta$

42. $\cos 2\theta + 2\sin^2\theta = 1$

43. $(\sin\theta-\cos\theta)^2 = 1-\sin 2\theta$

Find $\sin\dfrac{\theta}{2}$, $\cos\dfrac{\theta}{2}$, and $\tan\dfrac{\theta}{2}$ for each set of conditions.

44. $\sin\theta = \dfrac{3}{5}$ and $90° < \theta < 180°$

45. $\tan\theta = -\dfrac{7}{24}$ and $270° < \theta < 360°$

46. $\tan\theta = -\dfrac{\sqrt{5}}{2}$ and $\dfrac{\pi}{2} < \theta < \pi$

47. $\cos\theta = \dfrac{1}{5}$ and $0 < \theta < \dfrac{\pi}{2}$

Lesson 14-6

Find all of the solutions of each equation.

48. $2\cos\theta = \sqrt{2}$

49. $2\sin\theta + 5 = 6$

50. $3\tan\theta = 2\tan\theta - 1$

51. $\tan\theta = 2\tan\theta - \sqrt{3}$

Solve each equation for the given domain.

52. $\cos^2\theta - 3\cos\theta - 4 = 0$ for $0 \le \theta < 2\pi$

53. $2\sin^2\theta - 5\sin\theta + 2 = 0$ for $0 \le \theta < 2\pi$

54. $\sin^2\theta + 3\sin\theta + 1 = 0$ for $0° \le \theta < 360°$

55. $\cos^2\theta + 4\cos\theta - 2 = 0$ for $0° \le \theta < 360°$

Use trigonometric identities to solve each equation for the given domain.

56. $\cos 2\theta + 3\cos\theta = 1$ for $0 \le \theta < 2\pi$

57. $\cos 2\theta + 5\sin\theta = -2$ for $0 \le \theta < 2\pi$

58. $2\sin^2\theta = 3 - 3\cos\theta$ for $0° \le \theta < 360°$

59. $\sin 2\theta = \cos\theta$ for $0° \le \theta < 360°$

Extra Practice

Chapter 1 ■ Applications Practice

Sports **Use the following information for Exercises 1–3.**

In women's boxing, some of the official weight classes in pounds are defined in the table below. Each class includes the lightest weight in the range but not the heaviest weight. *(Lesson 1-1)*

Middleweight 154–160	Featherweight 122–126
Jr. Middleweight 147–154	Bantamweight 115–118
Super Lightweight 135–140	Welterweight 140–147

1. Order the weight classes from lightest to heaviest.
2. Use interval notation to represent the set of weights in pounds that define the Super Lightweight class.
3. Use set-builder notation to represent the set of weights in pounds that define the Welterweight class.
4. **Commerce** A sweater is on sale for 20% off. The regular price of the sweater is \$60. Use mental math to determine the sale price of the sweater. Explain how you determined your answer. *(Lesson 1-2)*
5. **Construction** A builder is covering a rectangular floor with square tiles. A row of 46 tiles fits along the length of the room, and a row of 26 tiles fits along the width of the room. If each tile covers 20.25 in^2, what are the dimensions of the room in feet? *(Lesson 1-3)*

Money **Use the following information for Exercises 6 and 7.**

Charles pays \$1.25 for a newspaper using only quarters and dimes. Let q represent the number of quarters he uses. *(Lesson 1-4)*

6. Write an expression in terms of q for the number of dimes Charles uses.
7. If Charles uses 3 quarters, how many dimes does he use?
8. **Astronomy** The diameter of the Sun is 1.392×10^6 km. What is the radius of the Sun in kilometers? Express your answer in scientific notation. *(Lesson 1-5)*
9. **Shipping** The table shows the cost of shipping packages that weigh up to 3 lb. Is the relation from weight to cost a function? Is the relation from cost to weight a function? Explain. *(Lesson 1-6)*

Weight (lb)	Cost (\$)
Up to 1 lb	3.69
More than 1 lb and up to 2 lb	3.85
More than 2 lb and up to 3 lb	4.65

Communication **Use the following information for Exercises 10 and 11.**

A cell phone plan charges \$2.99 per month for 300 text messages plus \$0.05 for each additional text message. *(Lesson 1-7)*

10. Write a function to represent the monthly charge in dollars for x text messages.
11. What is the value of the function for an input of 450, and what does it represent?
12. **Recreation** A bowling alley charges \$4.00 to rent shoes and \$2.75 per game. As part of a promotion, the alley lowers the price of shoes to \$3.00. What kind of transformation describes the change in the total cost of bowling x games per person? *(Lesson 1-8)*
13. **Sports** Each team in a soccer league plays each of the other teams one time during a season. Graph the relationship between the number of teams and the total number of games and identify which parent function best describes the data. Then use the graph to estimate the total number of games per season when there are 8 teams in the league. *(Lesson 1-9)*

Total Number of Games per Season					
Teams	4	6	10	12	14
Games	6	15	45	66	91

Chapter 2 ■ Applications Practice

1. **Poetry** An English sonnet is made up of 14 lines of text. Rebecca is working on a poetry project for her English class that must be at least 100 lines long. Rebecca has already written 61 lines of haiku, free-verse poems, and limericks, and she plans to write the rest in sonnet form. How many sonnets must she write to complete the project? *(Lesson 2-1)*

2. **Nutrition** Tom follows a strict diet in which he gets 22% of his daily Calories from fat. His diet contains 363 Calories of fat each day. What is his total daily caloric intake? *(Lesson 2-2)*

Home Economics **Use the following information for Exercises 3 and 4.**

Jonathan owes his parents $150 for car repairs. For each hour of chores he does, Jonathan's parents credit him $8 toward his debt. *(Lesson 2-3)*

3. Make a graph showing the amount that Jonathan owes his parents versus the number of hours he does chores.

4. What are the intercepts of the graph? What do they mean?

Consumer Economics **Use the following information for Exercises 5 and 6.**

The table below shows the cost of several long-distance calls made using the same calling plan. *(Lesson 2-4)*

Time (min)	7	12	15	24
Cost ($)	$1.80	$2.55	$3.00	$4.35

5. Find the function that represents the data and write it in slope-intercept form. What do the slope and y-intercept represent?

6. Make a graph to show the cost of all phone calls up to 20 minutes.

7. **Sports** The basketball team is losing by 8 points. They score either 2 points or 3 points for each basket they make. Write and graph an inequality for the number of 2-point and 3-point baskets the team needs to make in order to win the game if the other team scores 16 more points. *(Lesson 2-5)*

Careers **Use the following information for Exercises 8 and 9.**

At a spa each masseur earns $80 per day plus $15 per massage. Starting next month, the per-massage fee will be raised to $30. *(Lesson 2-6)*

8. Write $f(x)$ to represent the original earnings and $g(x)$ to represent the new earnings.

9. Graph $f(x)$ and $g(x)$ on the same coordinate plane. Describe the transformation.

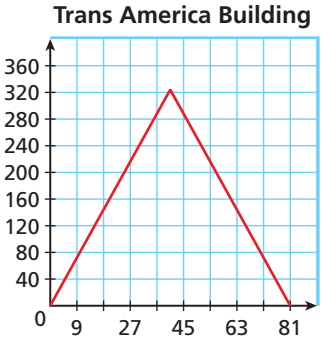
10. **Astronomy** The table below shows the distances from four planets to the Sun and the time it takes in Earth days for each planet to complete its revolution around the Sun.

Planet	Mercury	Venus	Earth	Mars
Distance (million km)	58	108	150	228
Revolution (Earth days)	88	225	365	687

Make a scatter plot using distance as the independent variable. Find the line of best fit and correlation coefficient. *(Lesson 2-7)*

11. **Meteorology** A meteorologist predicts that Sunday's high temperature will be 76°F. Her predictions are generally accurate to within 4°F. Write and solve an absolute-value inequality to find the possible high temperatures. *(Lesson 2-8)*

12. **Architecture** The TransAmerica Pyramid in San Francisco is 324 meters high and 81 meters wide at its base. The diagram shows a side view of the building. The vertex is at $(40.5, 324)$. Write an absolute-value equation to describe the graph. *(Lesson 2-9)*



Extra Practice

Chapter 3 ▪ Applications Practice

Oceanography **Use the following information for Exercises 1–3.**

A dolphin and a shark are swimming toward the same fish. The dolphin is 88 meters away and is swimming at a rate of 10.7 meters per second. The shark is 100 meters away and is swimming at a rate of 12.2 meters per second. *(Lesson 3-1)*

1. Write and graph a system of equations that could be used to model the distance of both the dolphin and the shark from the fish.
2. In how many seconds will the dolphin and the shark be the same distance from the fish?
3. What will that distance be?
4. **Chemistry** A chemist needs 16 ounces of a 10% potassium chloride solution. Solution A contains 12% potassium chloride, and solution B contains 8% potassium chloride. How much of each solution must the chemist combine? *(Lesson 3-2)*
5. **Consumer Economics** At the hardware store, Leslie bought 2 boxes of nails and 3 boxes of screws and paid $13.95 before tax. Michael bought 4 boxes of nails and 1 box of screws and paid $12.15 before tax. Find the cost of a box of nails and the cost of a box of screws. *(Lesson 3-2)*
6. **Entertainment** Hillary and Rob are at a multimedia store buying CDs and DVDs. The CDs cost $14 each, and the DVDs cost $12 each. Their total budget is $150, and Hillary wants to make sure that they buy at least as many DVDs as they buy CDs. Write and graph a system of inequalities that describes the possible number of CDs and DVDs they can buy. *(Lesson 3-3)*
7. **Construction** The Moua family is building a toy train using two types of cars. Passenger cars cost $30 and are 6 inches long. Freight cars cost $25 and are 9 inches long. They cannot spend more than $350, and the train must not be more than 9 feet long. Write and graph a system of inequalities that describes the possible number of each type of car they can use to make their train. *(Lesson 3-3)*

Business **Use the following information for Exercises 8 and 9.**

Leona owns a small business selling earrings and necklaces. Each pair of earrings takes 30 minutes to make and yields $3 of profit. Each necklace takes 1 hour to make and yields $7 of profit. Leona works no more than 7 hours per day, and she always makes at least 4 pairs of earrings each day. *(Lesson 3-4)*

8. Write a system of inequalities and graph the feasible region for how many earrings and necklaces Leona can make in a day.
9. Write an equation describing Leona's profit P. How many of each product should she make each day to maximize her profits?

Interior Design **Use the following information for Exercises 10 and 11.**

Mrs. Walsh has $150 to spend on decorations for her living room. She plans to buy candles, picture frames, and decorative pillows. The candles are 3 for $20, the picture frames cost $15 each, and the pillows cost $25 each. *(Lesson 3-5)*

10. Write a linear equation in three variables to represent this situation.
11. If Mrs. Walsh buys 6 candles and 4 frames, how many pillows can she buy?

Fitness **Use the following information for Exercises 12 and 13.**

Each day at the gym, Jaya cycles on the stationary bicycle, lifts weights, and swims laps. The table shows the number of minutes she spent doing each activity and the number of Calories she burned on 3 different days. *(Lesson 3-6)*

Day	Cycling	Weight Lifting	Swimmimg	Total Calories
1	30	30	20	455
2	25	45	15	432.5
3	25	20	30	495

12. How many Calories per minute does each activity burn?
13. How many Calories would Jaya burn if she spent 40 minutes cycling, 20 minutes lifting weights, and 20 minutes swimming?

Chapter 4 ■ Applications Practice

Transportation **Use the following information for Exercises 1 and 2.**

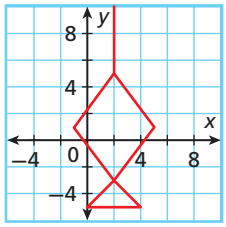
The table shows the costs for different bus tickets from San Antonio to Dallas. *(Lesson 4-1)*

Category	Child	Student	Adult	Senior
One-Way	$18.00	$23.50	$41.00	$34.50
Round-Trip	$34.00	$45.00	$79.00	$66.00

1. Next year, the bus company will raise the price of each ticket by 3%. Use a scalar product to find the new ticket prices.
2. Due to a rise in gas prices this month, the bus company is temporarily charging passengers an extra $2.50 per one-way ticket and $4.50 per round-trip ticket. Use matrix addition to find the new ticket prices.
3. **Taxes** Different portions of Mr. Waller's income are taxed at different rates. Use matrix multiplication to find the total amount of taxes he paid each year. *(Lesson 4-2)*

Tax Rates			Mr. Waller's Income		
Year	Job	Home Business	Source	2004	2005
2004	23%	31%	Job	$23,550	$25,750
2005	24%	31%	Home Business	$5,600	$5,200

4. **Design** The Fishing Club's new logo is shaped like a fish on a fishing line. On a coordinate plane, the vertices of the image are $(-1, 1)$, $(2, -3)$, $(0, -5)$, $(4, -5)$, $(5, 1)$, $(2, 5)$ and $(2, 10)$. Find the vertices of the figure after it is enlarged by a factor of 2 and reflected across the x-axis. *(Lesson 4-3)*



Gardening **Use the following information for Exercises 5 and 6.**

Mrs. Alarcón bought 3 geraniums, 2 ferns, and 2 petunias for $67.00. Mrs. Muñiz bought 4 geraniums, 4 petunias, and 1 fern for $89.50. The geraniums cost $3.50 more than the ferns. *(Lesson 4-4)*

5. Write the coefficient matrix for the problem.
6. Use Cramer's rule to find the cost of each type of plant.

Sports **Use the following information for Exercises 7–9.**

The gymnastics coach kept track of how many of each type of deduction some of his gymnasts received during the balance beam exercise at their last meet. *(Lesson 4-5)*

	Stepping Out of Bounds	Heavy Brush of Hands/ Feet	Out of Sync with Music	Total Points Lost
Amber	2	1	1	0.55
Marcia	1	2	1	0.75
Jenna	3	1	2	0.70

7. Write the appropriate matrix equation.
8. Find the inverse of the coefficient matrix.
9. Solve the matrix equation to find the amount of each deduction.

Manufacturing **Use the following information for Exercises 10 and 11.**

A manufacturing company makes wooden boxes in small, medium, and large sizes out of the same wood. The table shows how many of each box were made and how much wood was used on 3 different days. *(Lesson 4-6)*

	Small	Medium	Large	Wood Used
Mon	10	12	8	4468 in^2
Tue	7	15	10	5068 in^2
Wed	20	9	5	4298 in^2

10. Write an augmented matrix to describe the situation.
11. Use row operations to find the amount of wood used to make each size of box.

Chapter 5 ▪ Applications Practice

Construction **Use the following information for Exercises 1 and 2.**

A landscape designer is using square stepping stones in a backyard. The function $f(x) = 8x^2$ represents the area in square inches that will be covered by 8 stepping stones with side length x inches. *(Lesson 5-1)*

1. Write a function g for the area that will be covered by 16 stepping stones with side length x inches. Describe g as a transformation of f.
2. The landscape designer decides to use smaller stones with a side length of $(x - 2)$ inches. Write a function h for the area that will be covered by 8 of the smaller stones. Describe h as a transformation of f.

Entertainment **Use the following information for Exercises 3 and 4.**

Part of a roller coaster's path can be modeled by the function $f(x) = -\frac{4}{49}x^2 + \frac{40}{7}x$, where x is the horizontal distance in feet the roller coaster has traveled and f is its height in feet above the ground. *(Lesson 5-2)*

3. What is the roller coaster's maximum height above the ground on this part of the path?
4. How far has the roller coaster traveled horizontally when it reaches its maximum height?

Sports **Use the following information for Exercises 5–7.**

A kickball player kicks a ball from ground level with an initial vertical velocity of 24 ft/s.

5. Write a function in standard form for the ball's height h in feet, where t is the time in seconds after the ball is thrown. *(Lesson 5-3)*
6. How long is the ball in the air? *(Lesson 5-3)*
7. Complete the square to rewrite h in vertex form. What is the ball's maximum height? *(Lesson 5-4)*
8. **School** In a student's science fair project, he claims that the height h in feet above the ground of an object shot from a catapult can be modeled by $h(t) = 16t^2 - 32t + 32$, where t is the time in seconds after the object is shot. What are the zeros of this function? Explain why the values of the zeros indicate that the student's model is incorrect. *(Lesson 5-5)*
9. **Forestry** A wind gust blows a cone from a branch on a redwood tree. The cone's height h in meters above the ground can be modeled by $h(t) = -4.9t^2 - t + 75$, where t is the time in seconds since the cone broke from the branch. To the nearest tenth of a second, how long does the cone fall before hitting the ground? *(Lesson 5-6)*
10. **Business** The weekly profit p in dollars generated by a smoothie stand can be modeled by the function $p(c) = -302c^2 + 1635c - 1712$, where c is the cost in dollars per smoothie. For what range of smoothie costs will the stand generate at least \$450 per week? *(Lesson 5-7)*
11. **Law Enforcement** The table shows the cost of speeding tickets in a certain town, based on how many miles per hour over the speed limit the driver was traveling. Find a quadratic model for the fine given the number of miles per hour over the speed limit. Estimate the fine for a driver traveling 8 mi/h over the speed limit. *(Lesson 5-8)*

Miles per Hour over the Speed Limit	Fine ($)
5	55
10	70
15	95
20	130

12. **Fractals** A fractal can be generated from the formula $Z_{n+1} = (Z_n)^2 + 0.4$. Find the value of Z_2 for this fractal given that $Z_1 = 0.5 - 0.5i$ and $Z_2 = (Z_1)^2 + 0.4$.

Chapter 6 ■ Applications Practice

Manufacturing **Use the following information for Exercises 1 and 2.**

A company produces globes in two different sizes. The large globes have a radius of x inches, and the small globes have a radius of $x - 2$ inches. *(Lesson 6-1)*

1. Write functions to find the volume of each globe size.
2. Evaluate each function for $x = 8$.

Business **Use the following information for Exercises 3 and 4.**

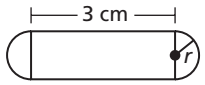
Mr. Schwartz models the number of items his business sold during its first 10 years as $N(x) = 0.07x^3 + 9x^2 - 16x + 80$. His average profit per item (in dollars) can be modeled as $P(x) = 0.5x + 10$. *(Lesson 6-2)*

3. Write a polynomial $T(x)$ that can be used to model the total profit for his company during these years.
4. Evaluate $T(4)$ and explain its significance.
5. **Entertainment** The concert attendance for a music group can be described by the function $F(x) = \frac{1}{4}x^3 + 2x^2 + 50$, where x is the number of concerts since its debut. Use synthetic division to find the number of people who attended the fourth concert. *(Lesson 6-3)*

Sports **Use the following information for Exercises 6 and 7.**

The manager of a basketball team charted the team's progress for the season. For each game, she took the team's points and subtracted the points that the other team scored. The team's performance can be modeled by the function $P(x) = x^3 - 9x^2 + 18x$, where x represents the number of games since the start of the season. *(Lesson 6-4)*

6. Find the zeros of the function. What do they represent?
7. Write the function in factored form.
8. **Packaging** A company packages its ink pens in a box whose length is 3 inches longer than its width and whose height is 2 inches shorter than its width. The volume of the box is 18 in^3. What are the dimensions of the box? *(Lesson 6-5)*
9. **Medicine** A medicine capsule is shaped like a cylinder with a hemisphere at each end. The cylindrical portion of the capsule is 3 cm long, and the volume is $\frac{9}{4}\pi$ cm^3. Find the radius of the capsule. *(Lesson 6-6)*



Investing **Use the following information for Exercises 10–12.**

Sharon tracked the closing value of a stock that she owns each day over a 25-day period. On average, the stock followed the curve $F(x) = -0.005x^3 + 0.05x^2 + x + 31.25$. *(Lesson 6-7)*

10. Graph the function on a graphing calculator.
11. What is the maximum value that the stock hit, and on approximately what day did it occur?
12. What is the y-intercept of the graph, and what does it signify?

School **Use the following information for Exercises 13 and 14.**

The enrollment of students at a school each year since 2000 can be modeled by the function $S(x) = -0.005x^5 + 0.07x^4 - 0.5x^2 + 278$. *(Lesson 6-8)*

13. Write the function $T(x) = S(x) - 50$.
14. Graph S and T on the same coordinate plane. Describe T as a transformation of S.
15. **Government** The table below shows the number of city employees during a 6-year period. Use a polynomial model to estimate the number of city employees in 2007. *(Lesson 6-9)*

Year	City Employees
2000	165
2001	168
2002	181
2003	210
2004	261
2005	340

Chapter 7 ▪ Applications Practice

School **Use the following information for Exercises 1–3.**

A school's honor society was founded in 1970 with 120 members. Since then, the society membership has increased by about 10% each year. *(Lesson 7-1)*

1. Write a function representing the number of members each year since the club's founding (1970 = year 0).
2. Graph the function through the year 2005.
3. In which year did the number of members exceed 1000?

Chemistry **Use the following information for Exercises 4–6.**

A glass was filled with 6 inches of water and left out on the counter. The amount of water in inches left in the glass after d days is $f(d) = 6 - 0.2d$. *(Lesson 7-2)*

4. Write the inverse function $f^{-1}(d)$.
5. After how many days was there 3.4 inches of water left in the glass?
6. After how many days was the glass empty?

Biology **Use the following information for Exercises 7 and 8.**

The number of bacteria in a culture after t hours is $f(t) = 3^{\frac{t}{2}}$. *(Lesson 7-3)*

7. How many bacteria are in the culture after 10 hours?
8. Replace $f(t)$ with y and write the function in logarithmic form.

Sound **Use the following information for Exercises 9 and 10.**

The loudness L of sound in decibels is given by $L = 10 \log\left(\frac{I}{I_0}\right)$, where I is the intensity of sound and I_0 is the intensity of the softest audible sound. *(Lesson 7-4)*

9. Rewrite this equation as the difference of two logarithms.
10. When is the equation undefined?

11. **Investing** A stock is losing value at a rate of 5% per month. An investor made an initial purchase of \$1500 worth of stock. The value of her shares of stock after m months is $A = 1500(0.95)^m$. Solve for m to find how many months it will take for the stockholder's shares to be worth less than \$1000. *(Lesson 7-5)*
12. **Economics** Ivy's parents invested \$2700 for college in an account that receives 3.5% interest compounded continuously. What will the total amount of their investment be when Ivy starts college in 8 years? *(Lesson 7-6)*
13. **Physics** Americium-241, a radioactive element used in smoke detectors, has a half-life of 7370 years. Find the decay constant, then use the decay function $N(t) = N_0\, e^{-kt}$ to determine the amount of atoms that remain from a sample of 1000 atoms after 20,000 years. *(Lesson 7-6)*

Art **Use the following information for Exercises 14–16.**

A small painting by Mondrian was valued at \$10,500 in the year 2000. Since then its value has been increasing by 3% each year. The value of the painting x years after the year 2000 is $V = 10{,}500(1.03)^x$. Write a function for each transformation described below and explain the effect on the graph of the parent function. *(Lesson 7-7)*

14. The initial value in 2000 is adjusted to \$9500.
15. The value is \$1500 more each year.
16. The value of the painting increases by 3% every 2 years.
17. **Business** The table gives the number of employees at a company in the years since it was founded. Find a logarithmic model for the data. Predict when the company will have 60 employees. *(Lesson 7-8)*

Company Employees

Years since Founding	1	2	3	4	5	6
Employees	12	26	34	40	44	48

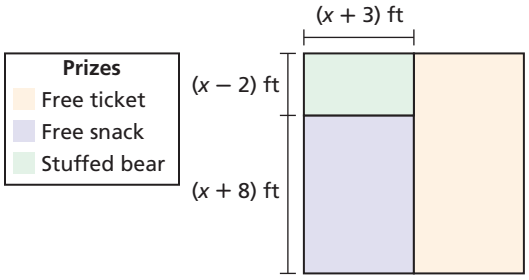
Chapter 8 ■ Applications Practice

1. **Physics** The amount of force F exerted by an object varies directly as the object's acceleration a. An object accelerating at 5 m/s^2 exerts a force of 10 Newtons. How much force would the same object exert at an acceleration of 7 m/s^2? *(Lesson 8-1)*

2. **Transportation** The time t required for a bus to travel a certain distance varies inversely as its average speed r. It takes the bus 2.2 h to travel between two cities at 50 mi/h. How long would the same drive take at 40 mi/h? *(Lesson 8-1)*

Recreation **Use the following information for Exercises 3 and 4.**

At a carnival booth, contestants can win a prize by throwing a dart at a square board. The total area of the board in square feet can be represented by the expression $4x^2 + 24x + 36$. *(Lesson 8-2)*



3. If a dart hits the board at random, what is the probability in terms of x of winning a bear?

4. If a dart hits the board at random, what is the probability in terms of x of winning a snack?

5. **Fitness** Geoff ran a 6 mi race for charity. During the first 4 mi of the race, he averaged 6 mi/h. During the last 2 mi, he averaged 5.5 mi/h. What was Geoff's average speed in miles per hour for the entire race? Round to the nearest hundredth. *(Lesson 8-3)*

School **Use the following information for Exercises 6 and 7.**

A science class is taking a field trip to a planetarium. Admission costs $8 per student, plus there is a tour charge of $80 per class. *(Lesson 8-4)*

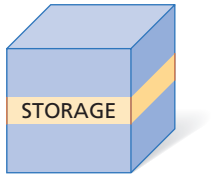
6. Write and graph a function to represent the total average cost of the field trip per student.

7. Find the total average cost per student if 25 students go on the field trip.

8. **Travel** A tour boat travels 12 mi up a river and 12 mi down the river in a total of 5.5 h. In still water, the boat travels at an average speed of 5.5 mi/h. Based on this information, what is the speed of the river's current? *(Lesson 8-5)*

9. **Carpentry** A carpenter can build a cabinet in 4 h. When his son assists him, they can build the same type of cabinet in 2.5 h. About how long would it take the carpenter's son to build a cabinet by himself? *(Lesson 8-5)*

10. **Measurement** A large cubic storage box has a volume of $166\frac{3}{8}$ ft^3. The box is labeled with a strip of tape that wraps once around the entire box. What is the length of the tape that labels the box? *(Lesson 8-6)*



Physics **Use the following information for Exercises 11 and 12.**

The period of a pendulum is the time it takes for the pendulum to complete one back-and-forth swing. The function $f(x) = 2\pi\sqrt{\frac{x}{32}}$ gives the period f of a pendulum in seconds where x is the length of the pendulum in feet. *(Lesson 8-7)*

11. Write a function g for the period of a pendulum of length $(x + 2)$ ft.

12. Describe the function g as a transformation of f.

13. **Geometry** The length of a diagonal d of a rectangular prism is given by $d = \sqrt{\ell^2 + w^2 + h^2}$, where ℓ is the length, w is the width, and h is the height. What is the minimum height in inches of a box with a length of 15 in. and a width of 12 in. that will hold a 20 in. baton? Round your answer to the nearest tenth. *(Lesson 8-8)*

Extra Practice

Chapter 9 ▪ Applications Practice

1. **Ecology** The table shows the population of a colony of penguins over a 6-year period. Use a graph and an equation to predict the number of penguins in the colony in 2010. *(Lesson 9-1)*

Penguin Colony Population	
Year	Population
2000	112
2001	123
2002	135
2003	149
2004	164
2005	180

Shipping **Use the following information for Exercises 2 and 3.**

A shipping company charges different rates depending on the weight of the package to be shipped. *(Lesson 9-2)*

Shipping Costs	
Weight (lb)	Cost ($)
Under 2	$3.50
2 to 7	$6.00
More than 7	$9.00

2. Write a piecewise function to represent shipping costs for packages up to 10 lb.
3. Graph the function.

Recreation **Use the following information for Exercises 4 and 5.**

The zoo charges $7.00 per person for admittance. For groups of 20 people or more, the zoo charges $6.00 per person plus a one-time administrative fee of $10. *(Lesson 9-3)*

4. Write a function to represent the cost of admittance to the zoo for x people.
5. The zoo decides to raise the group administrative fee by $5. Write the resulting function. How does this affect the graph of the function?

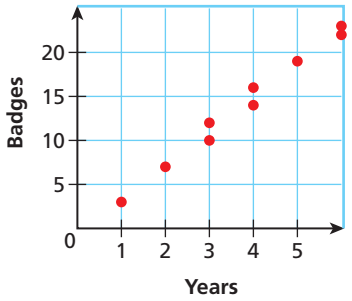
Politics **Use the following information for Exercises 6 and 7.**

Approximately 2 in 3 people surveyed support a bill to raise the salaries of local police officers. Of those who support the bill, 60% also support a raise in taxes to pay for the bill. *(Lesson 9-4)*

6. Write a composite function for the number of people who support the bill and think that taxes should be raised.
7. The total number of people who support both the bill and the tax is 90. How many people were surveyed?

Scouting **Use the following information for Exercises 8–10.**

The graph shows the number of merit badges that scouts from the same troop have earned, based on the number of years they have been in the troop. *(Lesson 9-5)*



8. Graph the inverse of $f(x)$.
9. Is $f(x)$ a function? Is its inverse a function?
10. Use your graph to predict how long a scout has been in the troop if he has earned 36 badges.

Nutrition **Use the following information for Exercises 11 and 12.**

The table shows the number of Calories and grams of fat in selected sandwiches. *(Lesson 9-6)*

Sandwich Nutrition Information					
Fat (g)	6	8	12	15	20
Calories	372	396	442	477	535

11. Write a function that models the data.
12. Use your model to predict the number of Calories in a sandwich containing 25 grams of fat.

Chapter 10 ▪ Applications Practice

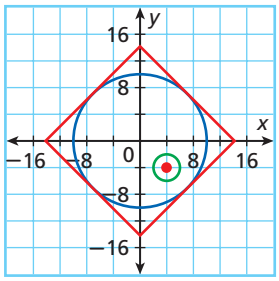
Geometry **Use the following information for Exercises 1–3.**

A circle has center $(7, 8)$ and contains the point $(11, 11)$. *(Lesson 10-1)*

1. Find the circumference of the circle.
2. Find the area of the circle.
3. Find the other endpoint of the diameter with one endpoint $(11, 5)$.

Design **Use the following information for Exercises 4–6.**

Grace is designing a courtyard for a client. The courtyard will include a small circular fountain inside a large circular patio, which will be surrounded by a square fence. The plans have been overlaid on a coordinate plane, as shown below. *(Lesson 10-2)*

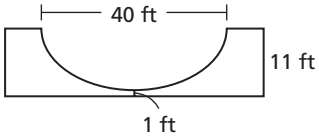


4. Find an equation for the fountain.
5. Find an equation for the circular patio.
6. Each side of the fence is tangent to the patio. Find the equation for the part of the fence that passes through the point $\left(-5\sqrt{2}, 5\sqrt{2}\right)$.

Architecture **Use the following information for Exercises 7 and 8.**

The Oval Office in the White House has a major axis 35 ft 10 in. long and a minor axis 29 ft long. *(Lesson 10-3)*

7. Suppose that the center of the floor of the Oval Office is located at the origin. Write an equation that can be used to model the office floor.
8. Find the coordinates of the foci.
9. **Sports** Two people watching a baseball game are seated 2000 feet apart. One person hears the crack of the bat 1 second before the other person. Because sound travels at 1100 feet per second, one person must be 1100 feet closer to the bat than the other. The possible locations of the batter form a hyperbola with the two people as foci. Write an equation that could be used to represent the possible locations of the batter. (*Hint:* Place the origin midway between the two people.) *(Lesson 10-4)*
10. **Recreation** A half-pipe, similar to those used by skateboarders, is parabolic in shape. Use the intersection of the ground and the center of the half-pipe as the origin and write an equation to model the shape of the curved interior of the structure. *(Lesson 10-5)*



Fitness **Use the following information for Exercises 11 and 12.**

A runner is running on a track. His path in yards can be modeled by the equation $x^2 - 160x + 4y^2 - 160y + 7840 = 0$. *(Lesson 10-6)*

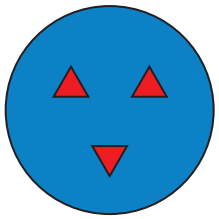
11. Write the equation in standard form by completing the square.
12. There is a drinking fountain in the center of the track. What are the coordinates of the fountain? What is the farthest distance that a runner would have to travel from the track to the fountain?
13. **Ecology** A water tank has spilled, and the flooded area in square feet can be modeled by the equation $x^2 + y^2 = 225$. Near the spilled tank, there is a garden whose shape can be modeled by the equation $\frac{(x-9)^2}{25} + \frac{(y-6)^2}{36} = 1$. At what points do the boundaries of the spill and the garden intersect? *(Lesson 10-7)*

Chapter 11 ▪ Applications Practice

Music **Use the following information for Exercises 1 and 2.**

Serialism is a form of music in which the composer arranges each of the 12 tones in an octave to form a musical phrase. *(Lesson 11-1)*

1. How many ways can the 12 tones of an octave be arranged?
2. How many different musical phrases could a composer create by arranging only 5 of the 12 tones of an octave?
3. **Drama** A drama class is performing the Greek tragedy *Antigone,* by Sophocles. Of the 15 students in the class, 6 will make up the chorus. How many different ways can the chorus be selected? *(Lesson 11-1)*
4. **Holidays** Of December's 31 days, the 25th and the 31st are holidays. What is the probability that a randomly chosen day in December is not a holiday? *(Lesson 11-2)*
5. **Games** If Sara's dart lands in a red equilateral triangle, she wins a prize. Each triangle has a base of 2 in. If all locations on the 12 in. diameter target are equally likely, what is the probability that Sara wins a prize? *(Lesson 11-2)*



Literature **Use the following information for Exercises 6 and 7.**

The works of Chilean poet Pablo Neruda have been published in many languages. The school library has copies of two of his books in both English and Spanish. The table shows how many times each book has been checked out. *(Lesson 11-3)*

Books Checked Out		
	Canto General	*Extravagario*
English	23	27
Spanish	17	14

6. What is the probability that *Canto General* was checked out in Spanish?
7. What is the probability that a student who checked out a Pablo Neruda book selected *Extravagario* in English?

Immigration **Use the following information for Exercises 8 and 9.**

A group of 100 immigrants was studied over a one-year period. During the study, 63 of the immigrants found jobs, and 14 returned to their country of origin. Of the immigrants who found jobs, 6 of them returned to their countries before the end of the study. *(Lesson 11-4)*

8. What is the probability that an immigrant found a job or returned to his or her country of origin?
9. What is the probability that an immigrant did not find a job or returned to his country of origin?

Basketball **Use the following information for Exercises 10–12.**

The table below shows the number of points scored by Tracy McGrady and Yao Ming of the Houston Rockets during the same 5 games of the 2005 season. *(Lesson 11-5)*

Points Scored					
Game	1	2	3	4	5
Tracy McGrady	28	36	25	37	27
Yao Ming	15	20	30	8	33

10. Find the mean of both sets of data.
11. Find the standard deviation of both sets of data.
12. Determine whether there is an outlier. If so, describe how it affects the mean and standard deviation.

Nutrition **Use the following information for Exercises 13 and 14.**

At a frozen yogurt store, 75% of customers ask for a cup, while the others ask for a cone. At closing time, the store has 7 people waiting and only 2 cones left. *(Lesson 11-6)*

13. What is the probability that exactly 2 people will want cones?
14. What is the probability that no more than 2 people will want cones?

Chapter 12 ▪ Applications Practice

Housing **Use the following information for Exercises 1 and 2.**

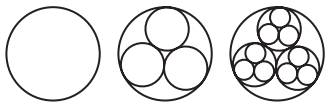
Lily moved into her apartment in 2001, when the rent was $650. Every year since then, the landlord has raised the rent by 5%. *(Lesson 12-1)*

1. Graph the sequence and describe its pattern.
2. How much will Lily's rent be in 2010?
3. **Fractals** Find the number of red circles in the next 2 terms of the fractal. *(Lesson 12-1)*

4. **Awards** A local charity started its Volunteer Hall of Fame by inducting the first honoree in 1997. The next year it inducted 2 new members, and in 1999 it inducted 3 new members. Each year since then, it has added one more member than it did the previous year. How many members will the Volunteer Hall of Fame have in 2009? *(Lesson 12-2)*

Fractals **Use the following information for Exercises 5 and 6.**

The number of circles in the first iteration of the fractal is $3^0 = 1$. The number of circles in the second iteration of the fractal is $3^0 + 3^1 = 4$. The number of circles in the third iteration of the fractal is $3^0 + 3^1 + 3^2 = 13$. *(Lesson 12-2)*



5. Use summation notation to write an expression for the number of circles in the nth iteration of the fractal.
6. Find the number of circles in the 5th iteration of the fractal.

Fitness **Use the following information for Exercises 7–9.**

When a member first joins a health club, he or she pays $240 for the first year. Each year after that, the yearly fee is reduced by $10. *(Lesson 12-3)*

7. What is the yearly fee for the 7th year?
8. How much will a member have paid after belonging to the health club for 10 years?
9. If a member has paid a total of $2450 in fees, how long has she been a member of the health club?

Communication **Use the following information for Exercises 10–12.**

The Parent Teacher Association spreads news using a phone tree. The president and vice president start the phone tree by calling 3 people each. Each of the 6 people called then have 3 new people to call, and so on, until every member of the PTA has been called. *(Lesson 12-4)*

10. Write a sequence to describe the phone tree.
11. How many people are on the 5th row of the phone tree?
12. It takes a total of 6 rows to finish the phone tree. Write an expression in summation notation to express the number of people called in the entire phone tree. How many members does the Parent Teacher Association have?

Business **Use the following information for Exercises 13 and 14.**

The table shows the annual revenue generated by a new product in its first 4 years. *(Lesson 12-5)*

Annual Revenue				
Year	2001	2002	2003	2004
Sales (thousand $)	375	225	135	81

13. Assume that the trend continues. Estimate the revenue generated in 2008.
14. Assume that the sales trend continues indefinitely. Estimate the total revenue the product will generate.

Extra Practice

Chapter 13 ■ Applications Practice

1. **Aviation** A plane is flying at an altitude of 6500 ft. The pilot sights the runway of an airport at an angle of depression of 6°. To the nearest tenth of a mile, what is the horizontal distance from the plane to the runway? *(Lesson 13-1)*

2. **Architecture** Thomas stands 250 m from the base of the Sears Tower in Chicago. His eye level is 1.75 m above the ground, and he measures the angle of elevation to the top of the tower to be 60.4°. Based on this information, what is the height of the Sears Tower to the nearest meter? *(Lesson 13-1)*

Recreation **Use the following information for Exercises 3 and 4.**

A Ferris wheel makes one complete revolution in 40 s. *(Lesson 13-2)*

3. Through what angle, in degrees, does a car of the Ferris wheel rotate in 70 s?

4. How long does it take a car of the Ferris wheel to rotate through an angle of 792° ?

5. **Landscape Design** A path through a park is shaped like an arc of a circle with a radius of 25 ft. The central angle that intercepts the path measures $\frac{\pi}{2}$ radians. To the nearest foot, how long is the path? *(Lesson 13-3)*

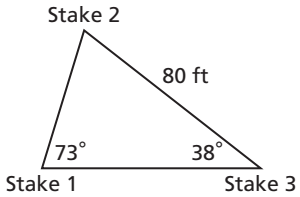
6. **Entertainment** A standard circus ring is 42 ft in diameter. A clown on a bicycle rides once around the circumference of the ring in 10 s. To the nearest tenth of a foot, how far does the clown travel in 1 second? *(Lesson 13-3)*

7. **Astronomy** Venus is approximately 108 million km from the Sun and takes 225 days to complete an orbit. Based on this information, how far does Venus travel in its nearly circular orbit around the Sun in 1 day? Round to the nearest million kilometers. *(Lesson 13-3)*

8. **Construction** The entrance to a store is 6 in. above the level of the sidewalk. A contractor is building an access ramp to the entrance that will cover a horizontal distance of 6 ft. To the nearest degree, what angle will the ramp make with the sidewalk? *(Lesson 13-4)*

9. **Safety** The "1-to-4" rule states that when a ladder is leaning against a wall, the bottom of the ladder should be 1 ft away from the wall for every 4 ft that the top of the ladder rises on the wall. To the nearest degree, what angle should the ladder make with the ground? *(Lesson 13-4)*

10. **Surveying** A surveyor is measuring a triangular plot of land, as shown. To the nearest foot, what is the distance between stakes 1 and 2? *(Lesson 13-5)*



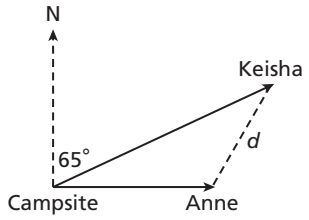
Hobbies **Use the following information for Exercises 11 and 12.**

Andrew uses pieces of wood to build triangular picture frames. Determine the number of triangles he can form using the given side and angle measurements. Then solve the triangles. Round to the nearest tenth. *(Lesson 13-5)*

11. $a = 10.5$ cm, $b = 12$ cm, $m\angle A = 60°$

12. $a = 8$ cm, $b = 15$ cm, $m\angle A = 44°$

13. **Hiking** Anne and Keisha leave their campsite at the same time. Anne hikes due east at 2 mi/h. Keisha heads 65° east of north at 3 mi/h. To the nearest tenth of a mile, what is the distance d between the hikers after 3 hours? *(Lesson 13-6)*



14. A museum has a triangular window with sides measuring 9 ft, 11 ft, and 14 ft. What is the area of the window to the nearest square foot? *(Lesson 13-6)*

Chapter 14 ▪ Applications Practice

1. **Sound** Use a sine function to graph a sound wave with a period of 0.006 second and an amplitude of 5 cm. Find the frequency in hertz for this sound wave. *(Lesson 14-1)*

Recreation **Use the following information for Exercises 2–4.**

As a cyclist rides her bike, the height in inches above the ground of one of the pedals is modeled by $H(t) = 6\cos 2\pi t + 12$, where t is the time in seconds. *(Lesson 14-1)*

2. Graph the height of the pedal for two complete periods.
3. What is the maximum and minimum height of the pedal?
4. How many complete revolutions does the pedal make in one minute?

Use the following information for Exercises 5 and 6.

As a swimming pool is drained, the depth of the water in feet is modeled by $D(t) = 1.05 \cot \frac{\pi}{8}\left(t + \frac{1}{2}\right)$, where t is the time in hours. *(Lesson 14-2)*

5. Graph the depth of the water in the swimming pool for $0 \le t \le 3$.
6. What is the starting depth of the water? Round to the nearest inch.

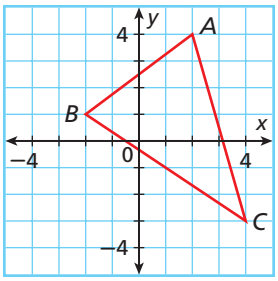
Use the following information for Exercises 7 and 8.

The minute hand of a clock begins on the 12 and moves around the dial. The slope of the line represented by the minute hand is given by the function $S(t) = -\tan 2\pi(t - 0.25)$, where t is the time in hours. *(Lesson 14-2)*

7. Graph the slope of the minute hand for six complete periods.
8. What is the period of the function?
9. **Physics** Use the equation $mg \sin\theta = \mu mg \cos\theta$ to determine the angle at which a steel table can be tilted before a copper pan on the table begins to slide. Assume $\mu = 0.53$ and round your answer to the nearest degree. *(Lesson 14-3)*

Geometry **Use the following information for Exercises 10 and 11.**

Find the coordinates, to the nearest hundredth, of the vertices of $\triangle ABC$ after the given rotation. *(Lesson 14-4)*



10. A 60° rotation about the origin
11. A 135° rotation about the origin

Physics **Use the following information for Exercises 12 and 13.**

The horizontal component of the acceleration of an object sliding down a frictionless inclined plane is $a(\theta) = 9.8 \sin\theta \cos\theta$, where θ is the angle of the inclined plane and where acceleration is measured in meters per second per second $\left(\frac{m}{s^2}\right)$. *(Lesson 14-5)*

12. Rewrite the function in terms of the double angle 2θ.
13. Graph the function for $0 \le \theta \le \frac{\pi}{2}$. For what angle does the object have the greatest acceleration in the horizontal direction?
14. The population in thousands of a seaside town is modeled by $P(t) = 10 \sin\frac{\pi}{180}(t - 160) + 15$, where t is the day of the year and $t = 0$ represents January 1. How many days after January 1 is the population equal to 22,000? *(Lesson 14-6)*
15. The temperature in New York City during one day in the summer is modeled by $F(t) = 16 \sin\frac{\pi}{12}(t - 8) + 68$, where F is the temperature in degrees Fahrenheit and t is the time in hours after midnight. At what times during the day is the temperature 80°F? *(Lesson 14-6)*

Problem Solving Handbook

Draw a Diagram

You can draw a diagram that represents the information in a problem to help you understand and solve the problem.

Problem Solving Strategies

Draw a Diagram	Make a Table
Make a Model	Solve a Simpler Problem
Guess and Test	Use Logical Reasoning
Work Backward	Use a Venn Diagram
Find a Pattern	Make an Organized List

EXAMPLE

Carmen is participating in an online contest to win a car. She is given a choice of three doors. Each door leads to another level with three doors, and each of those leads to another level with three doors. Behind one of those final doors is the grand prize of a new car. What is the probability of winning the car if she chooses doors at random?

1 Understand the Problem

List the important information.

- She begins with three doors.
- Each of those doors leads to three other doors, and each of those doors leads to another three doors.
- One of the final doors leads to the car.
- The probability of winning is the number of cars divided by the total number of final door choices.

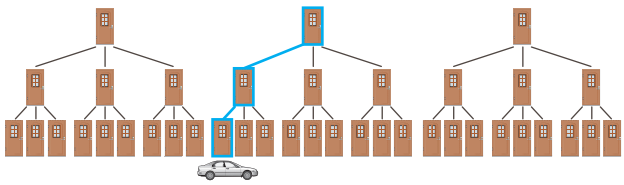
The answer will be the probability of finding the car.

2 Make a Plan

Use a tree diagram to show the possible door choices. This will show the number of possible paths Carmen could choose.

3 Solve

Draw the tree diagram. Draw three doors to represent the original three doors. Draw three more doors for each door and connect them with lines. Repeat to have three rows of doors.



The highlighted path shows that there is exactly one way to win the car.

The total number of paths is 27.

The probability of winning the car is $\frac{1}{27} \approx 3.7\%$.

4 Look Back

Check that you drew your diagram correctly. Does the diagram accurately represent the information given in the question?

PRACTICE

1. Bob has a green, a blue, a red, and a yellow marble in a bag. He randomly selects one marble at a time from the bag until the bag is empty. What is the probability that the blue marble is chosen immediately before the red one?

Make a Model

For problems that involve objects, it is sometimes useful to make a model to help you solve the problem.

Problem Solving Strategies

Draw a Diagram	Make a Table
Make a Model	Solve a Simpler Problem
Guess and Test	Use Logical Reasoning
Work Backward	Use a Venn Diagram
Find a Pattern	Make an Organized List

EXAMPLE

Ryan created a pyramid with a square base out of cans of soup for a store display. When he had finished, he needed to know the number of cans that he had used. The pyramid has four levels. The top level has one can, and each row beneath it has one additional can added to each side length. How many cans are in the pyramid?

1 Understand the Problem

List the important information.

- There are four levels.
- The top level has one can.
- The side length increases by one as you go down each level.

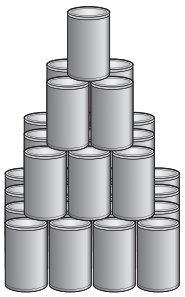
The answer will be the number of cans in the pyramid.

2 Make a Plan

You can use blocks to make a model of the problem. Use the blocks to create the pyramid in the problem. Remember to count the number of blocks as you go.

3 Solve

Since each level has one can added to the side length, the side length of the bottom level is 4. Make a 4-by-4 square of blocks for the base. Count the number of blocks used. The level above has side lengths of 3, so make a 3-by-3 square on top of the base. Count the number of blocks used in this level. Continue to the top of the pyramid. The total number of blocks is the sum of the blocks at each level: $16 + 9 + 4 + 1 = 30$.



4 Look Back

Make sure the pyramid matches the given information. There should be four levels increasing by one in side length as you go down the pyramid.

PRACTICE

1. Paul wants to make a pyramid with an equilateral triangle base out of cans. Paul has 25 cans. He wants the pyramid to have one can on the top, and he wants the number of cans on each side of the following triangle layers to increase by one. How tall can Paul make the pyramid? How many cans will he have left over?
2. A display of cereal boxes is arranged with 1 box on top and each row having an additional box. How many boxes are in a display of 9 rows?

Guess and Test

One way to solve a problem is to guess the answer and test to see whether it is correct. You can continue to guess and test until you find the correct answer.

Problem Solving Strategies

Draw a Diagram	Make a Table
Make a Model	Solve a Simpler Problem
Guess and Test	Use Logical Reasoning
Work Backward	Use a Venn Diagram
Find a Pattern	Make an Organized List

EXAMPLE

Tom is playing a game where he draws marbles out of a bag. Red marbles are worth 3 points, and blue ones are worth 2 points. Tom drew 8 marbles and won 20 points. How many marbles of each color does Tom have?

1 Understand the Problem

List the important information.

- Red marbles are worth 3 points.
- Blue marbles are worth 2 points.
- The total number of points is 20.
- The total number of marbles is 8.

2 Make a Plan

Start with a guess in which the total number of marbles is 8. Test to see whether the total number of points is 20.

3 Solve

Make a first guess of 3 red and 5 blue, and find the total number of points.

Guess: 3 red and 5 blue

Test: $(3 \times 3) + (5 \times 2) = 19$

The number of points is too small. Increase the number of red marbles and decrease the number of blue marbles.

Guess: 5 red and 3 blue

Test: $(5 \times 3) + (3 \times 2) = 21$

The number of points is too high. Decrease the number of red marbles and increase the number of blue marbles.

Guess: 4 red and 4 blue

Test: $(4 \times 3) + (4 \times 2) = 20$

Tom should have drawn 4 red marbles and 4 blue marbles.

4 Look Back

Test the answer to see whether the number of marbles satisfies the question.

4 red marbles and 4 blue marbles are 8 marbles and are worth 20 points.

PRACTICE

1. Fred has 7 coins. All the coins are nickels or dimes. The total value of the coins is \$0.55. How many of each type of coin does he have?
2. The sum of Beth's age and Brian's age is 20. Three times Beth's age plus 2 times Brian's age is 55. How old are Beth and Brian?

Work Backward

Sometimes in a problem you are given an end result and asked to find a fact that leads to the result. In these cases, you can work backward to solve the problem.

Problem Solving Strategies

Draw a Diagram	Make a Table
Make a Model	Solve a Simpler Problem
Guess and Test	Use Logical Reasoning
Work Backward	Use a Venn Diagram
Find a Pattern	Make an Organized List

EXAMPLE

Laura is delivering meals to retirement communities. She dropped off 2 less than $\frac{1}{2}$ of the meals at the first community. Then she dropped off $\frac{1}{3}$ of the remaining meals plus 2 at the second community. She has 8 meals left. How many meals did she have to start?

1 Understand the Problem

List the important information.

- Laura delivered $\frac{1}{2}$ of the meals minus 2 at the first community.
- Laura delivered $\frac{1}{3}$ of the meals plus 2 at the second community.
- She has 8 meals left.

The answer will be the number of meals that she had at the start.

2 Make a Plan

Start with the 8 meals and work backward through the given information to determine the beginning number of meals.

3 Solve

She has 8 meals at the end, so start with 8 meals.

She delivered $\frac{1}{3}$ of the meals plus 2 at the second community, so add 2 to the number of meals and multiply by $\frac{3}{2}$ to undo giving $\frac{1}{3}$ away. $\frac{3}{2}(8 + 2) = 15$

She had 15 meals before she visited the second community.

She delivered $\frac{1}{2}$ of the meals minus 2 at the first community, so subtract 2 and multiply by 2 to undo giving $\frac{1}{2}$ away. $2(15 - 2) = 26$

Laura started with 26 meals.

4 Look Back

Use the starting amount of 26 meals and work from the beginning of the problem following the steps.

Start: 26

Subtract $\frac{1}{2}$ of the meals plus 2 more: 15

Subtract $\frac{2}{3}$ of the meals minus 2 more: 8

PRACTICE

1. A tree is growing in Danny's yard. When Danny first observed the tree, he noticed that the number of branches on the tree had doubled that year. The year after, the number of branches tripled minus 3. The year after that, the tree doubled its number of branches, plus 6. How many branches did the tree originally have if it currently has 120 branches?

Find a Pattern

When the pieces of information in a problem have a relationship, you can find a pattern to help solve the problem.

Problem Solving Strategies

Draw a Diagram	Make a Table
Make a Model	Solve a Simpler Problem
Guess and Test	Use Logical Reasoning
Work Backward	Use a Venn Diagram
Find a Pattern	Make an Organized List

EXAMPLE

Fred has 3 homework problems the first day of school. The second day he has 5. The third day he has 7. The fourth day he has 9. If this pattern continues, how many homework problems will Fred have on the tenth day of school?

1 Understand the Problem

List the important information.

- On day 1 he has 3 homework problems, on day 2 he has 5 homework problems, on day 3 he has 7 homework problems, and on day 4 he has 9 homework problems.

The answer will be the number of homework problems Fred will have on day 10.

2 Make a Plan

Find a pattern by comparing the number of homework problems Fred has each day. Then use this pattern to determine the number of homework problems he will have on day 10.

3 Solve

Organize the data and find the pattern.

Day	Number of Homework Problems	Pattern
1	3	$3 + 2(1 - 1)$
2	5	$3 + 2(2 - 1)$
3	7	$3 + 2(3 - 1)$
4	9	$3 + 2(4 - 1)$

The pattern is that he gains 2 homework problems each day. Since he has 3 problems the first day and the number of days that have passed is the day number minus 1, the number of homework problems Fred has on the day n is $3 + 2(n - 1)$.

The number of homework problems Fred will have on day 10 is $3 + 2(10 - 1) = 21$.

4 Look Back

Since the pattern is that he gains 2 homework problems each day, continue the data in a table to make sure that he will have 21 homework problems on the tenth day. Check that the formula you developed satisfies the information given in the question.

PRACTICE

1. Joseph is making signs for his student council election campaign. He made 1 sign the first day, 4 signs the second day, 7 signs the third day, and 10 signs the fourth day. How many signs will he make on the tenth day?
2. A flower is growing in a field. In year 1 there are two flowers in the field, in year 2 there are 3, in year 3 there are 5, in year 4 there are 9, in year 5 there are 17, and in year 6 there are 33. How many flowers will there be in year 11?

Make a Table

When you are solving problems that involve a large amount of data, it is often useful to make a table to organize and analyze the data.

Problem Solving Strategies

Draw a Diagram	**Make a Table**
Make a Model	Solve a Simpler Problem
Guess and Test	Use Logical Reasoning
Work Backward	Use a Venn Diagram
Find a Pattern	Make an Organized List

EXAMPLE

Peter, Michael, and Lisa work at the same shop. Peter works every 2 days, Michael works every 4 days, and Lisa works every 5 days. They all worked today. In how many days will they all work together again? How many days will each person work between now and when they all work together next?

1 Understand the Problem

List the important information.

- Peter works every 2 days, Michael every 4 and Lisa every 5.
- They all worked together today.

The answers will be:

- the number of days until they work together again and
- the number of days each person will work between now and then.

2 Make a Plan

Make a table, using ✔'s to show the days each person works.

3 Solve

Start with a ✔ in each person's row on day 0. For Peter, place a ✔ every 2 days. For Michael, place a ✔ every 4 days. For Lisa, place a ✔ every 5 days.

Day	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Peter	✔		✔		✔		✔		✔		✔		✔		✔		✔		✔		✔
Michael	✔				✔				✔				✔				✔				✔
Lisa	✔					✔					✔					✔					✔

They will all work together again in 20 days. Peter will work nine, Michael will work 4, and Lisa will work 3 days between now and then.

4 Look Back

Check the information in the table. Make sure that no mistakes have been made in counting and that the data matches the information given in the question.

PRACTICE

1. If Peter works every 3 days, Michael works every 5 days, and Lisa works every 6 days, when will the next day be that they all work together if they all worked together today? How many days will each person work between now and when they all work together next?
2. A restaurant receives a shipment of produce every 2 days, a shipment of meat every 9 days, and a shipment of frozen food every 12 days. When will be the next day that all three shipments arrive if all three shipments arrived today? How many of each type of shipment will the restaurant receive between now and then.

Solve a Simpler Problem

When solving a complex problem, it is sometimes helpful to write a simpler problem, solve it, and then use a similar method to solve the complex problem.

Problem Solving Strategies

Draw a Diagram	Make a Table
Make a Model	**Solve a Simpler Problem**
Guess and Test	Use Logical Reasoning
Work Backward	Use a Venn Diagram
Find a Pattern	Make an Organized List

EXAMPLE

In a garden, there are 2 flowers, 1 red and 1 blue. Each year, the number of red flowers increases by 1 and the number of blue flowers increases by 2. What percent of the flowers will be red 10 years from now?

1 Understand the Problem

List the important information.

- The field begins with 1 red and 1 blue flower.
- Each year, the number of red flowers increases by 1.
- Each year, the number of blue flowers increases by 2.

The answer will be the percent of red flowers after 10 years have passed.

2 Make a Plan

Solve a simpler problem: Find the pattern in the number of red flowers and the total number of flowers in ten years.

3 Solve

Make a table. Separate the two patterns. Identify each pattern and develop a formula.

Year	Red Flowers	Pattern	Blue Flowers	Total Flowers	Pattern
0	1	$1 + (0)$	1	2	$2 + 3(0)$
1	2	$1 + (1)$	3	5	$2 + 3(1)$
2	3	$1 + (2)$	5	8	$2 + 3(2)$
3	4	$1 + (3)$	7	11	$2 + 3(3)$

If n is the nth year, then the number of red flowers is $1 + n$ and the total number of flowers is $2 + 3n$. The percent of flowers that are red is the ratio of the number of red flowers to the total number of flowers.

So the percent of red flowers in the nth year is $\frac{1 + n}{2 + 3n}$, and in 10 years the percent of flowers that are red is $\frac{1 + 10}{2 + 3(10)} = \frac{11}{32} = 34.375\%$.

4 Look Back

Check that the answer is reasonable. Since the blue flowers grow faster than the red flowers and the garden starts with an equal number of each, there should be more blue flowers than red flowers. Therefore, the percent of red flowers should be less than 50%.

PRACTICE

1. In a field of flowers, there are 2 flowers; 1 yellow and 1 orange. Each year, the number of yellow flowers increases by 3, and the number of orange flowers increases by 4. What percent of the flowers will be yellow in 20 years?

Use Logical Reasoning

Use logical reasoning to help you solve problems by identifying the facts and using them to draw conclusions.

Problem Solving Strategies

Draw a Diagram	Make a Table
Make a Model	Solve a Simpler Problem
Guess and Test	**Use Logical Reasoning**
Work Backward	Use a Venn Diagram
Find a Pattern	Make an Organized List

EXAMPLE

Friends Jeff, Luca, Linda, and Blair are each a different age between 11 and 14. Each person has a different favorite color and sport. The sports are baseball, football, tennis and hockey. The colors are red, blue, green, and yellow. Jeff is 11 and likes to play baseball. The football player is the oldest and dislikes red. The hockey player's favorite color is blue. Linda does not play football. Blair likes the color green, plays tennis, and is a year younger than Linda. Find each person's age, favorite color, and sport.

1 Understand the Problem

List the important information.

- Jeff is 11 and likes to play baseball.
- The football player is the oldest and dislikes red.
- The hockey player's favorite color is blue.
- Linda does not play football.
- Blair likes the color green, plays tennis, and is a year younger than Linda.

The answer will be each person's age, favorite color, and sport.

2 Make a Plan

Start with the given clues. Use logical reasoning to make a table of the facts.

3 Solve

Make a table. Work with the clues one at a time. Place a ✔ in a box if the clue matches the person and an ✗ if it does not.

	R	Bl	G	Y	Ba	F	T	H	11	12	13	14
Jeff	✔	✗	✗	✗	✔	✗	✗	✗	✔	✗	✗	✗
Luca	✗	✗	✗	✔	✗	✔	✗	✗	✗	✗	✗	✔
Linda	✗	✔	✗	✗	✗	✗	✗	✔	✗	✗	✔	✗
Blair	✗	✗	✔	✗	✗	✗	✔	✗	✗	✔	✗	✗

Jeff is 11, plays baseball, and likes red. Luca is 14, plays football, and likes yellow. Linda is 13, plays hockey, and likes blue. Blair is 12, plays tennis, and likes green.

4 Look Back

Compare your answer to the clues in the problem. Make sure none of the conclusions conflict with the clues.

PRACTICE

1. Friends Bob, Gary, Roxanne, and Robin have last names that begin with the letters *B, S, T,* and *H*. Their ages are 10, 12, 14, and 16, and their hair colors are blond, black, brown, and red. Bob's last initial is *B.* Robin is a teenager. Roxanne does not have red hair. Bob is 2 years older than Roxanne. The oldest has the last initial *S* and brown hair. Gary's last initial comes before Roxanne's in the alphabet. Gary is 10 and has black hair. Find each person's last initial, age, and hair color.

Use a Venn Diagram

Venn diagrams can be useful in solving problems with sets that overlap each other.

Problem Solving Strategies

Draw a Diagram	Make a Table
Make a Model	Solve a Simpler Problem
Guess and Test	Use Logical Reasoning
Work Backward	**Use a Venn Diagram**
Find a Pattern	Make an Organized List

EXAMPLE

There were three science lectures that students could attend, one on physics, one on chemistry, and one on biology. Four students attended all lectures, 6 students went to both the biology and physics lectures, 10 students went to both the chemistry and physics lectures, and 16 students went to both the chemistry and biology lectures. If a total of 30 students attended the physics lecture, 50 students attended the chemistry lecture, and 60 students attended the biology lecture, how many students went to at least one lecture?

1 Understand the Problem

List the important information.

- all lectures: 4
- biology and physics: 6
- chemistry and physics: 10
- chemistry and biology: 16
- physics: total of 30
- chemistry: total of 50
- biology: total of 50

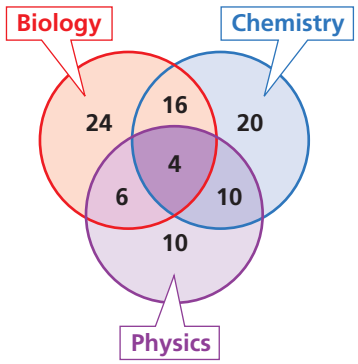
The answer will be the number of students that went to at least one lecture.

2 Make a Plan

Use a Venn diagram to show the number of students that attended each lecture.

3 Solve

Draw and label three overlapping circles. In the section where all the circles overlap, place a 4 because 4 students attended all the lectures. In each section where only two circles overlap, place the number of students that went to both those two lectures. Calculate the number of students that went to only one lecture by taking the number of students that attended each lecture and subtracting the number that also attended other lectures. The sum of the numbers in each circle should be the total number of students that attended that lecture.



The total number of students is the sum of the numbers in all the circles.

Therefore, the number of students is $24 + 6 + 10 + 16 + 4 + 10 + 20 = 90$.

4 Look Back

Check your Venn diagram against the initial data to make certain that the diagram agrees with the question asked.

PRACTICE

1. In summer school the math courses offered were Algebra, Geometry, and Calculus. Three students took all three courses, 5 students took only Algebra and Geometry, 1 student took only Calculus and Geometry, and 10 students took only Algebra and Calculus. If there were 28 students in Algebra, 24 students in Geometry, and 30 students in Calculus, how many students took at least one math course?

Make an Organized List

When you are solving a problem that contains a lot of information, it may be helpful to make an organized list to record the possible outcomes.

Problem Solving Strategies

Draw a Diagram	Make a Table
Make a Model	Solve a Simpler Problem
Guess and Test	Use Logical Reasoning
Work Backward	Use a Venn Diagram
Find a Pattern	**Make an Organized List**

EXAMPLE

Pete's Pizza has four toppings to choose from: pepperoni, ham, extra cheese, and mushrooms. How many possible pizzas are there if you can have 0, 1, 2, 3, or 4 toppings and cannot get the same topping twice?

1 Understand the Problem

List the important information.

- There are 4 possible toppings.
- A pizza can have 0 to 4 toppings.
- You cannot have the same topping twice.

The answer will be the number of pizzas that are possible.

2 Make a Plan

Make an organized list of the possible combinations of toppings. List all the possible combinations.

3 Solve

Make a column for each number of toppings on the pizza. Let P = pepperoni, H = ham, C = extra cheese, and M = mushrooms.

0 Toppings	1 Topping	2 Toppings	3 Toppings	4 Toppings
Plain Cheese	P	PH	PHC	PHCM
	H	PC	PHM	
	C	PM	PCM	
	M	HC	HCM	
		HM		
		CM		

Adding the number of choices yields 16 possible pizzas.

4 Look Back

Make sure all the possible choices are shown in the table and that none repeat.

PRACTICE

1. Pete's Pizza has decided that customers may repeat toppings but choose no more than a total of two toppings per pizza. How many pizzas are now possible?
2. Calvin has a bag with 5 balls inside. The balls are all distinct and labeled A through E. How many three-letter "words" can he create by randomly removing a ball and not replacing it? (Consider a word to be any permutation of three letters.)
3. Matty is going to run some errands. She may stop by the cleaners, the video store, and the grocery store. If she plans to make at least 1 stop, how many possible routes can she take?

Skills Bank

Estimation

You can use estimation to find approximate values and determine whether your answers are reasonable.

EXAMPLE 1 **Estimate 4218 + 788.**

Round each value to a number that is easy to add.

$4218 + 788 \approx 4200 + 800 = 5000$ *Round 4218 to 4200 and 788 to 800. Add.*

4128 + 788 is about 5000.

EXAMPLE 2 **Estimate 157 ÷ 28.**

Round each value to a number that is easy to divide and will leave no remainder.

$157 \div 28 \approx 150 \div 30 = 5$ *Round 157 to 150 and 29 to 30. Divide.*

157 ÷ 28 is about 5.

PRACTICE

Estimate.

1. 12.616 + 16.791 **2.** 11,624 + 396 **3.** 32.56 + 108.44 **4.** 12.84 − 6.11

5. 6581 − 477 **6.** 533 − 29.1 **7.** 106 • 69 **8.** 5.23 • 14.86

9. 215 • 19 **10.** 7.86 ÷ 1.94 **11.** 18,274 ÷ 1011 **12.** 561 ÷ 47

Percent Increase and Decrease

Percent change is an increase or decrease given as a percent of the original amount. Percent increase describes an amount that has grown. Percent decrease describes an amount that has been reduced.

EXAMPLE 1 **Find the percent increase or decrease from 24 to 31.2.**

$$\frac{\text{amount of change}}{\text{original amount}} = \frac{31.2 - 24}{24} = \frac{7.2}{24} = 0.3 = 30\%$$

From 24 to 31.2 is a 30% increase.

EXAMPLE 2 **Find the percent increase or decrease from 8.2 to 6.97.**

$$\frac{\text{amount of change}}{\text{original amount}} = \frac{8.2 - 6.97}{8.2} = \frac{1.23}{8.2} = 0.15 = 15\%$$

From 8.2 to 6.97 is a 15% decrease.

PRACTICE

Find each percent increase or decrease.

1. from 36 to 43.2 **2.** from 100 to 19 **3.** from 5.5 to 7.26 **4.** from 42 to 39.9

5. from 220 to 327.8 **6.** from 9 to 10.35 **7.** from 0.66 to 0.594 **8.** from 78 to 25.74

9. from 685 to 506.9 **10.** from 11 to 12.43 **11.** from 1.54 to 2.31 **12.** from 51 to 13.77

Accuracy, Precision, and Error

The accuracy of a measurement refers to how close the measurement is to the actual value of the quantity. The precision of a measurement refers to the number of significant digits in the measured value. Use relative error to make judgments about measurements. Recall that relative error $= \frac{\text{measurement} - \text{actual value}}{\text{actual value}}$.

EXAMPLE **A 1.29-foot-long object was measured as both 1.23 feet and 1.3 feet. Which measurement was more accurate? Which was more precise?**

Calculate the relative errors.

$$\text{Error for 1.23 ft} = \frac{1.23\text{ ft} - 1.29\text{ ft}}{1.29\text{ ft}} \approx -0.465 \approx -4.65\%$$

$$\text{Error for 1.3 ft} = \frac{1.3\text{ ft} - 1.29\text{ ft}}{1.29\text{ ft}} \approx 0.00775 \approx 0.775\%$$

The 1.3 ft measurement is more accurate because its relative error has a smaller magnitude. The 1.23 ft measurement is more precise because it is measured to more decimal places.

PRACTICE

Determine which measurements are more accurate and which are more precise.

1. measurements: 12.56 in. and 12.7 in.
   actual: 12.66 in.
2. measurements: 4.0 m and 4.24 m
   actual: 4.19 m
3. measurements: 0.67 s and 0.79 s
   actual: 0.73 s
4. measurements: 155 lb and 160 lb
   actual: 158 lb

Dimensional Analysis and Unit Conversions

Use unit conversion factors to change one unit of measure to another. Dimensional analysis requires choosing the appropriate conversion factor.

EXAMPLE **Convert 5 m/s to km/h.**

Use conversion factors relating meters to kilometers and seconds to hours. There are 1000 meters in a kilometer, so the conversion factor is $\frac{1\text{ km}}{1000\text{ m}}$. There are 60 seconds in a minute and 60 minutes in an hour, so the conversion factor is $\frac{3600\text{ s}}{1\text{ h}}$.

$$\left(5\frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{1\text{ km}}{1000\,\cancel{\text{m}}}\right)\left(\frac{3600\,\cancel{\text{s}}}{1\text{ h}}\right) = 18\text{ km/h}$$ *Multiply by the conversion factors.*

A speed of 5 m/s is equivalent to 18 km/h.

PRACTICE

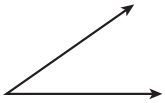
Convert.

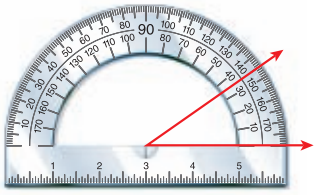
1. 19 ft/h to mi/s
2. 3.65 m^2/yr to m^2/day
3. 9.8 m/s^2 to ft/s^2
4. 12 mL/s to L/h
5. 6552 in. to yd
6. 9 gal to pt
7. 11,232 s to h
8. 792 ft^2 to in^2
9. 484 mg to kg
10. 18 in./day to mi/yr

Measure Angles

You can use a protractor to measure angles. Be sure to use the correct scale on the protractor when reading an angle's measure.

EXAMPLE **Find the measure of the angle by using a protractor.**



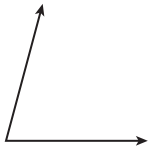


The angle measures 35°.

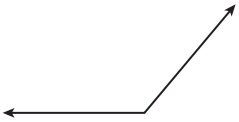
PRACTICE

Find the measure of each angle by using a protractor.

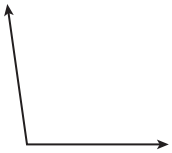
1.



2.

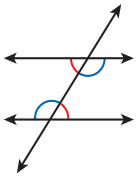


3.

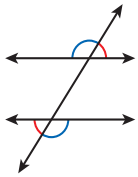


Parallel Lines and Transversals

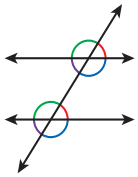
If two parallel lines are intersected by a transversal, the angle pairs shown below are congruent.



Alternate interior



Alternate exterior



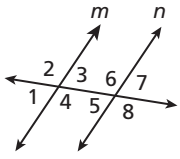
Corresponding

EXAMPLE **In the figure, line *m* is parallel to line *n*. Name three angles that are congruent to ∠1. Justify your answers.**

∠7: ∠1 and ∠7 are alternate exterior angles.

∠5: ∠1 and ∠5 are corresponding angles.

∠3: ∠1 ≅ ∠7, and ∠7 and ∠3 are corresponding angles.

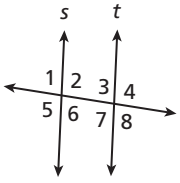


PRACTICE

In the figure, line *s* is parallel to line *t*. Name three angles that are congruent to each given angle. Justify your answers.

1. ∠2

2. ∠8

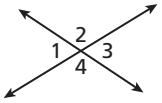


Angle Relationships

Complementary angles are two angles whose measures add to 90°. **Supplementary angles** are two angles whose measures add to 180°.

Vertical angles are a pair of opposite angles formed by intersecting lines. Vertical angles are congruent. **Adjacent angles** have a common vertex, a common side, and do not overlap.

In the figure shown, ∠1 and ∠3 are vertical angles, and ∠2 and ∠4 are vertical angles. Two angles adjacent to ∠1 are ∠2 and ∠4.

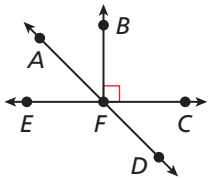


EXAMPLE

A **Name two angles adjacent to ∠*CFD*.**

∠*BFC* ∠*CFD* and ∠*BFC* share vertex *F* and side $\overrightarrow{FC}$.

∠*EFD* ∠*CFD* and ∠*EFD* share vertex *F* and side $\overrightarrow{FD}$.



B **If m∠*CFD* = 43°, find m∠*AFB*.**

m∠*AFE* = 43°	∠*AFE* and ∠*CFD* are vertical angles and are congruent.
m∠*EFB* = 90°	∠*EFB* is supplementary to ∠*BFC*, and m∠*BFC* = 90°.
m∠*AFE* + m∠*AFB* = m∠*EFB*	If point *A* is in the interior of ∠*EFB*, then m∠*AFE* + m∠*AFB* = m∠*EFB*.
43° + m∠*AFB* = 90°	Substitute.
m∠*AFB* = 47°	Solve for m∠*AFB*.

PRACTICE

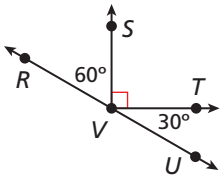
Find the measure of the complement of an angle with each given measure.

1. 23° **2.** 16° **3.** 42° **4.** 87° **5.** 35°

Find the measure of the supplement of an angle with each given measure.

6. 120° **7.** 74° **8.** 94° **9.** 27° **10.** 156°

Use the figure for Exercises 11–13.

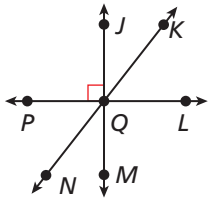


11. Name two angles adjacent to ∠*RVS*.

12. Name a pair of complementary angles.

13. Name two pairs of supplementary angles.

In the figure, m∠*JQK* = 38°. Find each angle measure.



14. m∠*PQK* **15.** m∠*MQN*

16. m∠*KQL* **17.** m∠*NQP*

18. m∠*LQM* **19.** m∠*KQM*

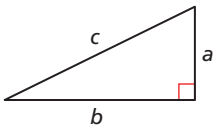
20. Name two angles adjacent to ∠*KQL*.

21. Name two pairs of vertical angles.

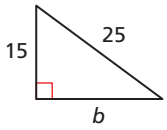
22. Name a pair of complementary angles.

Pythagorean Theorem

The **Pythagorean Theorem** states that in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse, $a^2 + b^2 = c^2$.



EXAMPLE **Find the unknown side length of the right triangle.**



$15^2 + b^2 = 25^2$ *Substitute 15 for a and 25 for c.*

$b^2 = 400$ *Solve for b^2.*

$\sqrt{b^2} = \sqrt{400}$ *Take the positive square root of both sides.*

$b = 20$

PRACTICE

Find the unknown side length of each right triangle with legs *a* and *b* and hypotenuse *c*.

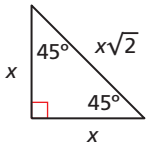
1. $a = 5, b = 12$ **2.** $a = 14, b = 8$ **3.** $b = 15, c = 17$

4. $b = 7, c = 20$ **5.** $a = 6, c = 10$ **6.** $a = 9, c = 12$

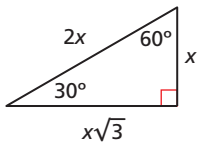
7. $a = 10, b = 15$ **8.** $b = 1, c = 3$ **9.** $a = 14, c = 20$

Special Right Triangles

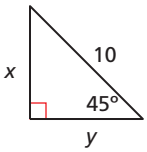
In a 45°-45°-90° triangle, the length of the legs are equal, and the length of the hypotenuse is $\sqrt{2}$ times the length of the legs.



In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg, and the longer leg is $\sqrt{3}$ times the length of the shorter leg.



EXAMPLE **Find the unknown side lengths of the right triangle.**



$\sqrt{2} \cdot \text{leg length} = \text{hypotenuse length}$ *45°-45°-90° triangle*

$\sqrt{2} \cdot x = 10$ *Substitute.*

$x = \frac{10}{\sqrt{2}} = 5\sqrt{2}$ *Solve for x.*

$y = 5\sqrt{2}$ *In a 45°-45°-90° triangle, the leg lengths are equal.*

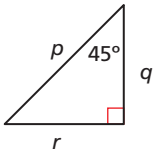
PRACTICE

For each length given below, find the unknown side lengths of the right triangle.

1. $p = 20$

2. $q = 6$

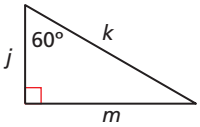
3. $r = 13$



4. $j = 15$

5. $k = 10$

6. $m = 8$

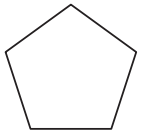


Properties of Polygons

A **polygon** is a closed plane figure formed by three or more segments. Polygons can be classified based on properties of their sides and angles.

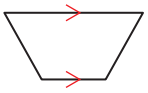
EXAMPLE **Give all names that apply to each polygon.**

A



pentagon — *The polygon has 5 sides.*

B



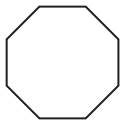
quadrilateral — *The polygon has 4 sides.*

trapezoid — *The quadrilateral has 1 pair of parallel sides.*

PRACTICE

Give all names that apply to each polygon.

1.



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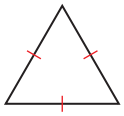

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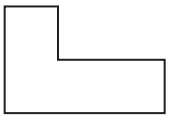

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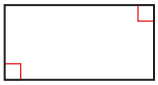


Area of Polygons, Circles, and Composite Figures

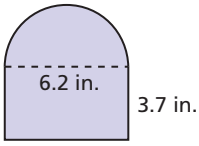
To find the area of a composite figure, break it down into simpler shapes and find the area of each shape.

Triangle: $A = \frac{1}{2}bh$ Rectangle: $A = \ell w$ Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Parallelogram: $A = bh$ Circle: $A = \pi r^2$

EXAMPLE **Find the area of the figure. Round to the nearest tenth.**

The figure is composed of a rectangle and a semicircle.



Step 1 Find the area of the rectangle.

$A = \ell w = 6.2(3.7) = 22.94 \text{ in}^2$

Step 2 Find the area of the semicircle.

$A = \frac{1}{2}\pi r^2$ *Area of a semicircle*

$= \frac{1}{2}\pi(3.1)^2 \approx 15.10 \text{ in}^2$ *The radius is half the diameter, or* $\frac{6.2}{2} = 3.1$ *in.*

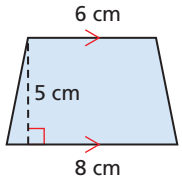
Step 3 Add to find the total area.

$22.94 + 15.10 \approx 38.0 \text{ in}^2$

PRACTICE

Find the area of each figure. Round to the nearest tenth.

1.



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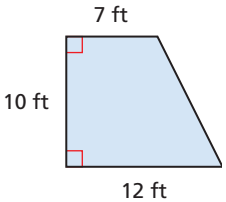

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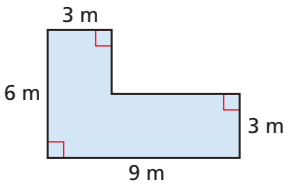

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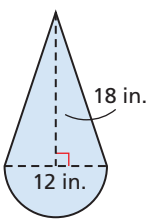
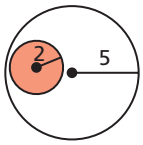


Relative Area

Relative area is the ratio of the area of a portion of a figure to the total area of the figure.

EXAMPLE **Find the relative area of the shaded portion of the figure compared to the total area of the figure. Write the relative area as a percent.**

$$\text{Relative area} = \frac{\text{Shaded area}}{\text{Total area}} = \frac{\pi(2)^2}{\pi(5)^2} = \frac{4}{25} = 0.16$$

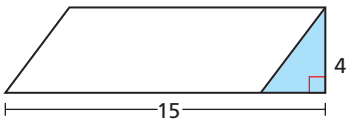


The relative area is 0.16, or 16%.

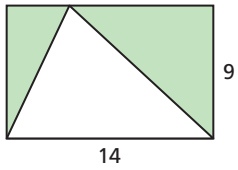
PRACTICE

Find the relative area of the shaded portion of the figure compared to the total area of the figure. Write the relative area as a percent.

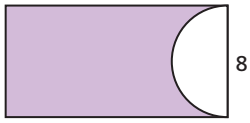
1.



2.



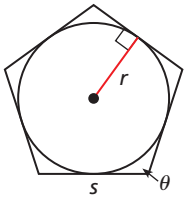
3.



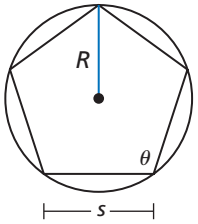
Angle Relationships in Circles and Polygons

A **regular polygon** has sides of equal length and has equal interior angles. The following formulas apply to regular polygons, where n is the number of sides.

$$\theta = \left(\frac{n-2}{n}\right)180^\circ \qquad s = 2R\sin\left(\frac{180^\circ}{n}\right) \qquad r = R\cos\left(\frac{180^\circ}{n}\right)$$



Inscribed circle



Circumscribed circle

EXAMPLE **A circle with a radius of 10 cm circumscribes a regular pentagon. Find the side length of the pentagon to the nearest tenth of a centimeter.**

$s = 2R\sin\left(\frac{180^\circ}{n}\right)$ *Use the side length formula.*

$= 2(10)\sin\left(\frac{180^\circ}{5}\right)$ *Substitute 10 for R and 5 for n.*

≈ 11.8 cm *Simplify.*

PRACTICE

Find the measure of an interior angle of each regular polygon.

1. octagon **2.** pentagon **3.** hexagon **4.** 15-gon

5. A circle with a radius of 20 m inscribes a square.

a. Find the radius of a circle that circumscribes the square.

b. Find the side length of the square.

Plane Figures and Coordinate Geometry

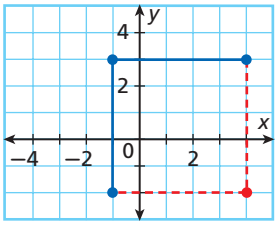
Coordinate geometry can be used to specify plane figures.

EXAMPLE **A square has vertices at $(-1, -2)$, $(-1, 3)$, and $(4, 3)$. Find the coordinates of the fourth vertex of the square.**

Plot the given vertices.

Connect the vertices to form two sides of a square.

Sketch the remaining sides.



The fourth vertex is located at $(4, -2)$.

PRACTICE

Give all the names that apply to each polygon with the given vertices.

1. $(-1, 2), (1, 2), (3, -1), (-2, -1)$
2. $(-1, 1), (3, -3), (-1, -3)$
3. A parallelogram has vertices at $(-2, -2)$, $(-1, 1)$, and $(3, 1)$. Find the coordinates of the fourth vertex of the parallelogram.

Geometric Formulas

In the following geometric formulas, s is the length of a side, and r is the radius.

Regular Hexagon	Regular Octagon	Tetrahedron	Octahedron	Sphere
$A = \frac{3s^2}{2}\sqrt{3}$	$A = 2s^2(\sqrt{2} + 1)$	$V = \frac{s^3}{12}\sqrt{3}$	$V = \frac{s^3}{3}\sqrt{2}$	$S = 4\pi r^2$ $V = \frac{4\pi r^3}{3}$

EXAMPLE

A **Find the area of a regular octagon with a side length of 8 cm.**

$$A = 2s^2(\sqrt{2} + 1)$$
$$= 2(8)^2(\sqrt{2} + 1)$$
$$= 128(\sqrt{2} + 1)$$
$$= (128\sqrt{2} + 128)$$

The area is $(128\sqrt{2} + 128)$ cm^2.

B **Find the side length of an octahedron with a volume of $9\sqrt{2}$ ft^3.**

$$V = \frac{s^3}{3}\sqrt{2}$$
$$9\sqrt{2} = \frac{s^3}{3}\sqrt{2}$$
$$27 = s^3$$
$$3 = s$$

The side length is 3 ft.

PRACTICE

Find each measure.

1. The volume of a sphere with a radius of 6 ft
2. The side length of a regular hexagon with an area of $54\sqrt{3}$ cm^2
3. The radius of a sphere with a surface area of 16π in^2
4. The side length of a tetrahedron with a volume of $18\sqrt{3}$ cm^3

Area and Volume Relationships

When the linear dimensions of a figure are increased or decreased by a constant factor of a, the area or surface area of the figure changes by a factor of a^2, and the volume changes by a factor of a^3.

EXAMPLE **The linear dimensions of a square with an area of 160 ft^2 are tripled. Find the new area.**

Step 1 Find the scale factor.

The linear dimensions increase by a factor of 3, so $a = 3$.

Step 2 Find the new area.

$160(3)^2 = 1440$ *Multiply the original area by a^2.*

The new area is 1440 ft^2.

PRACTICE

1. The linear dimensions of a cylinder with a surface area of 320 m^2 are doubled. Find the new surface area.
2. The linear dimensions of an octagon with an area of 26 ft^2 are reduced by a factor of $\frac{1}{4}$. Find the new area.
3. The linear dimensions of a cube with a volume of 2400 cm^3 are reduced by a factor of 50%. Find the new volume.

Surface Area and Volume

The table shows formulas for the surface area and volume of three types of solid figures.

Solid	Surface Area	Volume
Cylinder	$S = 2\pi r(r + h)$	$V = \pi r^2 h$
Cone	$S = \pi r\sqrt{r^2 + h^2} + \pi r^2$	$V = \frac{1}{3}\pi r^2 h$
Sphere	$S = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

EXAMPLE **Find the surface area and volume of a cylinder with a radius of 6 cm and a height of 12 cm.**

$S = 2\pi r(r + h)$ $V = \pi r^2 h$ *Use the formulas for a cylinder.*

$= 2\pi(6)(6 + 12)$ $= \pi(6)^2(12)$ *Substitute 6 for r and 12 for h.*

$= 216\pi$ $= 432\pi$ *Simplify.*

The surface area is 216π cm^2, and the volume is 432π cm^3.

PRACTICE

Find the surface area and volume of each solid.

1. A cone with a radius of 5 ft and a height of 12 ft
2. A cylinder with a radius of 4 m and a height of 3.5 m
3. A sphere with a radius of 27 in.
4. A sphere with a radius of 7 cm

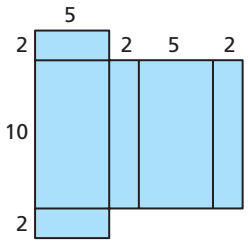
Nets

A **net** is a diagram of the faces of a three-dimensional figure arranged in such a way that the diagram can be folded to form the three-dimensional figure.

EXAMPLE **Find the surface area of the solid figure shown by the net.**

Step 1 Identify the figure.

The figure is a rectangular prism with length 5, width 2, and height 10.



Step 2 Find the surface area.

$S = 2\ell w + 2\ell h + 2wh$

$= 2(5)(2) + 2(5)(10) + 2(2)(10)$ *Substitute.*

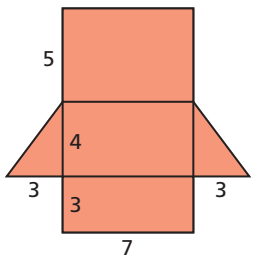
$= 160$ *Simplify.*

The surface area is 160 square units.

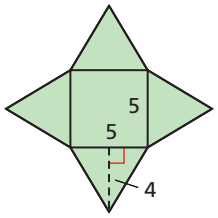
PRACTICE

Find the surface area of the solid figure shown by each net.

1.



2.



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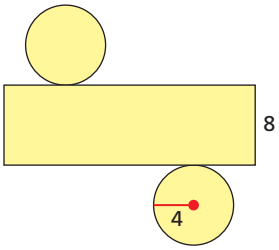
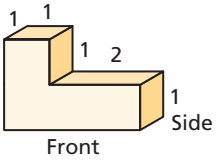
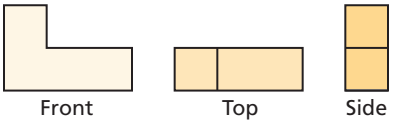


Views of Solid Figures

Solid figures can be represented with orthographic views.

EXAMPLE **Draw the front, top, and side views of the solid figure.**

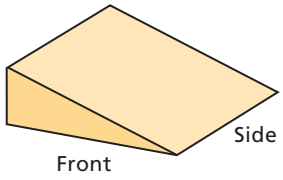




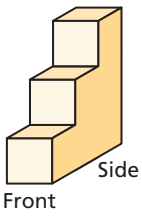
PRACTICE

Draw front, top, and side views of each solid figure.

1.



2.



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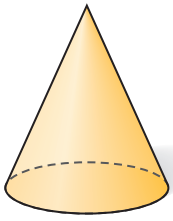


Geometric Patterns and Tessellations

A **tessellation** is a repeating pattern of plane figures that completely covers an area with no gaps or overlaps. Sequences involving tessellations and other **geometric patterns** can often be described numerically.

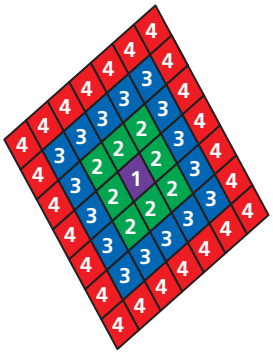
EXAMPLE 1 **The first four figures of a pattern are shown. Write a sequence for the number of dots in each stage. Describe the pattern in the sequence, and find the next term.**

Show the number of dots at each stage.

Stage	1	2	3	4
Dots	3	6	9	12

The terms of the sequence increase by 3 at each stage. The fifth stage probably contains 15 dots.

EXAMPLE 2 **A parallelogram can be used to tessellate a plane as shown. Write a sequence for the number of parallelograms in each stage. Describe the pattern in the sequence, and find the next term.**



Show the number of parallelograms in each stage.

Stage	1	2	3	4
Parallelograms	1	9	25	49

The terms of the sequence appear to be the squares of the positive odd numbers: 1^2, 3^2, 5^2, 7^2, ... The fifth stage probably contains 9^2, or 81, parallelograms.

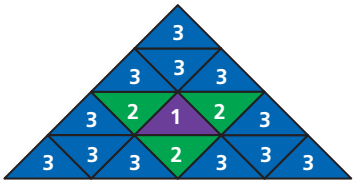
PRACTICE

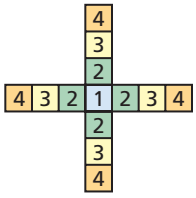
Write a sequence for the number of dots or polygons added at each stage. Describe the pattern in the sequence, and find the next term.

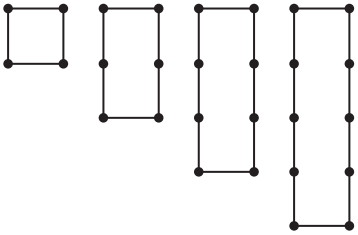
1. 

2. 

3. 

4. 

5. 

6. 

Factoring Quadratic Expressions

You can use several methods to factor quadratic expressions. Remember the standard form of a quadratic expression is $ax^2 + bx + c$.

EXAMPLE 1 **Factor $x^2 - 5x + 6$.**

Use a table to find the factors of 6 that add to -5. These factors are -2 and -3.

Factors of 6	Sum
1, 6	7 ✘
2, 3	5 ✘
-1, -6	-7 ✘
-2, -3	-5 ✔

Rewrite the expression as a product of binomial factors with -2 and -3 as constants.

$x^2 - 5x + 6 = (x - 2)(x - 3)$

Check your answer by multiplying.

$(x - 2)(x - 3) = x^2 - 3x - 2x + 6 = x^2 - 5x + 6$

EXAMPLE 2 **Factor $2x^2 - 18$.**

$2x^2 - 18$

$2(x^2 - 9)$ *Factor out the GCF, 2.*

$2(x^2 - 3^2)$ *Rewrite 9 as 3^2.*

$2(x - 3)(x + 3)$ *Factor the difference of squares: $a^2 - b^2 = (a - b)(a + b)$.*

Check your answer by multiplying.

$$\begin{aligned} 2(x - 3)(x + 3) &= (2x - 6)(x + 3) \\ &= 2x^2 + 6x - 6x - 18 \\ &= 2x^2 - 18 \end{aligned}$$

EXAMPLE 3 **Factor $6x^2 + x - 2$.**

Factor by guess and check. The coefficient of the x^2 term is 6, and the constant term in the trinomial is -2.

$(1x - 1)(6x + 2) = 6x^2 - 4x - 2$ ✘

$(1x - 2)(6x + 1) = 6x^2 - 11x - 2$ ✘

$(1x + 1)(6x - 2) = 6x^2 + 4x - 2$ ✘

$(1x + 2)(6x - 1) = 6x^2 + 11x - 2$ ✘

$(2x - 1)(3x + 2) = 6x^2 + x - 2$ ✔

The factored form of $6x^2 + x - 2$ is $(2x - 1)(3x + 2)$.

PRACTICE

Factor.

1. $x^2 - 5x - 14$

2. $x^2 - 81$

3. $6x^2 - 12x$

4. $x^2 + 7x + 12$

5. $4x^2 + 8x + 4$

6. $3x^2 - 147$

7. $7x^2 - 5x - 2$

8. $6x^2 - 5x + 1$

9. $x^2 + 2x - 99$

10. $21x^2 - 40x - 21$

11. $20x^2 + 11x - 42$

12. $10x^2 - 55x - 200$

13. $2x^2 + 6x$

14. $x^2 - 10x + 25$

14. $2x^2 + 14x + 12$

16. $5x^2 - 20$

17. $x^2 - 7x - 30$

18. $3x^2 + 16x + 5$

Mean, Median, Mode, and Range

The **mean** of a data set is the sum divided by the number of values. The **median** is the middle value in a numerically ordered set. The **mode** is the value or values that occur most often. The **range** is the difference between the greatest and the least values.

EXAMPLE **Find the mean, median, mode, and range of the data.** $\{4, 6, 1, 6, 2, 7, 8, 2, 7, 2\}$

Mean: Add all the values and divide by the number of values.

$\text{mean} = \frac{45}{10} = 4.5$ *The sum is 45. There are 10 values.*

Median: First, order the values from least to greatest. $\{1, 2, 2, 2, 4, 6, 6, 7, 7, 8\}$

$\text{median} = \frac{4 + 6}{2} = 5$ *Since there are an even number of values, 10, the median is the average of the 5th and 6th values.*

Mode: Find the value that repeats the most. $\{1, 2, 2, 2, 4, 6, 6, 7, 7, 8\}$

$\text{mode} = 2$ *The value 2 occurs most often.*

Range: Find the difference between the largest and smallest value: $\text{range} = 8 - 1 = 7$

PRACTICE

Find the mean, median, mode, and range of each data set.

1. $\{5, 1, 5\}$ **2.** $\{7, 5, 3, 25\}$ **3.** $\{2, 0, 0, 3, 0, 1, 0\}$ **4.** $\{15, 17, 171, 4, 0, 15, 2, 4\}$

Data Displays

Stem-and-leaf plots group the data by place value. Box-and-whisker plots show the least and greatest values, the median, the first quartile (the median of the lower half of the data), and the third quartile (the median of the upper half of the data).

EXAMPLE **Use the data.** $\{45, 47, 39, 30, 29, 37, 10, 50, 28, 49, 47, 36, 39, 28, 44\}$

A **Draw a stem-and-leaf plot of the data.**

Sort the data:

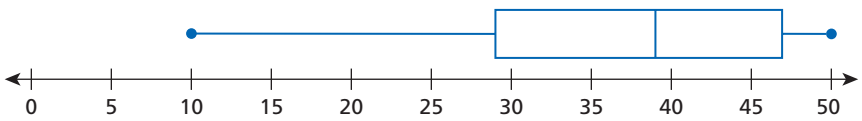
10, 28, 28, 29, 30, 36, 37, 39, 39, 44, 45, 47, 47, 49, 50

The stems represent the tens digit, and the leaves represent the ones digit—the number 10 is shown in red.

Stems	Leaves
1	0
2	8 8 9
3	0 6 7 9 9
4	4 5 7 7 9
5	0

B **Draw a box-and-whisker plot of the data.**

The least value is 10; the greatest value is 50; the median is 39; the first quartile is 29 (the median of 10, 28, 28, 29, 30, 36, and 37); and the upper quartile is 47.



PRACTICE

Draw a stem-and-leaf plot and a box-and-whisker plot of the data.

1. $\{35, 30, 21, 19, 15, 29, 28, 15, 24, 20, 34, 30, 23, 29, 16, 19, 24, 28, 31, 33, 22, 5\}$

Statistical Graphs

Statistical graphs include bar graphs, circle graphs, and line graphs. Bar graphs compare numerical amounts; circle graphs compare parts of a whole; and line graphs show changes in data, such as over time.

EXAMPLE **Make a line graph of the data.**

Year	2006	2007	2008	2009	2010	2011
Sales (million $)	5.2	4.9	6.8	7.1	12.9	11.4

Use "Year" evenly spaced on the horizontal axis and use "Sales (million $)" on the vertical axis. The vertical axis should go from 0 to at least 12.9 to accommodate all the data.

PRACTICE

1. Draw a bar graph and a circle graph of the data.

Students in 11th-Grade Classrooms						
Classroom	A	B	C	D	E	Total
Students	21	28	17	30	24	**120**

Sampling Methods and Bias

There are several sampling methods you can use when collecting a random sample of data from a population.

Random Sampling	Each member of the population has an equal chance of being selected.
Systematic Sampling	A rule or formula is used, such as surveying every *n*th person.
Stratified Sampling	The population is divided into groups, and then sampling is done within each group.

These sampling techniques are designed to reduce the **bias** in taking surveys. Bias is any factor that prevents an accurate representation of the population. Systematic and stratified sampling may or may not be random.

EXAMPLE **Hal asks every third person wearing a blue shirt for their favorite color. Identify the sampling method and explain whether the survey is biased.**

Surveying every third person is systematic sampling but is not random. The survey is biased because people wearing blue shirts might have a greater probability of liking blue than the rest of the population.

PRACTICE

Identify the sampling method or methods and explain whether the survey is biased.

1. Lisa asks the first 10 boys and the first 10 girls who arrive at school in the morning what method of transportation they take to get to school.

Pascal's Triangle

Pascal's triangle is a triangle of numbers.

- Each row has one more number than the row above it.
- Each number is the sum of the two numbers directly to the left and right above it.
- The first and last numbers of a row are each 1.
- Each row is symmetric.

Row 0 →					1					
Row 1 →				1		1				
Row 2 →			1		2		1			
Row 3 →		1		3		□		1		
Row 4 →	1		4		□		□		1	
Row 5 →	1		5		10		□		5	1

EXAMPLE **Find the missing number in row 3.**

Method 1 Add the two numbers above it in row 2.

Row 2 →		1	2	1	
Row 3 →	1	3	3		1

Method 2 Since the row is symmetric, it is the same as the second term in row 3.

Row 3 →	1	3 →	3	1

PRACTICE

1. Complete the first 5 rows above.
2. Part of Pascal's triangle is shown. Find the missing number.

120	210	□	
330	462	462	

3. Find the first two numbers in row 27.
4. The third number in row 12 is 220. Find the last three numbers in row 12.
5. What row gives the coefficients of $(x + 1)^2$ when written in standard form?

Exponents in Probability Formulas

If the probability of an event p is always the same, the probability that it occurs 3 times in succession is $p \cdot p \cdot p$, or p^3. The probability P that it occurs n times in succession is p^n.

EXAMPLE **What is the probability of getting 3 tails in a row when flipping a coin?**

$P = p^n$ *Write the probability formula.*

$P = 0.5^3$ *Substitute 0.5 for p and 3 for n.*

$P = 0.125$

The probability of getting 3 tails in a row is 0.125.

PRACTICE

1. Find P if $p = 0.7$ and $n = 4$.
2. There is a 60% chance Cal will get home on time each day. What is the probability he will get home on time all 5 days this week?
3. What is the probability of getting 2 ones in a row when rolling a number cube twice?

Proofs

A proof is an argument that uses logic to show that a conclusion is true. There are several different techniques that you can use to prove a conclusion is true.

The goal in a proof is to reach a statement that is either always or never true and either confirms or contradicts the desired conclusion.

EXAMPLE 1 **Prove $(x-3)(x+2) = x^2 - x - 6$.**

Begin with $(x-3)(x+2)$ and multiply to show it equals $x^2 - x - 6$.

$$\begin{aligned}(x-3)(x+2) &= x \cdot x + (-3) \cdot x + 2 \cdot x + (-3) \cdot 2 \\ &= x^2 - 3x + 2x - 6 \\ &= x^2 - x - 6 \ \checkmark\end{aligned}$$

EXAMPLE 2 **Prove $(x-3)(x-1) = (x-2)^2 - 1$.**

Simplify each side to obtain equivalent expressions.

Left Side	Right Side
$(x-3)(x-1)$	$(x-2)^2 - 1$
$x^2 - 3x - x + 3$	$x^2 - 2x - 2x + 4 - 1$
$x^2 - 4x + 3$	$x^2 - 4x + 3$

$(x-3)(x-1) = x^2 - 4x + 3 = (x-2)^2 - 1$ ✔

EXAMPLE 3 **Disprove $x(x-4) + 4 = (x+1)(x-5)$.**

Assume that $x(x-4) + 4 = (x+1)(x-5)$. Then simplify.

$$\begin{aligned}x(x-4) + 4 &= (x+1)(x-5) \\ x^2 - 4x + 4 &= x^2 - 4x - 5 \\ -4x + 4 &= -4x - 5 && \textit{Subtract } x^2 \textit{ from both sides.} \\ 4 &= -5 \ \times && \textit{Add } 4x \textit{ to both sides.}\end{aligned}$$

Since $4 \neq -5$, the assumption must be false.

$x(x-4) + 4 \neq (x+1)(x-5)$

PRACTICE

Prove or disprove each statement.

1. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
2. $(x+1)^3 = (x^3 + 1) + 3x(x+1)$
3. $\dfrac{1}{x+2} + \dfrac{1}{x-2} = \dfrac{2x}{x^2-4}$
4. $\sqrt{4x^6} = \sqrt[3]{8x^9}$, where $x \geq 0$
5. $(x+4)^2 = x^2 + 8x + 8$
6. $x^2(x+1) + 3(x-1) = x(x^2+3) + (x-2)(x+2)$
7. $(x+2)(x+8) = (x+5)^2 - 9$
8. $(x+1)(x-6)(x+7) = x^3 + 2x^2 - 41x - 42$

Logical Reasoning and Conditional Statements

Logical reasoning is a process of making conclusions based upon given information. When using logical reasoning it is sometimes best to write the given information as conditional statements. A conditional statement is a statement that can be written in the form "if p, then q," where p is the hypothesis and q is the conclusion.

EXAMPLE **Use logical reasoning to draw a conclusion from the given premises.**
If Aaron joins the baseball team, then Brandon will join the team.
If Brandon joins the baseball team, then Corey will not join the team.
Aaron joins the baseball team.

Because Aaron joins the team, Brandon also joins. Because Brandon joins the team, Corey will not join.

Conclusion: Corey does not join the team.

PRACTICE

Use logical reasoning to draw a conclusion from the given premises.

1. If it is raining, then the sun does not shine. It is raining.
2. If it is Friday, then we go out for dinner. If we go out for dinner, then we eat pasta. It is Friday.
3. If a quadrilateral is a square, then it has 4 right angles. If a quadrilateral has 4 right angles, then it is a rectangle.

Venn Diagrams

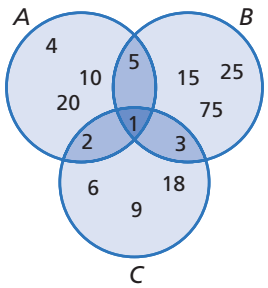
A Venn diagram is used to show relationships between sets.

EXAMPLE **Draw a Venn diagram of the relationships between the following sets:**
A: factors of 20 **B: factors of 75** **C: factors of 18**

Find the elements of each set:

A: $\{1, 2, 4, 5, 10, 20\}$ B: $\{1, 3, 5, 15, 25, 75\}$ C: $\{1, 2, 3, 6, 9, 18\}$

Draw three overlapping circles. Label one circle for each set. Place each element in the appropriate circle or overlapping region.



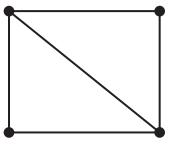
PRACTICE

Draw a Venn diagram for each of the following sets.

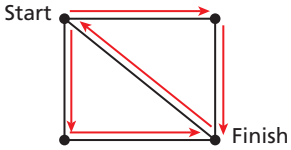
1. Set A: factors of 45 Set B: factors of 12 Set C: factors of 50
2. Set A: factors of 100 Set B: factors of 225 Set C: factors of 36

Graph Theory: Euler and Hamiltonian Paths

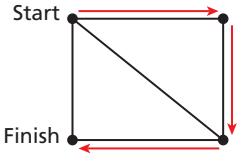
A vertex-edge graph is a set of points, or vertices, that are connected by a set of lines, or edges. If you trace through the graph from vertex to vertex along the edges, you create a path.



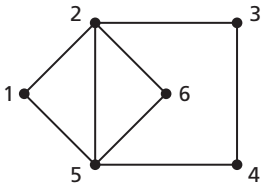
An Euler path includes every edge in the graph exactly once. Not every graph has an Euler path.



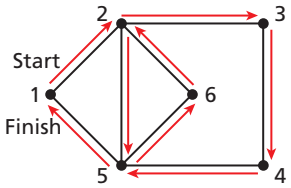
A Hamiltonian path includes every vertex in the graph exactly once. Not every graph has a Hamiltonian path.



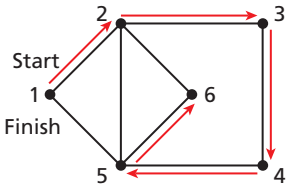
EXAMPLE **Find an Euler path and a Hamiltonian path for the graph, if possible.**



Try the vertices in sequence. To complete the path, revisit vertices 2, 5, and 1. This is an Euler path.



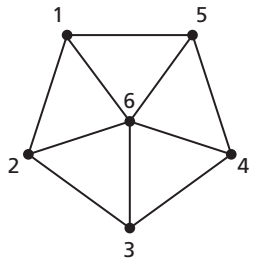
Try the vertices in sequence. Stop at vertex 6. This is a Hamiltonian path.



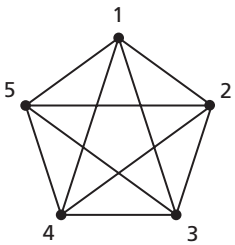
PRACTICE

Find an Euler path and a Hamiltonian path for each graph, if possible.

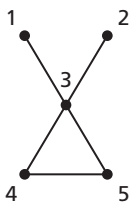
1.



2.



3.



Selected Answers

Chapter 1

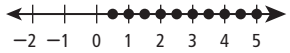
1-1

Check It Out! **1a.** $-2, -\sqrt{3}, -0.321, \frac{3}{2}, \pi$ **b.** -2: $\mathbb{R}, \mathbb{Q}, \mathbb{Z}$; $-\sqrt{3}$: $\mathbb{R}$, irrational; -0.321: $\mathbb{R}, \mathbb{Q}$; $\frac{3}{2}$: $\mathbb{R}, \mathbb{Q}$; π: $\mathbb{R}$, irrational **2a.** $(-\infty, -1]$ **b.** $(-\infty, 2]$ or $(3, 11]$ **3a.** even numbers between 1 and 9 **b.** $\{3, 4, 5, 6, 7\}$ **c.** $\{x \mid x \geq 99\}$

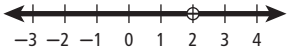
Exercises **1.** roster notation **3.** $-\frac{100}{4}, -6.897, \frac{1}{8}, \sqrt{4}, \sqrt{6}; -\frac{100}{4}$: $\mathbb{R}, \mathbb{Q}, \mathbb{Z}$; -6.897: $\mathbb{R}, \mathbb{Q}$; $\frac{1}{8}$: $\mathbb{R}, \mathbb{Q}$; $\sqrt{4}$: $\mathbb{R}, \mathbb{Q}, \mathbb{Z}, \mathbb{W}, \mathbb{N}$; $\sqrt{6}$: $\mathbb{R}$, irrational **5.** $(-10, 10]$ **7.** $[1, 20)$ or $(30, \infty)$ **9.** $\{x \mid -5 \leq x < 3\}$ **11.** $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ **13.** $-2, -\sqrt{2}, -1.\overline{25}, \frac{\sqrt{2}}{3}, \frac{1}{2}$; -2: $\mathbb{R}, \mathbb{Q}, \mathbb{Z}$; $-\sqrt{2}$: $\mathbb{R}$, irrational; $-1.\overline{25}$: $\mathbb{R}, \mathbb{Q}$; $\frac{\sqrt{2}}{3}$: $\mathbb{R}$, irrational; $\frac{1}{2}$: $\mathbb{R}, \mathbb{Q}$ **15.** $(-\infty, 5)$ or $(5, \infty)$ **17.** $[-3, 3]$ **19.** $\{11, 22, 33, 44, 55, 66, 77, \ldots\}$ **21.** $\{x \mid -9 \leq x \leq -1$ and x is odd$\}$ **23.** $\mathbb{Q}$ **27.** numbers greater than or equal to -4 and less than 8; cannot be expressed in roster notation; $\{x \mid -4 \leq x < 8\}$ **29.** numbers greater than 0 and less than 1; cannot be expressed in roster notation; $(0, 1)$ **31.** $(-\infty, 2)$ or $(2, \infty)$; $\{x \mid x \neq 2\}$ **33.** $(1, 10)$; $\{x \mid 1 < x < 10\}$ **35.** $(-\infty, 5)$ or $(5, 10]$; $\{x \mid x < 5$ or $5 < x \leq 10\}$ **37.** false **39.** true **41.** $\{x \mid 11 \leq x \leq 12\}$; $\{x \mid 12 \leq x \leq 13\}$; $\{x \mid 14 \leq x \leq 16\}$ **45.** $(-\infty, -1]$ or $(3, 6)$ or $[9, \infty)$

47.

49.



51.



53. $\mathbb{N}$ **55.** W **57a.** interior designer, police officer, pediatric nurse, marine biologist, astronaut **b.** The order would not change. **c.** The order would not change. **d.** $\{46{,}000, 52{,}900, 59{,}800, 79{,}350, 106{,}950\}$ **63.** D **65.** B **67.** finite; $\mathbb{Q}$ **69.** finite; $\mathbb{Q}, \mathbb{Z}, \mathbb{W}, \mathbb{N}$ **75.** \$20, \$20, and \$10

1-2

Check It Out! **1a.** $-500; \frac{1}{500}$ **b.** 0.01; -100 **2a.** Commutative Property of Multiplication **b.** Associative Property of Multiplication **3.** \$3.12 **4a.** always true by the Additive Inverse Property **b.** sometimes true; possible answer: true when $a = 0$, $b = 1$, and $c = 2$; false when $a = 1$, $b = 2$, and $c = 3$

Exercises **1.** 36; $-\frac{1}{36}$ **3.** $-2\sqrt{2}; \frac{1}{2\sqrt{2}}$ **5.** $\frac{1}{500}; -500$ **7.** Associative Property of Multiplication **9.** Commutative Property of Multiplication **11.** \$7.33 **13.** sometimes true **15.** 2.5; $-\frac{2}{5}$ **17.** $-2\pi; \frac{1}{2\pi}$ **19.** $-\frac{1}{20}$; 20 **21.** Distributive Property **23.** Additive Identity Property **25.** \$9.80 **27.** never true by the Multiplicative Inverse Property **29.** $4(11.99) - 2(8.88) = \$30.20$ **31.** $3(0.9)(9.96) + 5(0.75)(11.99) = 71.8545 \approx \$71.85$ **33.** ≈ 2 loops **35.** 5; Associative Property of Addition **37.** 0; Additive Identity Property **39.** $\frac{5}{4}$; Multiplicative Inverse Property **41.** yes **45.** Multiplicative Identity Property **47.** Distributive Property; Associative Property of Addition **49.** Distributive Property **53.** D **55.** C **57.** $n = 2$ **59.** 66.7% **61.** $-4\sqrt{2}$ **63.** $(-10, 0]$ **65.** cannot be notated

1-3

Check It Out! **1.** -7.4 **2a.** $4\sqrt{3}$ **b.** $\frac{3}{2}$ **c.** 10 **d.** 7 **3a.** $\frac{3\sqrt{35}}{7}$ **b.** $\frac{\sqrt{10}}{2}$ **4a.** $13\sqrt{5}$ **b.** $-\sqrt{5}$

Exercises **1.** radicand **3.** 4.5 **5.** 3.6 **7.** 12 **9.** $4\sqrt{5}$ **11.** $-5\sqrt{2}$ **13.** $-\frac{\sqrt{7}}{7}$ **15.** $5\sqrt{2}$ **17.** $\sqrt{2}$ **19.** -3.9 **21.** 9.9 **23.** $-\frac{1}{11}$ **25.** $-8\sqrt{5}$ **27.** $5\sqrt{17}$ **29.** $-3\sqrt{21}$ **31.** $\frac{9\sqrt{2}}{4}$ **33.** $\frac{\sqrt{3}}{30}$ **35.** $7\sqrt{7}$ **37.** $23\sqrt{3}$ **39.** $8\sqrt{7}$ **41.** $-3\sqrt{6}$ **43.** 33.9 cm **45.** 99.0 in. **47.** about 50 in. by 50 in. **49.** 180 **51.** $2\sqrt{5} - 5\sqrt{2}$ **53.** $\frac{3\sqrt{35} + \sqrt{5}}{5}$ **55.** $\frac{16\sqrt{10}}{5}$ **57.** 600 ft by 600 ft **59.** 7467.3 ft **61.** 8167.7 ft **63.** always true **65.** no **67a.** ≈ 7.81 s **67b.** ≈ 3.20 s **69.** H **71.** 8.9 **73a.** 6 in.; $6\sqrt{5}$ in. **b.** 54 in^2 **c.** $18 + 6\sqrt{2} + 6\sqrt{5}$ in. **75.** tetrahedron or triangular pyramid **77.** triangular prism **79.** $1.5 < x < 8$ **81.** $\frac{3}{4} < x < \frac{5}{2}$ **83.** Commutative Property of Addition **85.** Distributive Property

1-4

Check It Out! **1a.** $18 + y$ **b.** $3600h$ **2.** -15 **3.** $-6x - 8xy - 9y$ **4a.** $8000 - 30h$ **b.** \$ 7160

Exercises **1.** $0.79c$ **3.** 9 **5.** $-12a + 9$ **7.** $1 + 5ab - 25a - b^2$ **9.** $(180 - x)°$ **11.** -18 **13.** 115 **15.** $3x - 12y + 2$ **17.** $5 - 3m - 2n$ **19a.** $500 - 20m$ **b.** 460 min or 7 h 40 min **21.** $5g^2 - 6g + 1$; 28 **23.** $\frac{a^2 - 2b^2 + 2a}{2 + a}; -7$ **25.**

x	$(x-4)^2$	x^2+16	$x^2-8x+16$
1	9	17	9
0	4	20	4
2	1	25	1
4	0	32	0

$(x - 4)^2 = x^2 - 8x + 16$ **27.** $7a + 4b$ **29a.** $4125 - 175d$ **b.** \$3250 **c.** They save \$175 per day. **31.** $y = -40$; $y = -25$; $y = -7$; $y = -5$; $y = -10$ **33.** $y = 7$; $y = 15$; $y = 1$; $y = -13$; $y = -5$ **37.** G **39.** $a = 8$ **41.** $a = 22$ **43a.** 4; undefined; -48; undefined; 36; $\frac{147}{8}$ **b.** $x = 1$ and $x = 3$ **c.** $\{x \mid x \neq 1$ and $x \neq 3\}$ **45.** square pyramid **47.** $\mathbb{Q}$ **49.** irrational **51.** $3\sqrt{6}$ **53.** 14

1-5

Check It Out!
1a. $(2a)(2a)(2a)(2a)(2a)$ **b.** $3 \cdot b \cdot b \cdot b \cdot b$ **c.** $-(2x - 1)(2x - 1)(2x - 1) \cdot y \cdot y$ **2a.** 9 **b.** $-\frac{1}{3125}$ **3a.** $125x^{18}$ **b.** $-\frac{1}{8a^9b^3}$ **4a.** 2.5×10^{-4} **b.** 1.24×10^{-9} **5.** ≈ 8.33 min

Exercises **3.** $(12xy)(12xy)(12xy)(12xy)$ **5.** $\left(-\frac{1}{2}d\right)\left(-\frac{1}{2}d\right)\left(-\frac{1}{2}d\right)$ **7.** 1
9. $\frac{1}{10}$ **11.** cd^6 **13.** $\frac{10y^{10}}{x^4}$
15. $-4m^4n^6$ **17.** $\frac{y^3}{x^4}$
19. 3×10^{11} **21.** 400
23. $5 \cdot x \cdot x \cdot x$
25. $2a(-b^2 - a)(-b^2 - a)$ **27.** $-\frac{4}{3}$
29. -1 **31.** $-x^{20}y^{10}$ **33.** $-16a^5b^7$
35. 1.5×10^9 **37.** 1.55 min
39. $2^6, 2^2, 2^5, 2^{-8}$; $16^{-2}, 4^1, 2^5, 8^2$
41. $-2^6, 2^0, 2^4, 2^{-2}$; $-8^2, 2^{-2}, 4^0, 16^1$
43. $3m^5n^4$ **45.** $\frac{3x}{2y^3}$ **47.** $-24a^3b^7$
49. $\frac{9m^2}{25n^2}$ **51.** 1296 **53.** 1728
55a. 3742 km/h **b.** ≈ 288,609 times as fast **c.** ≈ 1.28 s **57.** $-\frac{7x^3}{4y^2}$
59. $\frac{100}{x^4z^6}$ **61.** $\frac{8m^4}{n^2}$ **63.** Laos; 25.6
65. Vietnam; 615.6
67. $\approx 2.84 \times 10^9$ beats
69. $\approx 1.27 \times 10^5$ hairs **71.** Power of a Product Property or Power of a Power Property **73.** Power of a Quotient Property or Power of a Power Property **77.** 6.5×10^{-15}
79. 3.5×10^{14} **81.** 1.1346×10^{24}
83. C **85.** C **87.** $\approx 2.0363 \times 10^{-4}$
91. $\frac{1}{3}$ **93.** 8 **95.** $-\frac{1}{3}$

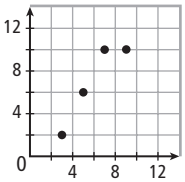
1-6

Check It Out! **1.** D: $\{-2, -1, 0, 1, 2, 3\}$; R: $\{-3, -2, -1, 0, 1, 2\}$
2a. function **b.** not a function
3a. function **b.** not a function; $(1, 2)$ and $(1, -2)$

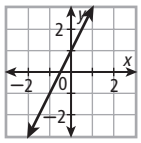
Exercises **1.** range **3.** D: {2000, 2001, 2002, 2003}; R: {5.39, 5.65, 5.80, 6.03} **5.** not a function
7. function **9.** D: {Irene, Anna, Lea, Kate}; R: {12, 16, 22}
11. function **13.** not a function
15. function **17.** D: $\{-2, -1, 0, 1, 2\}$; R: $\{-2, 0, 2\}$ **19.** D: {jumbo, extra large, large, medium}; R: {1.75, 2, 2.25, 2.5}
21a. function **b.** function
c. not a function **d.** function
e. not a function **23.** D: {a, b, c, d}; R: {1, 2, 4}; function
25. D: {1, 3, 5, 7, 9}; R: {3}; function **27.** D: {3, 4, 5, 6, 7}; R: $\{-1, 2, 3\}$; function **29.** D: {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}; R: {24}; function **31.** B to A
33. A to B **35.** B to A **37.** both
39. No; the relation is not a function.
41a. Yes, the relation is a function.
b. It is a function. **c.** 2d: ≈ 0.0183 oz; 3d: ≈ 0.0282 oz; 4d: ≈ 0.0506 oz; 5d: ≈ 0.0590 oz; 6d: ≈ 0.0884 oz
45. F **47.** $b \in \mathbb{R}$ and $a \neq \{-1, 0, 1, 2\}$
49. One to one; each length in feet corresponds to only one length in inches. **51.** 288 ft **53.** $36\pi \approx 113.1$ ft^2
55. 4.7 **57.** 9.5 **59.** $\frac{20}{w}$ **61.** $\frac{x^{21}}{z^7}$

1-7

Check It Out!
1a. $f(0) = 0$; $f\left(\frac{1}{2}\right) = -\frac{7}{4}$; $f(-2) = 12$
b. $f(0) = 1$; $f\left(\frac{1}{2}\right) = 0$; $f(-2) = 5$
2a.

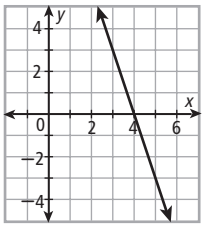


b.

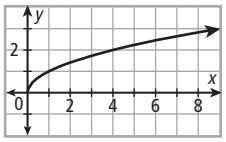


3a. $f(x) = 0.27x$ **b.** 6.48; the price to develop 24 prints, in dollars

Exercises **1.** independent
3. $f(0) = 9$; $f(1.5) = 11.25$; $f(-4) = 25$ **5.** $f(0) = 3$; $f(1.5) = 4$; $f(-4) = 4$ **7.** $f(0) = -5$; $f(1.5) = 1$; $f(-4) = 1$
9.



11. $f(x) = 125x$; 6250; the loss if 50 customers purchase the living room set, in dollars
13. $f(0) = 0$; $f\left(\frac{3}{2}\right) = -\frac{3}{4}$; $f(-1) = -2$
15. $f(0) = 2$; $f\left(\frac{3}{2}\right) = 5$; $f(-1) = 0$
17. $f(0) = 0$; $f\left(\frac{3}{2}\right) = 3$; $f(-1) = \frac{1}{2}$
19.

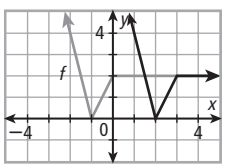


21. $f(m) = 160 + 4m$; 192; a fine of $192 for driving 8 mi/h over the speed limit
23. $f(-3.5) = -16.5$; $f(-1) = -9$; $f\left(\frac{1}{4}\right) = -5.25$; $f(2) = 0$; $f(11) = 27$
25. $f(-4) = -3$; $f(0) = -\frac{1}{3}$; $f\left(\frac{1}{2}\right) = 0$; $f(5) = 3$
27. $f(-2)\,0 = -1$; $f(-1) = 2$; $f(1) = 2$; $f(2) = -1$ **29.** D: $\{A \mid A \geq 0\}$; R: $\{y \mid y \in W\}$ **31.** D: $\{t \mid t \geq 0\}$; R: $\{y \mid -16 < y \leq 32.8\}$ **33.** $t = 35$; the number of years it takes for plan h to reach a value of $7500
35. $t = 40$; the time when plan g is worth $\frac{1}{2}$ the value of plan h
37. $h(40) - g(40) = 5000$; the difference in the value of the plans after 40 years **39.** When $x = 3$, $f(x) = \frac{1}{x-3} = \frac{1}{0}$, but division by 0 is undefined.
41. For $-5 < x < 0$, x represents negative hours, and distance traveled would be negative.
43. independent: number of shirts; dependent: total cost; domain: $x \geq 15$ **45.** $f(x) = 2.37x$
47. $f(x) = 0.8x$ **51.** H **53.** 31
55. $g\left(-\frac{h}{4}\right) = 1$ **57.** $r(t^4) = \frac{\sqrt{t^{16} + 4}}{t^4}$
59. $12x - xy + 8$ **61.** $\frac{c-2}{c}$
63. b is any value. **65.** yes

1-8

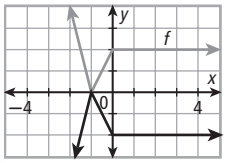
Check It Out!
1a. $(3, 3)$ **b.** $(-2, 1)$
2a.

$x + 3$	x	y
1	−2	4
2	−1	0
3	0	2
5	2	2



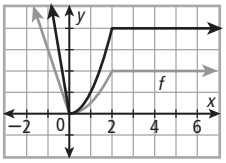
b.

x	y	$-y$
-2	4	-4
-1	0	0
0	2	-2
2	2	-2

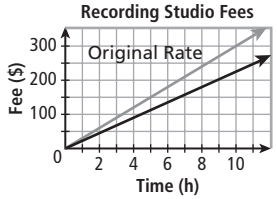


3.

x	y	$2y$
-1	3	6
0	0	0
2	2	4
4	2	4

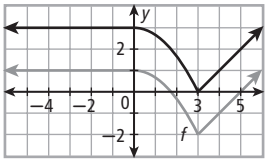


4. vertical compression by a factor of $\frac{3}{4}$

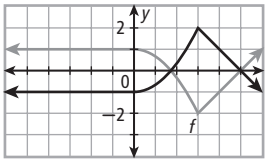


Exercises **1.** compression **3.** $(4, -1)$

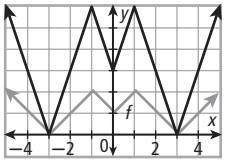
5.



7.

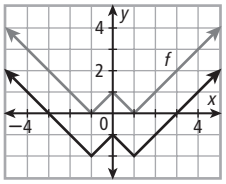


9.

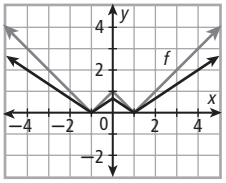


11. vertical compression by a factor of $\frac{1}{2}$ **13.** horizontal shift right 5 units **15.** $(3, 5)$

17.

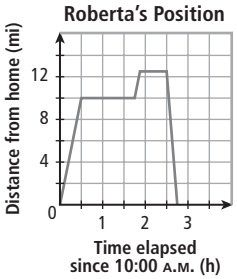


21.

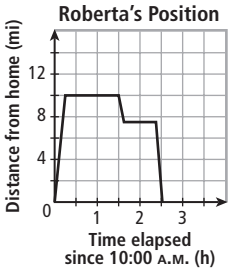


25. vertical shift down 5 units **27.** horizontal stretch by a factor of 2 **29.** 10 square units; the same as the original **31.** 7 square units; smaller than the original **33.** 10 square units; the same as the original **35.** 30 square units; larger than the original **37a.** vertical translation **b.** horizontal compression **c.** the increase in the per-hour labor rate

39.



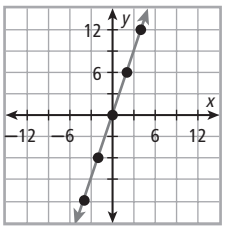
41.



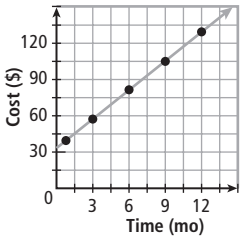
43. The library is half as far from Roberta's house. **47.** H **49.** H **53a.** $c(n) = 0.37n$ **b.** vertical stretch **c.** 15 in 1999 and 13 in 2002 **d.** The number of letters that can be mailed for $5.00 must be rounded down to the nearest whole number. **55.** 110 **57.** yes **59.** $f(1) = -\frac{1}{2}$; $f(-3) = -\frac{17}{2}$; $f\left(\frac{1}{4}\right) = -2$ **61.** $f(1) = 0$; $f(-3) = 64$; $f\left(\frac{1}{4}\right) = \frac{225}{256}$

1-9

Check It Out! **1a.** cubic; translation 2 units up **b.** quadratic; reflection across the y-axis **2.** linear; vertical stretch by a factor of 3



3. linear; about $72



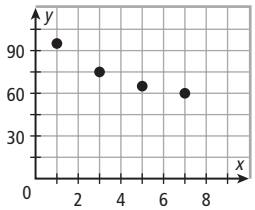
Exercises **3.** quadratic; translation 1 unit left **5.** square root; translation 3 units left **7.** linear; translation $\sqrt{2}$ units down **9.** cubic; vertical compression or horizontal stretch **11.** quadratic; translation 1 unit down **13.** cubic; translation 3 units up **15.** square root; vertical stretch or horizontal compression **17.** D: $\{x \mid x \geq 0\}$; R: $\{y \mid y \geq 0\}$; vertical stretch by a factor of 3 **19.** D: $\{x \mid x \geq 0\}$; R: $\{y \mid y \leq 0\}$; reflection across the x-axis **21.** D: $\{x \mid x \in \mathbb{R}\}$; R: $\{y \mid y \leq 1\}$; reflection across the x-axis and then a vertical shift up 1 unit **23.** $195 **25.** quadratic; horizontal shift right 7 units **27.** linear; reflection across the y-axis and a vertical shift down 1 unit **29.** linear; $\approx$ 1500 pixels **31.** quadratic; $\approx$ 1417 pixels **33.** Cubic; D: $\{\ell \mid \ell \geq 0\}$; R: $\{y \mid y \geq 0\}$; the domain and range are restricted. **35.** Linear; D: $\{n \mid n \in \mathbb{N}\}$; R: $\{y \mid y \in \mathbb{N}\}$; the domain and range are restricted. **37.** Square root; D: $\{a \mid a \geq 0\}$; R: $\{y \mid y \geq 0\}$; the domain and range are the same. **39a.** linear **b.** cubic **c.** quadratic **d.** square root

e. linear; horizontal stretch by a factor of 2 and a vertical shift up 3 units **43.** H **45.** G **47.** quadratic **49.** linear **51.** 7.5×10^{9} **53.** 2.0×10^{-25} **55.** $f(-5) = 15; f\left(-\frac{2}{3}\right) = -\frac{8}{9}; f(1.6) = 5.76; f(4) = 24$ **57.** $(4, -10)$

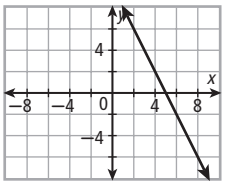
Study Guide: Review

1. domain; range **2.** $\{x \mid x \geq -5\}$ **3.** $(-1, 5]$ **4.** $\{4, 5, 6, 7, \ldots\}$ **5.** $\{x \mid x < -2 \text{ or } x > 5\}$ **6.** integers greater than -4 and less than or equal to 5 **7.** $[5.5, 5.6]$ **8.** Commutative Property of Multiplication **9.** Distributive Property **10.** $-0.55; \frac{1}{0.55}$ **11.** $\frac{7}{8}; -\frac{8}{7}$ **12.** $-1.\overline{2}; \frac{1}{1.\overline{2}}$ or $\frac{9}{11}$ **13.** 3.5 **14.** 7.4 **15.** 8.6 **16.** 5.4 **17.** $4\sqrt{2}$ **18.** 4 **19.** $-4\sqrt{2}$ **20.** $3\sqrt{7}$ **21.** $\frac{7\sqrt{2}}{2}$ **22.** $\frac{\sqrt{10}}{5}$ **23.** -96 **24.** 14 **25.** $\frac{1}{8}$ **26.** $11x - 3y$ **27.** $18 - 5a + b$ **28.** $-3x - 12y$ **29.** $a^2c + 2bc$ **30.** $\frac{-8x^{15}}{y^9}$ **31.** $\frac{-12x^7}{7y^9}$ **32.** $\frac{r^4}{s^4}$ **33.** $\frac{4m^6}{n^4}$ **34.** 7×10^{7} **35.** 5.4×10^{1} **36.** D: $\{3, 5, 7\}$; R: $\{-1, 0, 9\}$; not a function **37.** D: $[-2, \infty)$; R: $[-4, \infty)$; not a function **38.** D: $\{-2, 0, 3, 4\}$; R: $\{3, 4\}$; function **39.** D: $\{5, 10, 15, 20, 25\}$; R: $\{-5, -4, -3, -2, -1\}$; function **40.** D: $\{a, b, c\}$; R: {Alabama, Alaska, Arizona, Arkansas, California, Colorado, Connecticut}; not a function **41.** $f(2) = -2; f\left(\frac{1}{2}\right) = \frac{7}{4}; f(-2) = -2$ **42.** $f(2) = -16; f\left(\frac{1}{2}\right) = -\frac{17}{2}; f(-2) = 4$ **43.** $f(2) = -1; f\left(\frac{1}{2}\right) = 1; f(-2) = 2$ **44.** $f(2) = \frac{1}{2}; f\left(\frac{1}{2}\right) = 2; f(-2) = -\frac{1}{2}$

45.



46.

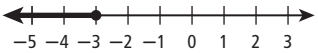


47. $A(s) = 6s^2$, where A is the surface area in square units and s is the side length in linear units; $A(10) = 600$; the surface area for a cube of side length 10 cm is 600 cm^2. **48.** $(0, -5)$ **49.** $(5, 1)$ **50.** vertical compression by a factor of $\frac{1}{2}$ **51.** vertical stretch by a factor of 1.1 **52.** translation 1 unit up **53.** quadratic function; translation 1 unit down **54.** square-root function; reflection across the x-axis **55.** linear function; about 90 psi

Chapter 2

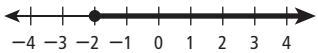
2-1

Check It Out! 1. 44 **2a.** $p = -4$ **b.** $r = \frac{1}{2}$ **3.** $w = 3$ **4a.** $\varnothing$ **b.** $\mathbb{R}$

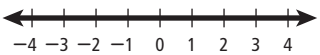


5. $x \leq -3$

Exercises 1. identity **3.** $x = 14$ **5.** $x = -6$ **7.** $x = -3$ **9.** $x = 2$ **11.** $r = -6$ **13.** $\varnothing$ **15.** $\mathbb{R}$ **17.** $\varnothing$ **19.** $x > \frac{7}{3}$ **21.** 28 s **23.** $x = 16$ **25.** $x = \frac{19}{2}$ **27.** $n = 6$ **29.** $x = 2$ **31.** $t = -8$ **33.** $\varnothing$ **35.** $\varnothing$ **37.** $x \geq -2$



39. $x \leq -12$ **41.** \$0.04 **43a.** no more than 10 **b.** no more than 21 **c.** no more than 13 **45.** $m\angle D = 70°$; $m\angle E = 20°$ **47.** 10 tragedies; 17 comedies; 10 histories. **49.** $k = -10$ **53.** C **55.** D **57.** 12 **59.** $\mathbb{R}$

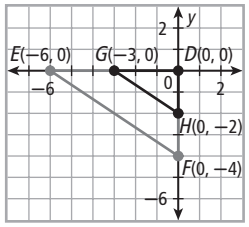


61a. ≈ 111 min **b.** ≈ 75 min **63.** $8\sqrt{10}$ **65.** $2\sqrt{15}$ **67.** function **69.** Yes

2-2

Check It Out! 1a. $y = 11$ **b.** $x = 42$ **2.** 1240 students **3.** ≈ 53 in.

4.



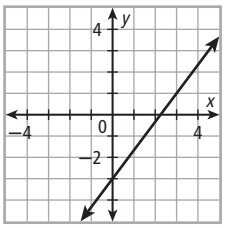
5. 27 ft

Exercises 1. rate **3.** $n = 8$ **5.** $t = 54$ **7.** $x = 20$ **9.** $x = -21$ **11.** 24 mi/gal **13.** 52.5 ft **15.** $y = 6.2$ **17.** $x = 10$ **19.** \$1.22 **21.** 36 ft **23.** $t = 0.168$ **25.** $u = 8$ **27.** $h = -30$ **29.** $x = 0$ **33.** The person would be 22 ft tall. **35a.** ≈ 128.57 **35b.** ≈ 128.59 times as long **37.** ≈ 0.0018% **39.** 5.4 in. **41.** $AB = 16\frac{2}{3}; EF = 22\frac{1}{2}$ **43.** HO **45.** 145 ft **49.** G **51.** G **53.** $h = 8.6$ **55.** $z = 8$ or $z = -8$ **57.** no **59.** 3 **61.** 350 **63.** 0.025 **65.** cone **67.** hexagonal prism or cylinder **69.** $f(x) = x$; vertical translation, up 4, then stretch by a scale factor of 3 **71.** $f(x) = \sqrt{x}$; horizontal translation, left 1, then reflection.

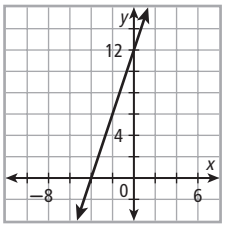
2-3

Check It Out! 1a. yes **b.** no

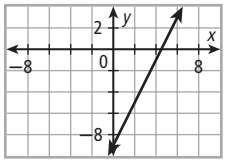
2.



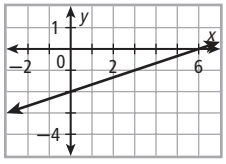
3. $x = -4; y = 12$



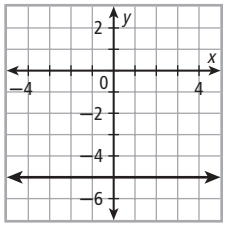
4a. $y = 2x - 9$



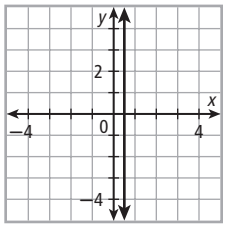
b. $y = \frac{1}{3}x - 2$



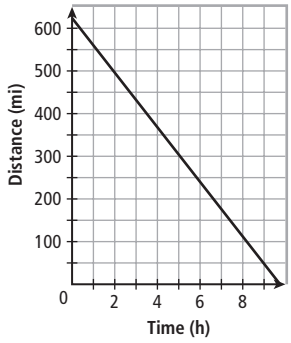
5a. horizontal;



b.



6. 64 mi/h

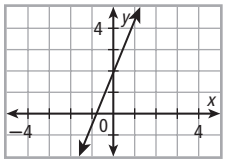


Exercises **1.** *y*-intercept: *y*-value of the point on the *y*-axis where *x* is 0; function value when *x* is 0

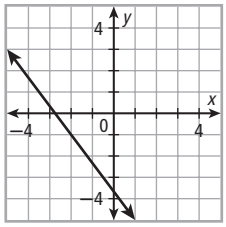
x-intercept: *x*-value of the point on the *x*-axis where *y* is 0; the input value when the function value is 0

3. yes

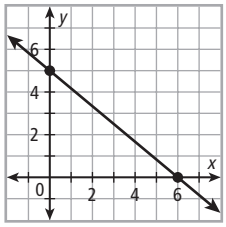
5.



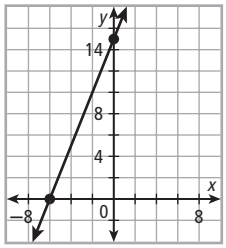
7.



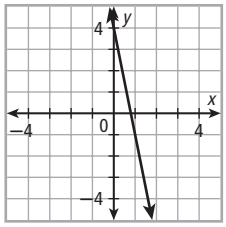
9. $x = 6$; $y = 5$



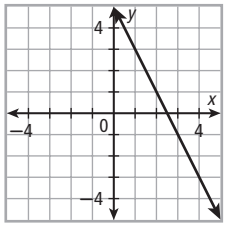
11. $x = -6$; $y = 15$



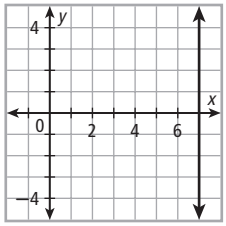
13. $y = -5x + 4$



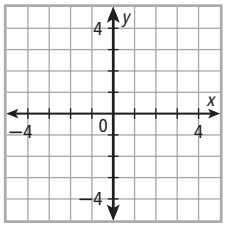
15. $y = -2x + 5$



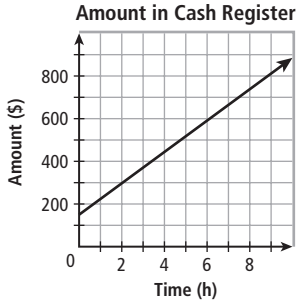
17. vertical



19. vertical

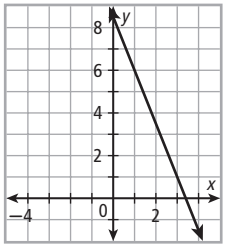


21. $73.50/h



23. yes

25.



29. $x = 4$; $y = -8$
31. $x = -2$; $y = 3$
33. $y = 3x - 2$
35. $y = \frac{4}{3}x + 2$ **37.** horizontal
39. horizontal
47a. yes **b.** $1.99 **c.** 1.19
49. sometimes **51a.** ≈ 1.97
51c. the number of leagues per foot
51d. ≈ 364.6 million ft; 10,172
53a. $-\frac{A}{B}; \frac{C}{B}$ **b.** 3; $-\frac{9}{2}$ **55a.** adult $5, student $2 **b.** *y*-intercept: 110; *x*-intercept: 44 **59.** D **61.** B
63. 0.125 **65a.** $\frac{9}{4}$; −9; 4
b. *y*-intercept: *b*; *x*-intercept: *a*
c. $\frac{x}{6} + \frac{y}{15} = 1$ **67.** $\frac{1}{2}$ **69.** $-\frac{27}{32}$
71. 81.9 **73.** $f(0) = 7$; $f(-3) = 6$
75. $f(0) = 0$; $f(-3) = -9$
77. $x = -24$ **79.** $t = 6$

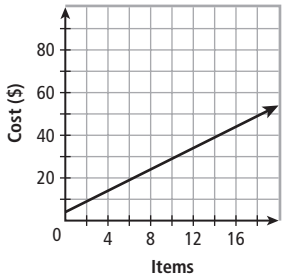
2-4

Check It Out! **1.** $y = \frac{3}{4}x + 3$

2a. 1 **b.** 0 **3a.** $y = -5x + 8$
b. $y = 2x + 1$

4a. $c = 2.5n + 4$; \$49

b.



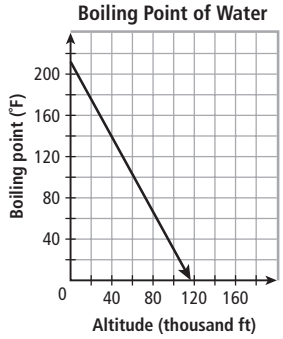
5a. $y = 5x - 1$ **b.** $y = -\frac{6}{5}x - 2$

Exercises **1.** $y = 2x + 1$

3. $y = \frac{1}{4}x + 3$ **5.** $\frac{7}{5}$ **7.** $y = -\frac{4}{3}x - \frac{8}{3}$

9a. $t = -\frac{1}{550}x + 212$

b.



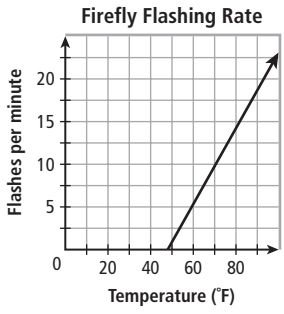
c. 192° **11.** $y = -\frac{9}{5}x - 4$

13. $y = \frac{5}{3}x - 2$ **15.** $\frac{2}{3}$

17. $y = \frac{7}{3}x + 4$

19a. $f = \frac{4}{9}T - \frac{64}{3}$

19.



c. 104.25°F **d.** ≈ −5.8 times; no

21. $y = -\frac{1}{3}x + 3$ **23.** neither

25. parallel **27.** $f(x) = 2x - 1$

29. $y = 4x + 3$

31. $y = -\frac{11}{8}x + \frac{1}{8}$ **33.** $y = 21x - 40$

35. $y = \frac{1}{6}x + \frac{4}{3}$ **37.** $y = -3x - 8$

39. slope $\overline{AB}$ = slope $\overline{DC} = -\frac{1}{3}$; slope $\overline{AD}$ = slope $\overline{BC} = 3$; rectangle

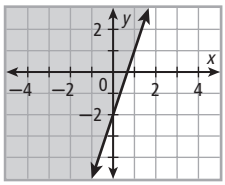
41. trapezoid **45.** G **49.** no

53. $(-\infty, -4)$ **55.** no **57.** yes

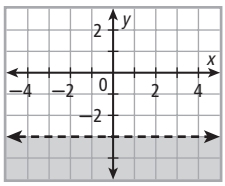
2-5

Check It Out!

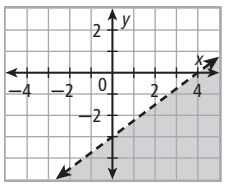
1a.



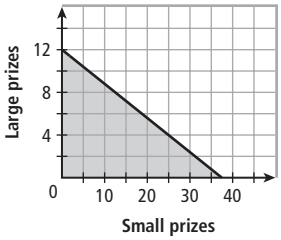
b.



2.

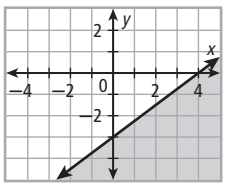


3. $40x + 125y \leq 1500$



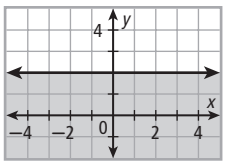
no more than 25

4.

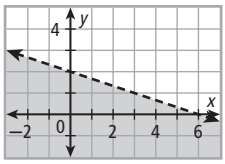


Exercises

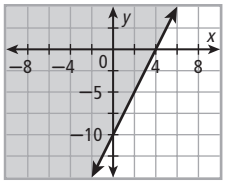
3.



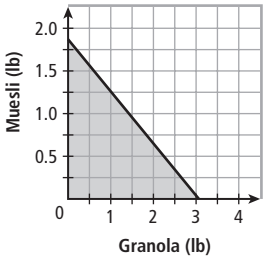
5.



7.

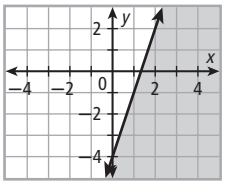


9a. $2.29x + 3.75y \leq 7.00$

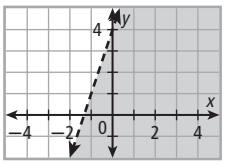


b. no more than 0.6 lb

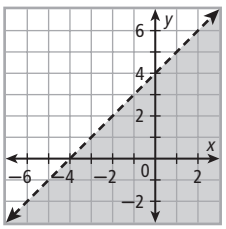
11. $y \leq +3x - 4$



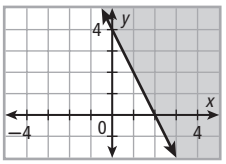
13. $y < 3x + 4$



15.



17.



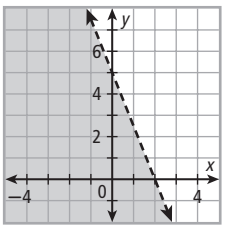
19. $200x + 500y \leq 10{,}000$

21a. $8x + 12y \leq 200$;

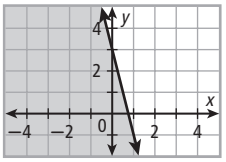
b. no more than 10 h

23. $y \leq 5x - 4$

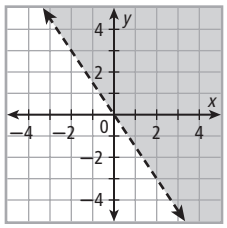
25.



29.



31.

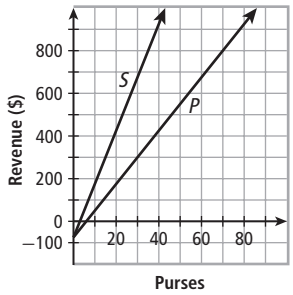


35a. $1.25x + 0.50y \geq 150$
b. yes **37a.** $> \approx 18.5$ h
b. possible distances from port
c. between 176 and 380 mi
39. $y > -\frac{4}{3}x + 2$ **43a.** $8x + 6y \geq 220$
b. $8x + 6y \leq 300$ **45.** G **47.** J **55.** no
57. $(-5, 3)$ **59.** $(-4, 6)$ **61.** $x = 1$
63. $y = 0.25x - 7.25$

2-6

Check It Out!

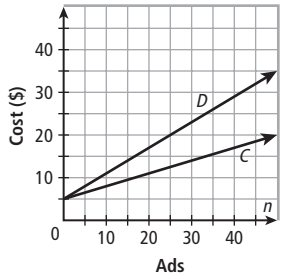
1a. $g(x) = 3(x - 2) + 1$
b. $g(x) = -(x + 2)$
2. $g(x) = \frac{1}{4}(3x + 2)$
3. $g(x) = \frac{1}{2}(x + 8)$
4a. $S(n) = 25n - 75$
b.



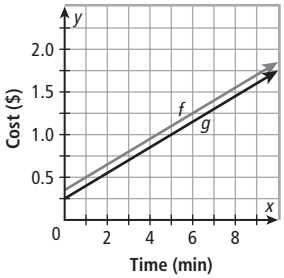
4c. horizontal compression by a factor of $\frac{1}{2}$

Exercises

1. $g(x) = -\frac{3}{2}x + 2$
3. $g(x) = x - 6$
5. $g(x) = \frac{2}{3}x - 6$
7a. $D(n) = 0.60n + 5.00$
b.



c. horizontal compression by a factor of $\frac{1}{2}$
9. $g(x) = \frac{1}{2}x - 4$
11. $g(x) = 1.2(-0.5x + 0.5)$
13. $g(x) = \frac{1}{2.75}(x + 1)$
15a. $g(x) = 0.15x + 0.35$
b.

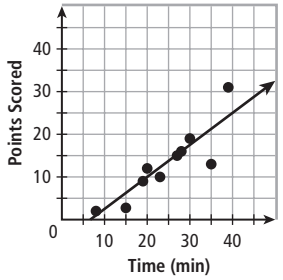


c. vertical shift up 0.1 unit
17. $g(x) = 2x$
19. $T(n) = 0.10\left(\frac{n}{15}\right) = \frac{n}{150}$; vertical stretch by a factor of 1.6
21a. $g(x) = -x - 2$
b. $h(x) = -x + 2$
23a. 22.125; 20; 23; 59 **b.** Mean, median, and mode are increased by 7. Range stays the same.
c. All are multiplied by 4.
d. Mean, median, and mode are multiplied by 2, and 5 is added. Range is multiplied by 2. **25.** H
27. F **31.** $\left(\frac{3}{5}d \cdot d\right)\left(\frac{3}{5}d \cdot d\right)\left(\frac{3}{5}d \cdot d\right)$
33. $-(2n)(2n)(2n)(2n)$
35. horizontal
37. neither
39. $B(a) = 32.5(a - 10)$; 26 ads

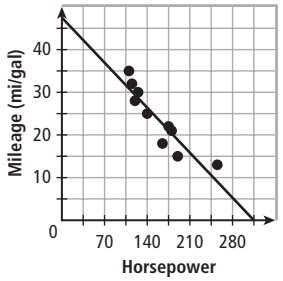
2-7

Check It Out!

1.



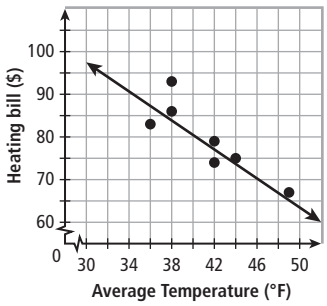
Possible answer: $p = 0.75m - 5$
2a.



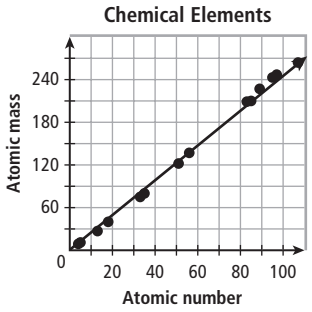
b. $r \approx -0.916$; $y \approx -0.15x + 47.5$; for a 1-unit increase in hp, gas mileage drops ≈ 0.15 mi/gal
c. ≈ 16.0 mi/gal **3.** ≈ 10 g; not close to the 15 g in the table.

Exercises

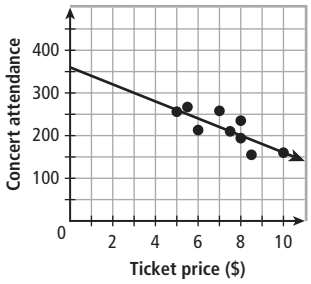
1a. a weak positive linear correlation between data sets
b. a strong negative linear correlation between data sets
c. virtually no correlation between the data sets
3a–b.



$r \approx -0.864$; $h \approx -1.68t + 148.88$
c. \$81.68; the correlation coefficient is fairly close to -1, so the prediction is somewhat close to the actual value.
5.

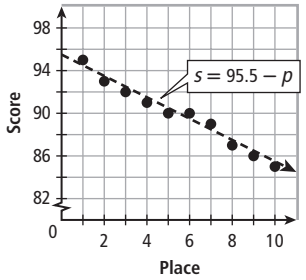


Possible answer: positive; $w = 2.5n - 5.5$
7a–b.

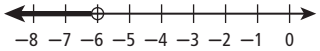


$r \approx -0.801$ $a \approx -20.95p + 368.89$
c. 180 people; fairly accurate.
9. $r \approx 0$ **11.** $r \approx 0.9$
13. Possible answers:
13a. $s = 95.5 - p$

b. $s = 100.5 - p$;



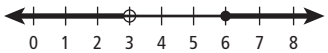
15a. $r = 0.994$; $y \approx 1.20x - 3.66$
b. A 1 cm increase in femur length corresponds to a 1.2 cm increase in humerus length. **c.** 44.7 cm; the data is nearly linear, so the prediction is probably accurate.
19. C **21.** B **23a.** $r = 0$
b. The data appear related but not linear. **25.** $8x^2 - 10x^2y + 4xy - 6$
27. $-g^2 + g - 12$
29. $x < -6$



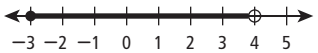
31. ∅ **33.** $f(x) = x + 6$; $g(x) = -x - 6$; $g(x) = -f(x)$; reflection across the x-axis

2-8

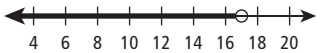
Check It Out! 1a. $\{x \mid x < 3 \cup x \geq 6\}$



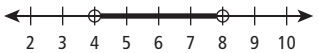
b. $\{x \mid x \geq -3 \cap x < 4\}$



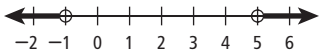
c. $\{x \mid x < 17\}$



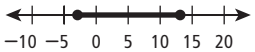
d. $\{x \mid 4 < x \leq 8\}$



2a. −22, 4 **b.** −5, 5
3a. $\{x \mid x < -1 \cup x > 5\}$

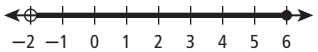


b. ∅
4a. $\{x \mid -3 \leq x \leq 13\}$

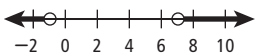


b. ∅

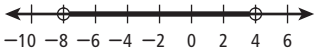
Exercises 1. disjunction
3. $-2 < x \leq 6$



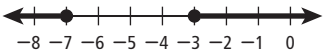
5. −3, −7 **7.** 3, −3
9. $x < -1$ or $x > 7$



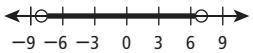
11. $-8 < x < 4$



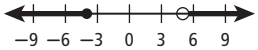
13. ∅ **15.** $-1 \leq x \leq 5$
17. −5, −9 **19.** 3, −3
21. $x \leq -7$ or $x \geq -3$



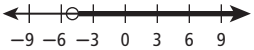
23. $x \leq -7$ or $x \geq 3$
25. $x < -8$ or $x > 3$
27. $-8 < x < 7$



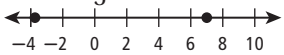
29. $x < -6$ or $x \geq -1$
31. $-4 \leq x < 3$
33. $x \leq -4$ or $x > 5$



35. $x > -5$

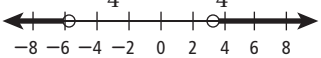


37. $7, -\frac{19}{5}$



39. $x \leq 2$ or $x \geq 3$

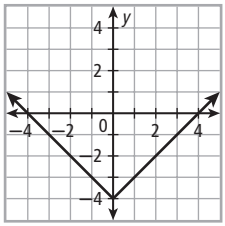
41. $x < -\frac{23}{4}$ or $x > \frac{13}{4}$



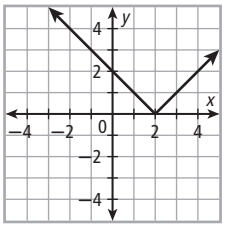
45. $|20x - 3400| \leq 100$; $165 \leq x \leq 175$
47. sometimes **55.** B **57.** D
58. F **59.** $x = 1$ or -4 **61.** B
63a. Associative Property
b. no **65.** ≈ 27 mi/gal
67. $n = 6$; Distributive Property
69. 60°; 80°; 100°; 120° **71.** 95°; 110°; 75°; 80°

2-9

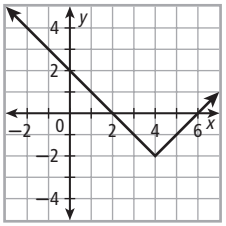
Check It Out!
1a. $g(x) = |x| - 4$



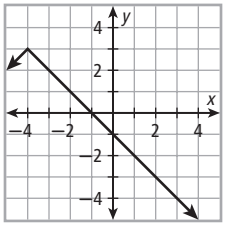
b. $g(x) = |x - 2|$



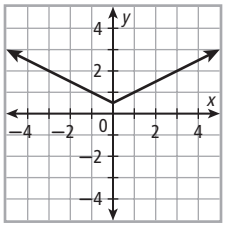
2. $g(x) = |x - 4| - 2$



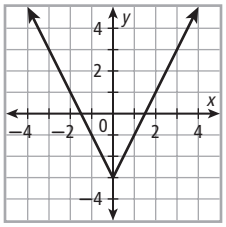
3a. $g(x) = -|-x - 4| + 3$



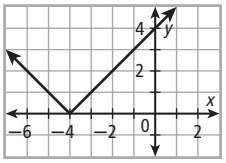
b. $g(x) = \frac{1}{2}(|x| + 1)$



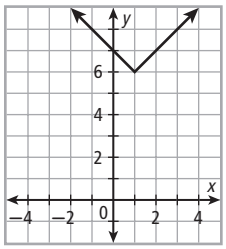
c. $g(x) = |2x| - 3$



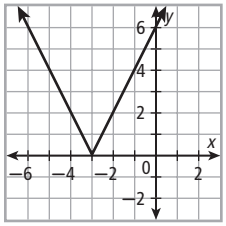
Exercises 1. The graph is the line $y = x$ where negative x-values are reflected over the x-axis, creating a **V**.
3. $g(x) = |x + 4|$



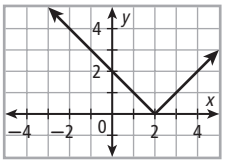
5. $g(x) = |x - 1| + 6$



7. $g(x) = 2|x + 3|$



9. $g(x) = |x - 2|$



11. $g(x) = |x + 4|$

13. $g(x) = |x - 1.5| + 4.5$

15. $g(x) = -|x - 5| - 2$

17. $f(x) = \left|2\left(\frac{2}{3}x\right)\right| - 3$

19. translated down 6 units

21. translated right 1 unit and vertically stretched by a factor of 2

23. $(-5, 9)$ **27.** $f(x) = |x - 2| - 4$

29. $f(x) = -|x - 4|$ **33.** D **35.** B

37. B **41.** $f(x) = 2|x + 3|$

43. 7.5×10^9 **45.** 6.561×10^7

47. 2.0×10^{-25} **49.** $(1, 1)$

51. $(6, -5)$ **53.** $(4, -10)$

55. $\varnothing$

Study Guide: Review

1. contradiction **2.** point-slope form **3.** correlation **4.** $x = \frac{13}{2}$

5. $\mathbb{R}$ **6.** $x = -\frac{4}{7}$ **7.** $x = \frac{31}{2}$ **8.** $x = \frac{51}{41}$

9. 140 **10.** $x \leq 7$ **11.** $x < -\frac{32}{3}$

12. $x \leq 9$ **13.** $19.95 + 2.75x < 50$; fewer than 11 times **14.** $x = 33$

15. $x = -15$ **16.** $x = -\frac{11}{3}$

17. $x = \frac{19}{18}$ **18.** 4.5 ft **19.** yes

20. $(5, 0); (0, 2)$

21. $(3, 0); (0, -2)$

22. $(-2.25, 0); (0, 1.5)$

23. $(1.5, 0); (0, 6)$

24. $y = -2x + 5$

25. $y = \frac{5}{3}x + 3$

26. $y = -\frac{3}{2}x + 2$

27. $y = -\frac{2}{3}x + 9$

28. vertical **29.** horizontal

30. $-27.5t + 500$

31. $y = \frac{1}{2}x + 4$

32. $y = 3x$

33. $y = \frac{3}{2}x - 8$

34. $y = -\frac{2}{3}x + 2$

35. $y > -3$

36. $y \leq x + 3$

37. $y > -\frac{1}{2}x - 3$

38. $y < 3x - 4$

39. $y < -2x + 3$

40. $12x + 21y \leq 2520$

41. $g(x) = x - 8$

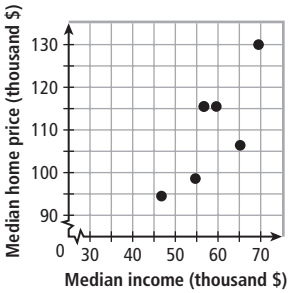
42. $g(x) = 3x + 15$

43. $g(x) = x - 4$

44. $g(x) = -x - 5$

45. $g(x) = -x - 12$

46a.



b. $r = 0.800$; $P = 1.279I + 35.074$

47. $x = 28$ or $x = -12$

48. $x = 66$ or $x = -54$

49. $\varnothing$

50. $\{x \mid x < -5 \cup x > 3\}$

51. $\{x \mid -2 \leq x \leq 5\}$

52. $\{x \mid 1 < x < 3\}$

53. $\{x \mid x \leq -8 \cup x \geq 4\}$

54. $g(x) = |x + 5| + 7$

55. $g(x) = |x - 6| - 9$

56. $g(x) = |-x - 4| + 1$

57. $g(x) = \frac{|3x + 1|}{3}$

58. $g(x) = -|x - 3| - 5$

Chapter 3

3-1

Check It Out! 1a. solution. **b.** not a solution. **2a.** $(0, -3)$ **b.** $(4, 4)$ **c.** $(-1, 4)$ **3a.** consistent, dependent; infinite number of solutions **b.** inconsistent; no solution **4.** 10 min

Exercises 1. inconsistent **3.** not a solution **5.** solution **7.** $(-2, 5)$ **9.** $(2, 3)$ **11.** consistent, dependent; infinite number of solutions **13.** inconsistent; no solution **15.** solution **17.** not a solution **19.** $(3, 1)$ **21.** $(1, -4)$ **23.** consistent, dependent; infinite number of solutions **25.** consistent, independent; one solution **27.** 10 system sales **29.** solution **31.** not a solution; $(-3, 0)$

33a. $\ell = 10{,}000 - 200x$
$m = 5{,}000 + 50x$

b. 20 min **c.** 6000 ft

35. $\begin{cases} y = 2x - 3 \\ y = -x + 6 \end{cases}$

consistent, independent; $(3, 3)$

37. $\begin{cases} y = 3x - 3 \\ y = 3x + 1 \end{cases}$

inconsistent; no solution

39. $(-0.25, 4)$ **41.** $(2.831, -30.403)$

45. Consistent, independent

47. D **49.** B **51.** $\left(\frac{100}{7}, \frac{6200}{7}\right)$

53. infinite number of solutions

55. The solution has no meaning in the real world. **57.** $\frac{2\sqrt{3}}{3}$

59. $\frac{\sqrt{2}}{2}$ **61.** -3 **63.** 40h

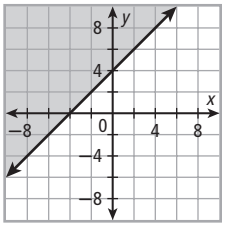
3-2

Check It Out! 1a. $(4, 7)$ **b.** $(3, -4)$ **2a.** $\left(\frac{3}{4}, -4\right)$ **b.** $(6, -4)$

3a. consistent, dependent; infinite number of solutions **b.** inconsistent; no solution **4.** 18.75 lb of Sumatra beans and 31.25 lb of Kona beans

Exercises 1. elimination **3.** $(8, -11)$ **5.** $(-2, -1)$ **7.** $(-55, -21)$ **9.** $(-2, 5)$

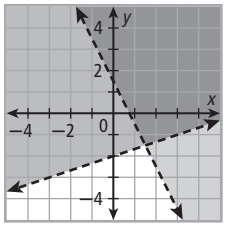
11. inconsistent; no solution
13. consistent, dependent; infinite number of solutions
15. $\left(-6, \frac{3}{2}\right)$ **17.** $\left(\frac{1}{4}, 1\right)$ **19.** $(-7, -6)$
21. $(-2.45, -4.8)$ **23.** consistent, dependent; infinite number of solutions **25.** consistent, dependent; infinite number of solutions **27.** $x + y = 1200$; $x = \frac{1}{2}y$; (400,800) **29.** $(6, 2)$
31. $(20, -3)$ **33a.** 22
b. The total number of coins increases. **c.** 12 dimes and 18 nickels **35a.** 266.5 mi
b. $y = 128.43x$ **c.** ≈ 7.12 mi
37. student = \$5.50 and adult = \$7.50 **41.** G **43.** $(2, -1)$; independent, consistent
47a. $p = 22\frac{7}{9}$; $q = 8\frac{8}{9}$
b. $p = 17\frac{2}{3}$; $q = 29\frac{1}{3}$
49. $28c^2 + 1$; 253 **51.** $\frac{2}{9y^2}$; $\frac{2}{81}$
53. $f(x) = -1.5x - 0.5$
55.



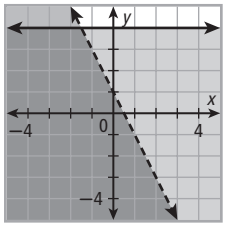
3-3

Check It Out!

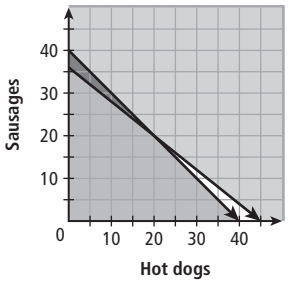
1a.



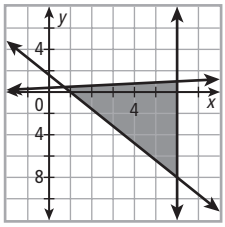
b.



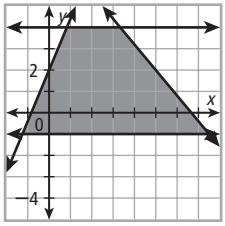
2. $\begin{cases} d + s \le 40 \\ 2d + 2.5s \ge 90 \end{cases}$



3a. triangle

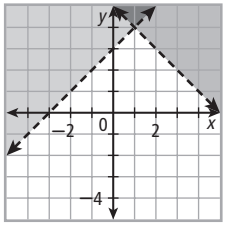


b. trapezoid

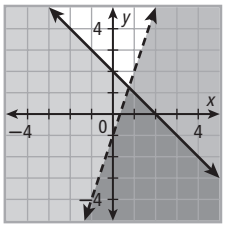


Exercises

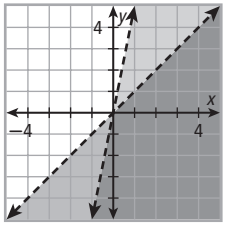
3.



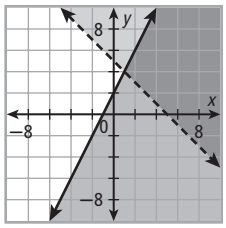
5.



7. rectangle **9.** isosceles triangle
11.



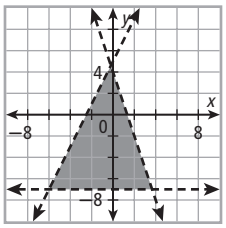
13.



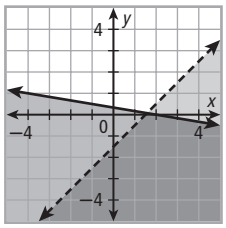
15. $\begin{cases} x + y \le 10{,}000 \\ y \le 0.2x \\ x \ge 0 \\ y \ge 0 \end{cases}$

17. trapezoid
19. isosceles right triangle
27. $\begin{cases} x \ge 0 \\ y \ge 0 \\ x + y \le 114{,}650 \\ x + y \ge 56{,}801 \\ y \ge x + 2000 \end{cases}$

29.



31.

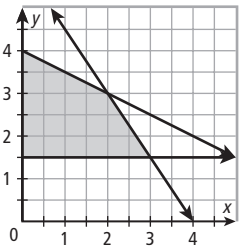


35. G **39.** \$20,000 **41.** $\frac{3}{4}$; $-\frac{4}{3}$
43. 1; −1 **45.** $y = -3$
47. $y = -\frac{1}{3}x + 9$ **49.** $y = -x + 5$

3-4

Check It Out!

1.

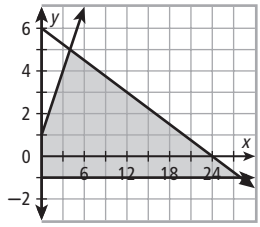


2. $P = 140$ **3.** 8 of bookcase A and 4 of bookcase B

Exercises

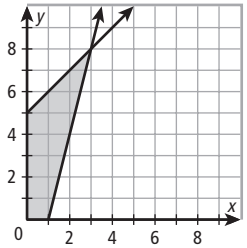
1. Constraints

3.

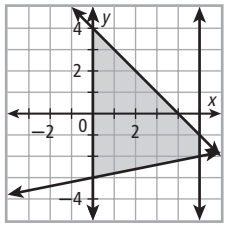


5. $P = 106$ **7.** $P = 3.9$

9.



11.



13. $P = -36$ **15.** 60 radio and 24 prime-time television commercials **17.** 32 h

19. right triangle; $\begin{cases} y \le x + 2 \\ y \ge 2x \\ y \ge -\frac{1}{2}x \end{cases}$

21. 20 stops **23.** 40 Soy Joy and 20 Vitamin Boost **27.** D **29.** G

31. $f(7) = \frac{1}{11}; f\left(-\frac{1}{2}\right) = -\frac{1}{4}$

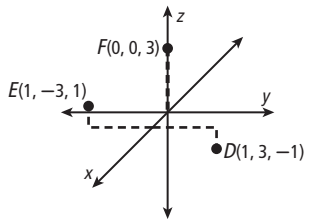
33. $f(7) = 8; f\left(-\frac{1}{2}\right) = \frac{1}{2}$

37. right triangle

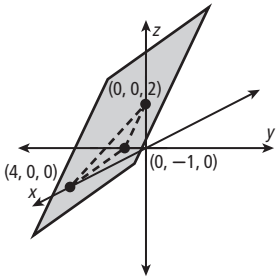
3-5

Check It Out!

1.



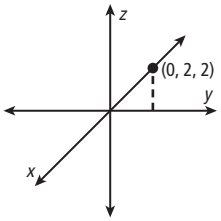
2.



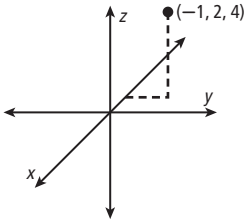
3a. $3.5x + 1.5y + 0.75z = 61.5$ **b.** 15

Exercises

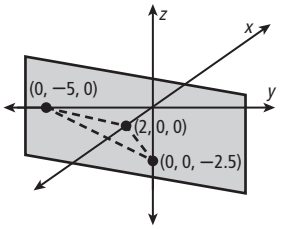
3.



5.



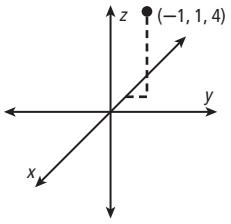
7.



9a. $225x + 150y + 300z = 3000$

b. 1; 6; 6; 0 **c.** 20

11.



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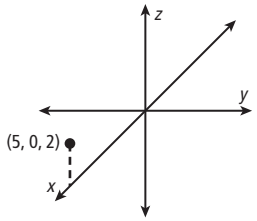
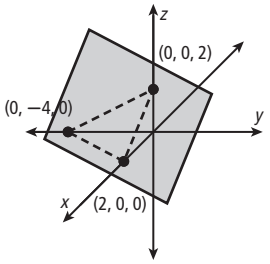
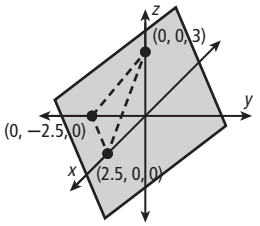


19.



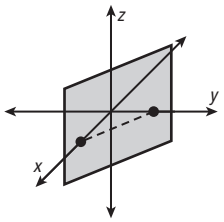
23.



29. No. **31b.** $(7, 12, 8.5)$

c. $(7, 12, 12.5)$ **33.** A **35.** H

37.



41. $x + 2y - 4z = 4$ **43.** square pyramid **45.** sphere **47.** $x = 10$; $y = 2$ **49.** $x = 5; y = \frac{1}{3}$

3-6

Check It Out!

1. $x = -2, y = 1, z = 2$

2. first place—3 points; second place—2 points; third place—1 point

3a. consistent; infinite number of solutions **b.** inconsistent; no solution

Exercises

1. $x = -4, y = 3, z = 3$

3. $x = 1, y = 3, z = 1$

5. inconsistent; no solution

7. inconsistent; no solution

9. $x = 3, y = 1, z = 6$

11. Talent—30%; Presentation—20%; Star Quality—50%

13. inconsistent; no solution

15. $m\angle A = 120°; m\angle B = 45°; m\angle A = 15°$ **19a.** $(5, -2, 50)$

b. 50 ft **c.** $(5, -2, 0)$ **21.** J

23. $w = 1, x = -2, y = -1, z = 3$

25. $(3, 3)$ **27.** 7.15 m by 5.2 m

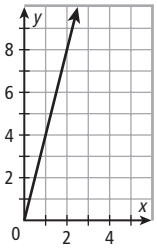
29. $y = \frac{2}{3}x - 4$

Selected Answers

Extension

Check It Out!

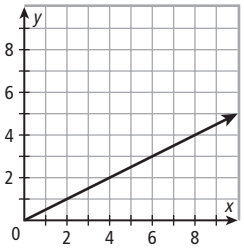
1a. $\begin{cases} x = 5t \\ y = 20t \end{cases}$



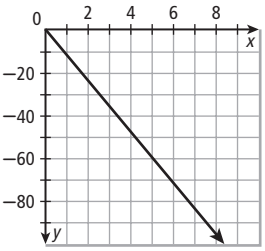
b. At $t = 10$, the helicopter has a ground distance of 50 ft from its takeoff point and an altitude of 200 ft. **2.** $y = 4x$

Exercises

1.



3.



5. $y = \frac{2}{3}x$ **7.** $y = 10x$

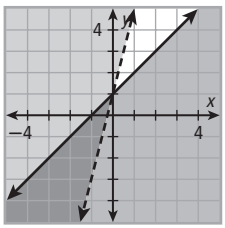
9a. $\begin{cases} x = 1.8t \\ y = -0.9t \end{cases}$

b. −45 m **c.** −77,760 m

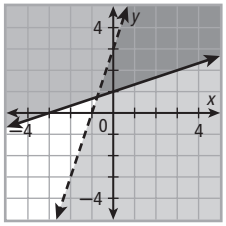
Study Guide: Review

1. dependent **2.** elimination **3.** system of linear inequalities; feasible region **4.** three-dimensional coordinate system; ordered triple **5.** consistent **6.** $(5, 10)$ **7.** $(4, 2)$ **8.** $(-4, -1)$ **9.** $(0, -2)$ **10.** independent; one solution **11.** dependent; infinitely many solutions **12.** inconsistent; no solution **13.** independent; 1 solution **14.** 3 locks **15.** $(1, 3)$ **16.** $(6, 5)$ **17.** $(-2, -8)$ **18.** $(4, -2)$ **19.** $(4, 5)$ **20.** $(3, 4)$ **21.** $(5, 2)$ **22.** $(2, 1)$ **23.** 48 oz pine; 32 oz lavender

24.



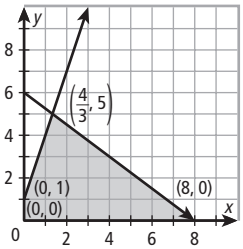
25.



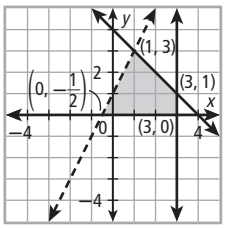
26. right triangle **27.** trapezoid

28. $\begin{cases} x + y \leq 120 \\ 8x + 11.5y < 1200 \end{cases}$

29.



30.



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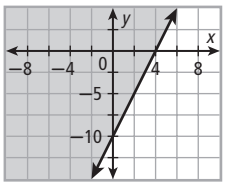
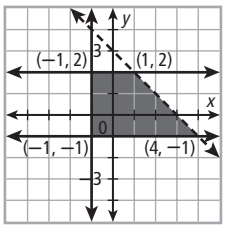


32.



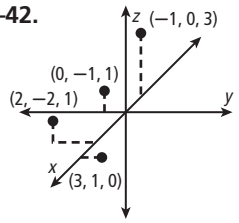
33. 58 **34.** −4.5

35. $\begin{cases} x \geq 0 \\ y \geq 0 \\ 6x + 4y \leq 720 \\ x \geq 2y \end{cases}$

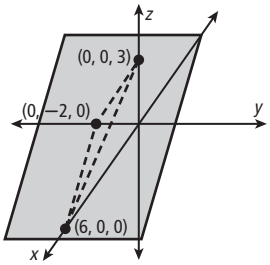
36. $P = 8x + 9y$ **37.** \$1125

38. 25 phones with contracts and 5 without contracts

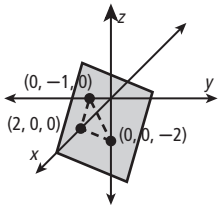
39–42.



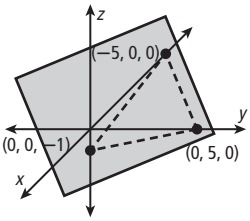
43.



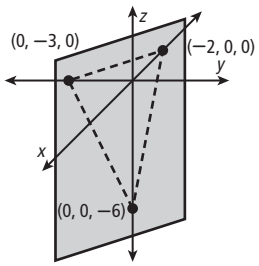
44.



45.



46.



47. $2d + 9p + 4c = 35$, where d = drinks, p = pizza, c = ice cream **48.** $(1, 2, 3)$ **49.** $(1, -1, 2)$ **50.** inconsistent, no solution **51.** dependent, infinitely many solutions

Chapter 4

4-1

Check It Out!

1a. 3×4 **b.** 11 **c.** m_{14} and m_{23}

2a. $\begin{bmatrix} 4 & 0 & -8 \\ 6 & 2 & 18 \end{bmatrix}$

b. not possible

c. $\begin{bmatrix} -4 & 2 & 2 \\ 0 & -2 & 2 \end{bmatrix}$

3.

Ticket Service Prices		
Day	**Plaza**	**Balcony**
Days 1–2	$120	$70
Days 3–8	$100	$56
Days 9–10	$160	$90

4a. not possible

b. $\begin{bmatrix} -1 & -10 \\ -6 & 47 \end{bmatrix}$ **c.** $\begin{bmatrix} -9 & 4.5 & 12 \end{bmatrix}$

Exercises

1. entry **3.** $\begin{bmatrix} 1.5 & 7.8 & 4 \\ -1.2 & 0.4 & 1 \end{bmatrix}$

5. $\begin{bmatrix} -1.5 & 0.2 & -2 \\ 1.2 & -4.4 & 1 \end{bmatrix}$

7. $P_T = \begin{bmatrix} 9.74 & 14.07 & 15.16 \\ 6.50 & 10.28 & 11.91 \\ 16.24 & 22.73 & 24.90 \\ 5.41 & 8.12 & 9.20 \end{bmatrix}$

9. $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 3 \\ 2 & 0 & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \end{bmatrix}$ **11.** not possible

13. $F - E = \begin{bmatrix} -7.4 & 0 \\ 37 & -2.4 \end{bmatrix}$

15. not possible

17. $\begin{bmatrix} 29{,}061 & 13{,}483 & 20{,}147 \end{bmatrix}$

19. not possible **21.** not possible

23. never true **27.** always true

29. $a = -6$; $b = 11$; $c = 4$

31. no **33.** F **35.** $\frac{1}{2}$

39. $\begin{bmatrix} 2.5 & -4 \\ 1 & -7 \end{bmatrix}$ **41.** $20n$ **43.** no

4-2

Check It Out!

1a. no **b.** no **c.** 4×3

2a. $\begin{bmatrix} 67 & 5 \\ 62 & 72 \\ 66 & -6 \end{bmatrix}$ **2b.** $\begin{bmatrix} 3 & 41 & 97 \\ -30 & 78 & 128 \end{bmatrix}$

3. $\begin{bmatrix} 2436 & 1196 & 1240 \\ 1605 & 786 & 819 \end{bmatrix}$; 819

4a. not possible

b. $A^3 = \begin{bmatrix} 259 & 129 \\ -86 & -42 \end{bmatrix}$

c. $B^3 = \begin{bmatrix} 82 & 103 & 2 \\ 125 & 33 & -39 \\ 17 & -12 & 34 \end{bmatrix}$

d. $I^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Exercises

1. multiplicative identity matrix

3. no **5.** yes; 5×5 **7.** yes; 2×2

9. $\begin{bmatrix} -2 & -18 & -9 \\ 4 & 29 & 15 \\ -2 & 3 & 0 \end{bmatrix}$ **11.** not possible

13. $\begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix}$ **15.** $\begin{bmatrix} -1 & 2 \\ -1 & -2 \end{bmatrix}$

17. not possible **19.** no

21. yes; 3×1 **23.** no

25. not possible

27. $\begin{bmatrix} 27 & -10 \\ -10 & 3 \\ -5 & 1 \end{bmatrix}$ **29.** $\begin{bmatrix} -2 & 3 & -4 \\ 1 & -1 & 1 \\ 4 & 1 & 3 \end{bmatrix}$

31. $\begin{bmatrix} -1 & 2 \\ -1 & -2 \end{bmatrix}$ **33.** $\begin{bmatrix} 6 & 2 & 1 \\ 5 & 4 & 1 \\ 7 & 3 & 3 \end{bmatrix}$

35. not possible

37. $\begin{bmatrix} 5 & 11 \\ 7 & 27 \end{bmatrix}$ **39.** $\begin{bmatrix} -2 & 8 & 12 \\ 6 & 0 & 4 \end{bmatrix}$

41a. $SD = \begin{bmatrix} 122.8 & 124.5 & 160.2 & 148.1 \\ 161.85 & 160.4 & 207.8 & 191.95 \\ 131.45 & 130.3 & 168.4 & 155.85 \\ 168.95 & 169.0 & 217.8 & 201.65 \end{bmatrix}$

b. Ted: 122.8; Chloe: 160.4; Biko: 168.4; Hana: 201.65

43. sometimes true

45. sometimes true **47.** -8

49. Madison: 122; Devyn: 113.5; Ali: 69.5 **51.** Tristan Island **55.** G

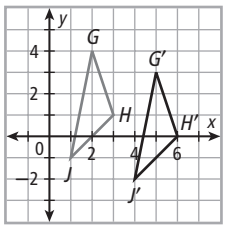
57. BA **59a.** yes **b.** p_{12} and p_{21}

61. 10 **63.** 11 **69.** not possible

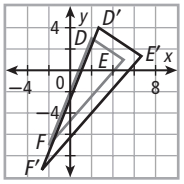
4-3

Check It Out!

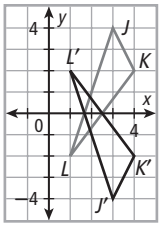
1. $G'(5, 3)$, $H'(6, 0)$, $J'(4, -2)$



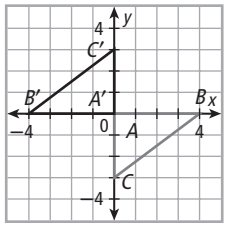
2. $D'\left(2\frac{2}{3}, 4\right)$, $E'\left(6\frac{2}{3}, 1\frac{1}{3}\right)$, $F'\left(-2\frac{2}{3}, -9\frac{1}{3}\right)$



3. $J'(3, -4)$, $K'(4, -2)$, $L'(1, 2)$



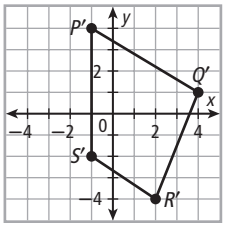
4. $A'(0, 0)$, $B'(-4, 0)$, $C'(0, 3)$; the image is rotated 180°.



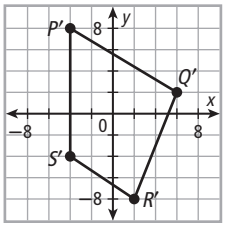
Exercises

1. reflection matrix

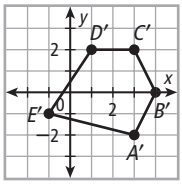
3. $P'(-1, 4)$, $Q'(4, 1)$, $R'(2, -4)$, $S'(-1, -2)$



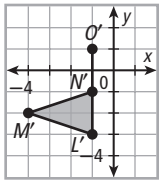
5. $P'(-4, 8)$, $Q'(6, 2)$, $R'(2, -8)$, $S'(-4, -4)$



7. $A'(3, -2)$, $B'(4, 0)$, $C'(3, 2)$, $D'(1, 2)$, $E'(-1, -1)$

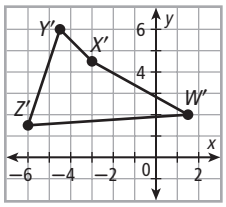


9.

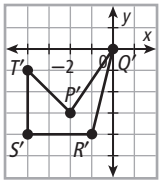


The image is rotated 180°.

11. $W'\left(\frac{3}{2}, 3\right)$, $X'\left(-3, \frac{9}{2}\right)$, $Y'\left(-\frac{9}{2}, 6\right)$, $Z'\left(-6, \frac{3}{2}\right)$



13.



The image is rotated 90° counterclockwise.

15a. $\begin{bmatrix} -2.5 & 2.5 & 4.5 & 0.5 & -4 \\ 3 & 3 & -2 & -4.5 & -1.5 \end{bmatrix}$

15b. counterclockwise

19. $(-8, 3)$, $(-6, 3.5)$, $(-4, 3)$, $(-2, 2.5)$, $(2, 3)$, $(1.5, 0)$, $(-1, 0)$

21. $(-5, -4)$, $(-3, -4.5)$, $(-1, -4)$, $(1, -3.5)$, $(5, -4)$, $(4.5, -1)$, $(2, -1)$

23. $(-4, -5)$, $(-4.5, -3)$, $(-4, -1)$, $(-3.5, 1)$, $(-4, 5)$, $(-1, 4.5)$, $(-1, 2)$

25. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$; $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

27. $\mathbb{B}$ **29.** $\mathbb{A}$ **33.** yes **35.** yes **37.** not possible

4-4

Check It Out!

1a. 10 **b.** $-2\frac{1}{4}$ **c.** $\pi - \frac{1}{2}$

2. $D = \begin{vmatrix} 6 & -2 \\ 3 & -1 \end{vmatrix} = 0$

$\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = \begin{vmatrix} 14 & -2 \\ 7 & -1 \end{vmatrix} = 0$

dependent; infinitely many solutions.

3. 75 **4.** 240 g protein, 380 g carbohydrates, and 80 g fat

Exercises

1. A 0 entry in a coefficient matrix corresponds to a coefficient of 0 for one variable in one equation.

3. 2.25 **5.** 0 **7.** no solution

9. $\left(2\frac{1}{2}, -\frac{1}{4}\right)$ **11.** 0

13. trail mix \$2.99; mixed nuts \$6.98; dried fruit \$3.99 **15.** −1

17. $\pi r^2 - 2r^2$ **19.** $(\approx 1.27, \approx 1.11)$

21. $\left(-4\frac{2}{3}, \frac{7}{9}\right)$ **23.** 1.2

25. bicycling: 420 Cal/h; racquetball: 540 Cal/h; swimming: 600 Cal/h **27.** 17 square units

29. 1 point: 9 voters; 2 points: 11 voters; 3 points: 18 voters **31.** −16

33. a. 235 dimes, 190 nickels

b. \$33 **37.** $\mathbb{D}$ **39.** 2.5, or $\frac{5}{2}$ **43.** 254

45. $\left(-\frac{4}{3}, -4\right)$ **47.** $\left(-\frac{1}{3}, -\frac{2}{3}\right)$

49. $D'(1, -1)$, $E'(4, 2)$, $F'(-2, 3)$, $G'(-1, 1)$

51. $D'(3, 3)$, $E'(12, -6)$, $F'(-6, -9)$, $G'(-3, -3)$

4-5

Check It Out!

1. yes

2. $\begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ \frac{1}{4} & -\frac{1}{4} \end{bmatrix}$ **3.** $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$; $(3, 1)$

4. smarty pants

Exercises

1. Set the product of the coefficient matrix and the variable matrix equal to the constant matrix. **3.** no

5. $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ **7.** no inverse **9.** $\begin{bmatrix} 8 & -7 \\ -9 & 8 \end{bmatrix}$

11. $\begin{bmatrix} 5 & 9 \\ -4 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$; $(-25, 14)$

13. $\begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$; in New Delhi

15. yes **17.** $\begin{bmatrix} 8 & -2 \\ -6 & 1 \end{bmatrix}$

19. $\begin{bmatrix} 8 & -3 \\ -5 & 2 \end{bmatrix}$ **21.** $\begin{bmatrix} -11 & -3 \\ 7 & 2 \end{bmatrix}$

23. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$; $(4, 1)$

25. $\begin{bmatrix} 4 & -3 \\ -9 & 7 \end{bmatrix}$; Monday early

27a. $\begin{bmatrix} 6 & 2 \\ 1 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 6 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 34 \\ 7 \end{bmatrix}$

c. $\begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{3}{2} \end{bmatrix}$

d. 5 six-person boats and 2 two-person boats

31. 46 \$50 bills and 27 \$100 bills

33a. They are halved. **37.** $\begin{bmatrix} \frac{1}{a} \end{bmatrix}$

39. The two matrices are inverses of each other. **41.** $\mathbb{H}$ **43.** F

45. points/game: 2; assists/game: 5; turnovers/game: −4; steals/game: 3

47a. play to win

b. $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} -3 & -12 & 23 & -11 \\ -6 & -13 & 0 & 20 \\ 18 & 25 & 0 & 0 \end{bmatrix}$

49. 0.175 **51.** $(1, 1, 0)$

53. 8.5 **55.** −43

4-6

Check It Out!

1a. $\left[\begin{array}{cc|c} -1 & -1 & 0 \\ -1 & -1 & -2 \end{array}\right]$

b. $\left[\begin{array}{ccc|c} -5 & -4 & 0 & 12 \\ 1 & 0 & 1 & 3 \\ 0 & 4 & 3 & 10 \end{array}\right]$

2a. $\left[\begin{array}{cc|c} 4 & 4 & 32 \\ 1 & 3 & 16 \end{array}\right]$; $(4, 4)$

b. $\left[\begin{array}{cc|c} 9 & 3 & 15 \\ -6 & -2 & 10 \end{array}\right]$; no solution

3a. $\left[\begin{array}{ccc|c} 3 & -1 & 5 & -1 \\ 1 & 0 & 2 & 1 \\ 1 & 3 & -1 & 25 \end{array}\right]$; $(5, 6, -2)$

b. $t = 820$; $d = 1600$; 1600 days.

Exercises

1. The coefficients are in columns to the left of the vertical bar that separates them from the constant terms.

3. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 2 & 0 & 1 & 12 \\ 0 & -1 & 1 & 3 \end{array}\right]$

5. $\left[\begin{array}{ccc|c} -3 & 1 & 0 & -2 \\ 0 & \frac{1}{4} & -1 & -1 \\ -\frac{1}{2} & 0 & 1 & 8 \end{array}\right]$

7. $\left[\begin{array}{cc|c} -1 & 8 & 7 \\ \frac{1}{2} & 3 & 0 \end{array}\right]$; $\left(-3, \frac{1}{2}\right)$

Selected Answers

9. $\left[\begin{array}{cc|c} 1 & -1 & -4 \\ 4 & -4 & -3 \end{array}\right]$;
the system is inconsistent.

11. $\left[\begin{array}{ccc|c} \frac{1}{2} & \frac{3}{2} & -1 & 0 \\ -2 & 1 & 0 & 4 \\ 1 & 1 & 1 & 3 \end{array}\right]$

13. $\left[\begin{array}{ccc|c} 0.1 & 0.2 & 0.15 & 1.0 \\ 1 & 1 & -1 & 0 \\ 1.3 & -2 & 0 & 0 \end{array}\right]$

15. $\left[\begin{array}{cc|c} 5 & -1 & 2 \\ 1 & -1 & -4 \end{array}\right]$; $(1.5, 5.5)$

17. 12 roosters, 4 hens, and 84 chicks.

23. $\left[\begin{array}{cc|c} 3 & -1 & -9 \\ -4 & 7 & 12 \end{array}\right]$; $(-3, 0)$

25. $\left[\begin{array}{ccc|c} 2 & 5 & -1 & 0 \\ -1 & 3 & 0 & -7 \\ 1 & 0 & 7 & 25 \end{array}\right]$; $(4, -1, 3)$

27b. first-place: 3 points, second-place: 2 points, third-place: 1 point.

29. $(-1, 4, 2)$ **31.** $\left(\frac{5}{6}, \frac{5}{6}, -\frac{5}{2}\right)$

33a. $\begin{cases} 2x + 7y = 24 \\ 4x + 13y = 46 \end{cases}$

33b. $\left[\begin{array}{cc|c} 2 & 7 & 24 \\ 4 & 13 & 46 \end{array}\right]$ **33c.** $(5, 2)$

33d. meal: 5 tickets, ride: 2 tickets

35. $\mathbb{A}$ **37.** $\mathbb{C}$ **41.** a reduction by a factor of $\frac{3}{8}$ **43.** $P = 6$ at $(0, 3)$

45. $\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \end{bmatrix}$; $(1, 3)$

Extension

Check it Out!

1. $A^3 = \begin{bmatrix} 6 & 1 & 0 & 0 & 5 \\ 3 & 4 & 1 & 3 & 5 \\ 3 & 1 & 1 & 1 & 3 \\ 4 & 1 & 2 & 6 & 2 \\ 2 & 2 & 1 & 4 & 2 \end{bmatrix}$;
5 3-step paths from B to F and from F to itself

Exercises

1. $\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 & 0 \end{bmatrix}$

3. A to D, A to E, B to A, B to C, C to D, C to E, D to C, E to F, F to B

5. For vertex A, one 1-step roundtrip is possible. For A, C, D, E, and F, at least one 2-step roundtrip is possible. For A and C, at least one 3-step roundtrip is possible.

Study Guide: Review

1. scalar **2.** constant matrix
3. square matrix **4.** not possible

5. $\begin{bmatrix} 0.4 & 0.6 \\ 0.8 & 1 \end{bmatrix}$ **6.** $\begin{bmatrix} 2 & -\frac{7}{3} & \frac{4}{3} \\ -\frac{11}{3} & \frac{2}{3} & 1 \end{bmatrix}$

7. $\begin{bmatrix} 6 & -9 & 4 \\ -9 & 2 & 5 \end{bmatrix}$ **11.** $\begin{bmatrix} -4 & -3 & 5 \\ 4 & 2 & -8 \\ -2 & -4 & -5 \end{bmatrix}$

12. $\begin{bmatrix} -7 & 9 \\ -3 & -3 \\ -8 & -1 \end{bmatrix}$ **13.** not defined

14. $\begin{bmatrix} -3 & 5 & 10 \\ -12 & 2 & 9 \end{bmatrix}$ **15.** not possible

16. $\begin{bmatrix} 15 & 1 & 7 \\ -1 & 5 & 5 \\ -7 & 5 & 9 \end{bmatrix}$ **17.** $\begin{bmatrix} 71 & -7 \\ 70 & -6 \end{bmatrix}$

18a. $A = \begin{bmatrix} 5 & 2.5 \\ 7.5 & 4.25 \\ 9 & 5.75 \end{bmatrix}$;
$B = \begin{bmatrix} 67 & 196 & 245 \\ 104 & 75 & 154 \end{bmatrix}$

18b. Thursday: \$595; Friday: \$1788.75; Saturday: \$3090.50
18c. adults: \$4010; students: \$1464.25 **19.** $(0, 0)$, $(1, 4)$, $(4, 5)$, and $(2, 1)$ **20.** $(-3, -1.5)$, $(-1.5, 4.5)$, $(3, 6)$, and $(0, 0)$
21. $(-2, 1)$, $(-1, -3)$; $(2, -4)$, $(0, 0)$; reflected across the x-axis.
22. $(-1, 2)$, $(3, 1)$, $(4, -2)$, and $(0, 0)$; rotated 90°clockwise.
23. 2 **24.** 0 **25.** $\frac{1}{2}$ **26.** 22 **27.** -31
28. 0 **29.** $(5, 4)$ **30.** $(2, -5)$
31. infinitely many solutions
32. $(2, 4, -3)$ **33.** no solution
34. $(0.5, 2, 1.5)$ **35a.** $\begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = 5$
35b. $(1, 2)$ **36b.** 72 small, 18 medium, and 12 large

37. $\begin{bmatrix} 0.15 & -0.1 \\ 0.05 & 0.3 \end{bmatrix}$ **38.** $\begin{bmatrix} \frac{4}{3} & \frac{8}{3} \\ 0 & 5 \end{bmatrix}$

39. no inverse

40. $\begin{bmatrix} -0.25 & -0.25 & 0.5 \\ 1.5 & 0.5 & -1 \\ -2.25 & -0.25 & 1.5 \end{bmatrix}$

41. $\begin{bmatrix} -2 & 0 & 2 \\ -5 & -1 & 7 \\ 6 & 2 & -8 \end{bmatrix}$

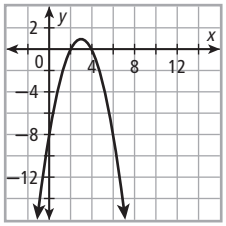
42. does not exist **43.** $(20, 10)$
44. $(5, 4)$ **45.** $(4, 3, 2)$
46. $(-1, -2.5, 3.5)$
47. $(0.25, -0.5)$ **48.** $(-2, -6)$
49. $(0.1, -0.05, 0.2)$ **50.** $(1, 1, 2)$
51. 12 first-place, 4 second-place, 11 third-place

Chapter 5

5-1

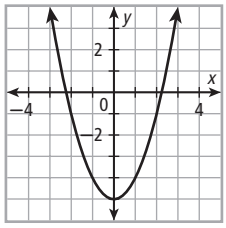
Check It Out!

1.



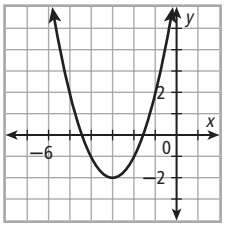
x	−1	1	3	5	7
$g(x)$	−15	−3	1	−3	−15

2a.



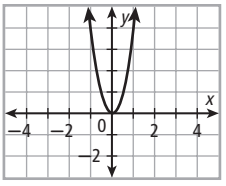
g is f translated 5 units down.

b.



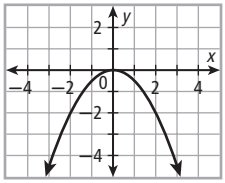
h is f translated 3 units left and 2 units down.

3a.



g is a horizontal compression of f by a factor of $\frac{1}{2}$.

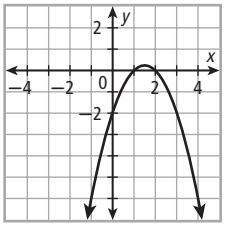
b.



h is f reflected across the x-axis and vertically compressed by a factor of $\frac{1}{2}$. **4a.** $g(x) = \frac{1}{3}(x-2)^2 - 4$ **b.** $g(x) = -(x+5)^2 + 1$ **5.** Vertical compression by a factor of $\frac{13}{15}$; the braking distance will be less with optimally inflated new tires than with tires having more wear.

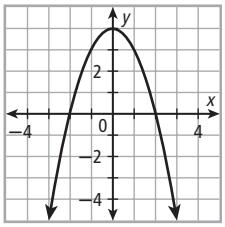
Exercises **1.** vertex

3.

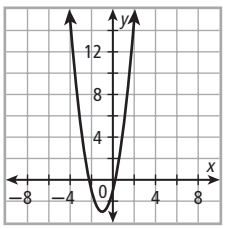


5. d is f translated 4 units right. **7.** h is f translated 1 unit left and 3 units down. **9.** h is a horizontal stretch of f by a factor of 8. **11.** h is f reflected across the x-axis and horizontally compressed by a factor of $\frac{1}{5}$. **13.** d is f reflected across the x-axis and vertically compressed by a factor of $\frac{2}{3}$. **15.** $h(x) = -x^2 - 6$

17.



19.

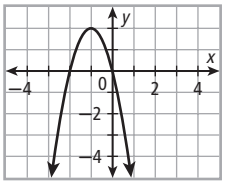


21. h is f translated 5 units left. **23.** g is f translated 4 units left and 3 units down. **25.** j is f translated 4 units right and 9 units down. **27.** h is f reflected across the x-axis and vertically stretched by a factor of 20. **29.** $g(x) = -\frac{1}{2}(x-1)^2$ **31.** Vertical translation; at any given speed, the gas mileage for an SUV is 18 mi/gal less than for a compact car. **33.** p is f reflected across the x-axis and translated 4 units right. **35.** h is f vertically stretched by a factor of 4 and translated 2 units down. **37.** g is f horizontally compressed by a factor of $\frac{1}{3}$ and translated 1 unit up. **39.** C **41.** A **43.** horizontal line; linear or constant function **45a.** vertical compression by a factor of 0.38 and translation 3.5 units right and 59 units up **b.** $y = -6.08(t-4)^2 + 95$ **47.** J **49.** G **51.** translation 6 units right and 6 units up: $y = -3(x-3)^2 + 3$ **55.** $f(x) = \sqrt{x}$ **57.** $y = 2x + 10$

5-2

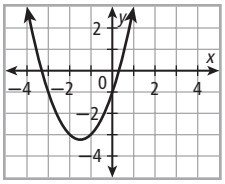
Check It Out! **1.** $x = 3$

2a.



downward; $x = -1$; $(-1, 2)$; 0

b.



upward; $x = -\frac{3}{2}$; $\left(-\frac{3}{2}, -\frac{13}{4}\right)$; -1

3a. minimum: -6; D: $\mathbb{R}$; R: $\{y \mid y \geq -6\}$ **b.** maximum: -4; D: $\mathbb{R}$; R: $\{y \mid y \leq -4\}$ **4.** 30.0 mi/gal at 49 mi/h

Exercises **1.** minimum **3.** $x = 0$ **5.** downward; $x = -1$; $(-1,-7)$; -8 **7.** downward; $x = 2$; $(2, 3)$; -1 **9.** maximum: $\frac{1}{4}$; D: $\mathbb{R}$; R: $\left\{y \mid y \leq \frac{1}{4}\right\}$ **11.** 3.125 m **13.** $x = 1$ **15.** upward; $x = -\frac{1}{2}$; $\left(-\frac{1}{2}, -\frac{9}{4}\right)$; -2 **17.** upward; $x = 2$; $(2, -6)$; -4 **19.** upward; $x = -\frac{1}{3}$; $\left(-\frac{1}{3}, -\frac{25}{3}\right)$; -8 **21.** downward; $x = 0$; $(0,-2)$; -2 **23.** upward; $x = -2$; $(-2,1)$; 2 **25.** maximum: 9; D: $\mathbb{R}$; R: $\{y \mid y \leq 9\}$ **27.** maximum: -4; D: $\mathbb{R}$; R: $\{y \mid y \leq -4\}$ **29.** minimum: 0; D: $\mathbb{R}$; R: $\{y \mid y \geq 0\}$ **31.** 64 ft **33a.** 562.5 mm **b.** 93.75 to 1 **c.** 168.75 m **35.** minimum: ≈ -3.029771 **37.** minimum: ≈ -1.253333 **41a.** about 1.6 s **b.** about 45 ft **43.** G **45.** G **51.** $60\sqrt{2}$ **53.** $\frac{3\sqrt{5}}{5}$ **55.** $f(0) = 10$; $f\left(\frac{1}{2}\right) = \frac{29}{4}$; $f(-2) = 26$ **57.** $f(0) = -20$; $f\left(\frac{1}{2}\right) = -22$; $f(-2) = -12$ **59.** $y + 4 = 3(x-1)$ **61.** $y - 5 = -2(x-3)$

5-3

Check It Out! **1.** $-3, 1$ **2a.** $-1, 6$ **b.** $0, 8$ **3.** 3 s **4a.** $x = 2$ **b.** $x = -\frac{3}{5}$, $x = \frac{3}{5}$ **5.** Possible answer: $f(x) = x^2 - 25$

Exercises **1.** roots **3.** 2, 4 **5.** 1, 6 **7.** $-4, 0$ **9.** $-2, 8$ **11.** 4 s **13.** $x = 2$ **19.** $-3, 2$ **21.** $-8, -3$ **23.** 0, 9 **25.** $-8, 1$ **27.** 4 s **29.** $x = -\frac{9}{2}$, $x = \frac{9}{2}$ **31.** $x = -\frac{1}{2}$, $x = \frac{1}{2}$ **33.** $x = \frac{2}{7}$ **37.** 0, 6 **39.** 6 **41.** 11 **43.** 5, 6 **45.** $-7, -2$ **47a.** $h(t) = -16t^2 + 16t + 5$ **b.** 1.25 s **49.** $x = -5$, $x = -1$ **51.** $x = -\frac{1}{3}$ **53.** $x = -2$, $x = 3$ **55a.** $(0, -16)$ **b.** -16 **c.** $-4, 4$ **57a.** $(1, 2)$ **b.** 0 **c.** 0, 2 **59a.** $\left(-\frac{1}{6}, -4\frac{1}{12}\right)$ **b.** -4 **c.** $-1\frac{1}{3}$, 1 **61.** 20 ft by 4 ft **63.** 10 m by 5 m **67.** B **69.** C **71.** $x = 0$, $x = \frac{3}{2}$ **73.** $x = \frac{1}{4}$, $x = \frac{1}{2}$ **75a.** $(a+b)(a^2 - ab + b^2) = a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3 = a^3 + b^3$ **b.** $(2x+3)(4x^2 - 6x + 9)$ **c.** $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ **d.** $(x-1)(x^2 + x + 1)$ **77.** 8.64×10^{12} **79.** 6.5×10^{-10} **81.** $w = 2.2$ **83.** h is a vertical compression of f by a factor of 0.5. **85.** g is f translated 1 unit left.

5-4

Check It Out! **1a.** $x = \pm\frac{5}{2}$ **b.** $x = -11$, $x = 3$ **2a.** $x^2 + 4x + 4 = (x+2)^2$ **b.** $x^2 - 4x + 4 = (x-2)^2$ **c.** $x^2 + 3x + \frac{9}{4} = \left(x + \frac{3}{2}\right)^2$ **3a.** $x = \frac{9 \pm \sqrt{89}}{2}$ **b.** $x = -1$, $x = 9$

4a. $f(x) = (x + 12)^2 + 1; (-12, 1)$
b. $g(x) = 5(x - 5)^2 + 3; (5, 3)$

Exercises **1.** $\left(\frac{b}{2}\right)^2$ **3.** $x = 1, x = 9$
5. $x^2 + 14x + 49 = (x + 7)^2$
7. $x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$
9. $x = 2, x = 4$ **11.** $x = -2 \pm 2\sqrt{7}$
13. $x = -2 \pm \frac{\sqrt{46}}{2}$ **15.** $g(x) = (x - 5)^2 - 14; (5, -14)$ **17.** $f(x) = (x + 4)^2 - 26; (-4, -26)$ **19.** $h(x) = 3(x - 2)^2 - 16; (2, -16)$ **21.** $x = -7, x = 13$ **23.** $x^2 - 18x + 81 = (x - 9)^2$ **25.** $x^2 - \frac{1}{2}x + \frac{1}{16} = \left(x - \frac{1}{4}\right)^2$ **27.** $x = 2 \pm \sqrt{3}$ **29.** $x = 2, x = 6$ **31.** $x = -1 \pm \frac{2\sqrt{3}}{3}$ **33.** $g(x) = (x + 7)^2 + 22; (-7, 22)$ **35.** $f(x) = (x + 2)^2 - 11; (-2, -11)$ **37.** $h(x) = 2(x + 1.5)^2 + 20.5; (-1.5, 20.5)$
39a. about 12.3 s **b.** about 2.3 s
41. $x = \pm\sqrt{3}$ **43.** $x = \pm 5$ **45.** $x = -13 \pm \sqrt{7}$ **47.** $x = \frac{-3 \pm 5\sqrt{2}}{2}$
49. $x = \frac{-3 \pm \sqrt{5}}{3}$ **51.** $x = -5, x = -3$
53. $x = \frac{-2 \pm \sqrt{7}}{3}$ **55.** $x = \frac{7 \pm \sqrt{57}}{2}$
57. $x = -3 \pm \sqrt{5}$ **59.** $x = 4 \pm 2\sqrt{10}$
61a. 1.7 s **b.** 71 ft/s **65.** $x = \pm 7.416$
67. $x = \pm 4.192$ **69.** $x = \pm 1.528$
73. B **75.** A **77.** 2.5 **79.** $b = \pm 24$
81. $b = \pm 18$ **83.** $2\sqrt{5} \pm 1$
85a. 135,000 ft^2 **b.** 450 ft by 300 ft
c. 129,600 ft^2 **87.** $\{x | -6 \le x \le 14\}$
89. $\{x | -1 \le x \le 5\}$ **91.** 3×3
93. 1368; the amount in dollars the Hernandez family budgeted for housing **95.** $x = 0; (0, -1)$

5-5

Check It Out! **1a.** $2i\sqrt{3}$ **b.** $12i$
c. $-i\sqrt{7}$ **2a.** $x = \pm 6i$ **b.** $x = \pm 4i\sqrt{3}$
c. $x = \pm\frac{5}{3}i$ **3a.** $x = -4; y = -\frac{3}{10}$
b. $x = -\frac{8}{5}; y = -\frac{\sqrt{6}}{6}$ **4a.** $-2 \pm 3i$
b. $4 \pm i\sqrt{2}$ **5a.** $9 + i$ **b.** $\sqrt{3} - i$ **c.** $8i$

Exercises **1.** imaginary **3.** $2i$
5. $12i$ **7.** $x = \pm 6i$ **9.** $x = \pm 11i$
11. $x = 1; y = -1$ **13.** $-3 \pm 5i$
15. $\sqrt{5} - 5i$ **17.** $6 - i\sqrt{2}$ **19.** $-i\sqrt{10}$
21. $5i\sqrt{2}$ **23.** $x = \pm 4i$ **25.** $x = \pm 8i$
27. $x = -3; y = -5$ **29.** $\frac{3 \pm i\sqrt{7}}{8}$
31. $\frac{3 \pm i\sqrt{21}}{3}$ **33.** $-\frac{\sqrt{3}}{2} + 2i$
35. $-1 - \frac{i}{10}$ **37.** $1 - 14i$
39. $-2\sqrt{5} - 4i$ **41.** $9 + i\sqrt{2}$
43. $c = 0, d = 5$ **45.** $c = \pm 2, d = 4$
47. $x = \pm 9i$ **49.** $x = \pm 12i$
51. $x = \pm 2i\sqrt{2}$ **53.** $x = -5 \pm 2i$
55. $x = -1 \pm 2i$ **57.** $x = 12 \pm i\sqrt{5}$
59. always true **61.** sometimes true **63.** sometimes true
65. sometimes true **67.** $-1 \pm 4i$
69. $-8 \pm 3i$ **71.** $8 \pm 2i$
73. The complex conjugate of a real number a is the number a.
75a. $t = \frac{7}{2} \pm \frac{\sqrt{3}}{2}i$ **b.** no **c.** 196 ft
77. F **79.** G **81.** When $a < 0$, the 2 solutions are imaginary and complex. When $a > 0$, the 2 solutions are real and complex.
85. $T^2 = \begin{bmatrix} 12 & -3 & 5 \\ 2 & 7 & -1 \\ -4 & 4 & -2 \end{bmatrix}$
87. not defined **89a.** upward
b. $x = -2.5$ **c.** $(-2.5, -11.25)$
d. -10 **91a.** upward **b.** $x = -1$
c. $(-1, -5)$ **d.** -3 **93.** $x = -7, x = 2$
95. $x = \frac{3}{4}, x = 3$ **97.** $x = -8, x = -3$

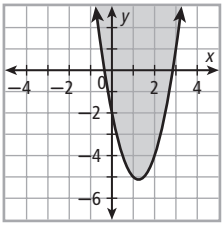
5-6

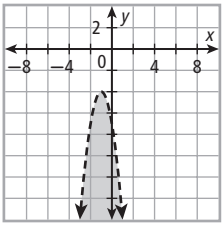
Check It Out! **1a.** $\frac{-3 \pm \sqrt{37}}{2}$
b. $4 \pm \sqrt{6}$ **2.** $\frac{1}{6} \pm \frac{\sqrt{95}}{6}i$
3a. 1 distinct real solution
b. 2 distinct nonreal complex solutions **c.** 2 distinct real solutions **4.** 449 ft

Exercises **3.** $\frac{2 \pm \sqrt{7}}{3}$ **5.** $-6, 1$
7. $\pm\frac{\sqrt{38}}{2}$ **9.** $-3 \pm i\sqrt{3}$ **11.** $-2 \pm i\sqrt{6}$
13. $\frac{-7 \pm i\sqrt{111}}{20}$ **15.** 2 distinct real solutions **17.** 14 in. and 20 in.
19. $-6, 0$ **21.** $-1 \pm \sqrt{10}$ **23.** $\pm\frac{\sqrt{21}}{7}$
25. $\frac{-1 \pm i\sqrt{3}}{2}$ **27.** $\frac{-7 \pm 3\sqrt{17}}{4}$
29. $\frac{2 \pm 2i\sqrt{2}}{3}$ **31.** 2 distinct real solutions **33.** 1 distinct real solution
35. 2 distinct real solutions
37a. 9 s **b.** 6 s **39.** $\frac{1 \pm \sqrt{3}}{2}$
41. $\frac{-3 \pm \sqrt{17}}{4}$ **43.** $\frac{1 \pm i\sqrt{87}}{2}$
45. $x = -2, x = 5$ **47.** $x = -2.5, x = 1.5$ **49.** $x = -3, x = 7$
51. $x = \pm 5$ **53.** $x = 8$ **55.** 3 in.
57. $c = -36$ **61.** B **63.** C
65. 15 cm and 8 cm
73. $\left[\begin{array}{cc|c} 2 & -3 & -7 \\ 1 & -6 & 1 \end{array}\right]; x = -5; y = -1$
75. $\left[\begin{array}{cc|c} 4 & 5 & -1 \\ 2 & -7 & 9 \end{array}\right]; x = 1; y = -1$
77. $x = 4 \pm \sqrt{14}$

5-7

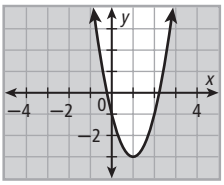
Check It Out!

1a.


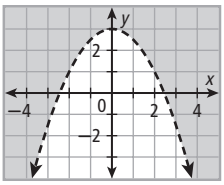
b.


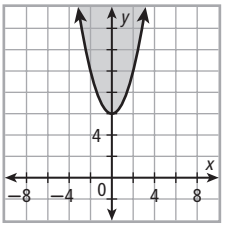
2a. $-1 < x < 2$ **b.** $x \le 0$ or $x \ge 2.5$
3a. $x \le 2$ *or* $x \ge 4$ **b.** $x < -1$ or $x > 2.5$
4. fewer than 14 or more than 36 people

Exercises

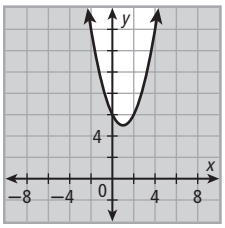
3.


5. $0 \le x \le 5$ **7.** $1.5 \le x \le 3$
9. $-8 < x < -5$ **11.** a range of costs between about \$21.78 and \$48.22

13.


15.


17.

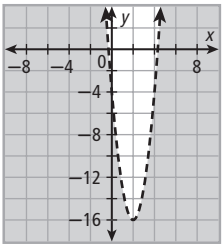


19. $x \le -1$ or $x \ge -0.5$

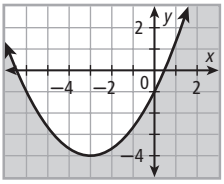
21. $x < -2$ or $x > 4$ **23.** $2 < x < 10$

25. $-1 \le x \le 2.5$ **27.** at distances less than about 3 ft and at distances greater than about 102 ft

29.



33.



35. $-3 \le x \le 8$

37. $\frac{-1-\sqrt{17}}{4} < x < \frac{-1+\sqrt{17}}{4}$

39. $-1-\frac{\sqrt{6}}{3} < x < -1+\frac{\sqrt{6}}{3}$

41. $\frac{-5-\sqrt{61}}{6} \le x \le \frac{-5+\sqrt{61}}{6}$

43. $x \le \frac{1-\sqrt{6}}{5}$ or $x \ge \frac{1+\sqrt{6}}{5}$

45. $x < 3$ or $x > 9$ **47.** a distance between 0 ft and about 31 ft

49. A **51a.** $A(x) = -\frac{1}{2}x^2 + 10x$

b. $3.7 \le x \le 16.3$ **c.** $0 < x \le 5.5$ or $14.5 \le x \le 20$ **53b.** a width between 5 ft and 15 ft **c.** a width of 10 ft **55.** $0.9 \le x \le 14.1$

57. $x \le -2.2$ or $x \ge 0.7$ **59.** It is not **63.** J **65.** $x < -3$ or $x > -1$

67. $\frac{5-\sqrt{17}}{4} < x < \frac{5+\sqrt{17}}{4}$

69. 121.5 square units

75. $c = 9$

5-8

Check It Out! 1a. Quadratic; second differences are constant for equally spaced x-values.

b. Not quadratic; first differences are constant so the function is linear. **2.** $f(x) = -x^2 + 4x - 3$

3. $L(d) \approx 14.3d^2 - 112.4d + 430.1$; about 446 ft

Exercises 3. Not quadratic; second differences are not constant for equally spaced x-values.

5. $y = x^2 - 2x - 3$ **7.** $y = x^2 - 3x + 4$

9. $y = -\frac{1}{2}x^2 + 4x - 3$ **11.** $C(x) \approx 0.0098x^2 + 0.62x + 3.8$; about $31.20

13. Quadratic; second differences are constant for equally spaced x-values. **15.** $y = \frac{4}{3}x^2 - x - \frac{7}{3}$

17. $y = 0.5x^2 - 3x - 8$

19. $y \approx -3.7x^2 + 216x + 781$; about $3290 million, or $3.29 billion

21. The function is $A(b) = \left(\frac{1}{2}h\right)b$, which is linear. **23.** The function is $A(s) = s^2$, which is quadratic.

25. -3 **27.** -1

29a. $p(s) = -0.125s^2 + 5.25s - 35.05$ **b.** $18.95 **c.** a maximum point; the price and size of the most expensive pizza

d. $9.95; −$1.05 **31.** not quadratic

33. quadratic; $y = 5x^2 + 2x$

35. not quadratic **39a.** $y \approx -10.7x + 208.1$ **b.** no **c.** linear; $y = 4x + 8$

41. $t(n) = \frac{1}{2}n^2 + \frac{1}{2}n$

43b. $y \approx 0.5x + 3$

c. $y \approx -0.13x^2 + 2.8x - 6$

45. B **47.** D

49. $y = 2x^2 - 5$ **53.** yes

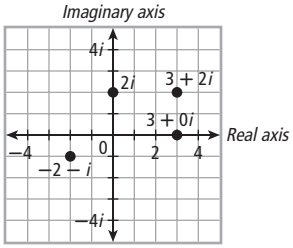
55. The matrix is undefined.

57. $\begin{bmatrix} \frac{1}{3} & -\frac{8}{3} \\ 0 & -2 \end{bmatrix}$ **59.** $\pm 3i$

5-9

Check It Out!

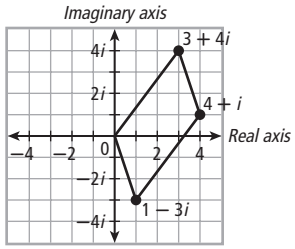
1.



2a. $\sqrt{5}$ **b.** $\frac{1}{2}$ **c.** 23 **3a.** $-3 - i$

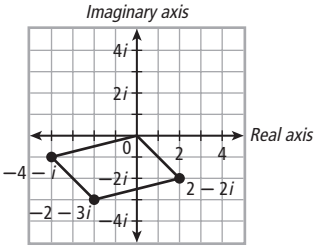
b. $-3 - 3i$ **c.** 8

4a.



$4 + i$

b.



$-2 - 3i$

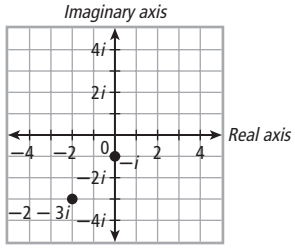
5a. $10 + 6i$ **b.** $20 - 28i$

c. 13 **6a.** $-\frac{1}{2}i$ **b.** -1

7a. $-8 + 3i$ **b.** $\frac{7}{5} + \frac{1}{5}i$

Exercises 1. real; imaginary

3, 5.



7. 33.3 **9.** 13 **11.** 15 **13.** $3 - 5i$

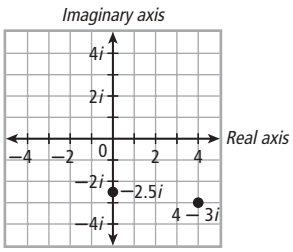
15. $-9 + 15i$ **17.** $-28 - 19i$

19. $-3 - i$ **21.** 5 **23.** $37 - 5i$

25. $8 + 6i$ **27.** $-i$ **29.** -1 **31.** $\frac{21}{10} + \frac{17}{10}i$

33. $4 - i$ **35.** $-2 + \frac{1}{2}i$

37, 39.



41. 18 **43.** 10 **45.** $2\sqrt{29}$ **47.** $-11 + 7i$

49. $28 + 41i$ **51.** $-4 + 9i$ **53.** $4 + 6i$

55. $48 + 12i$ **57.** 53 **59.** $24 + 78i$

61. $-i$ **63.** -5 **65.** $\frac{13}{10} - \frac{11}{10}i$

67. $4 + i$ **69.** $\frac{3}{4} + \frac{9}{4}i$ **71.** $3i$

73. $-2 - i$ **75.** $\sqrt{10}$ **77.** $2\sqrt{10}$

79. 0 **81.** $\frac{\sqrt{10}}{2}$ **83.** $\sqrt{11}$ **85.** $9.5 + 2.9i$

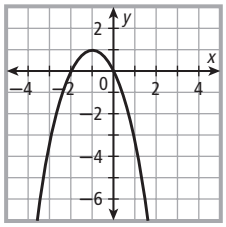
87. $-9.7 + 1.3i$ **89.** $-1 + 4i$

91. $-5 + 12i$ **93.** $10 - 5i$ **95.** 0

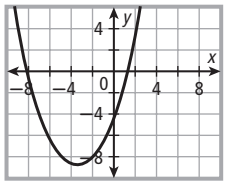
97. $1 - 12i$ **99.** $\frac{26}{37} + \frac{8}{37}i$ **101.** $\frac{8}{13} + \frac{12}{13}i$ **103.** $Z_{eq} = \frac{7}{4} - i$
105. always true **107.** always true
109. A is incorrect. **113.** D **115.** C
119. $\frac{ac + bd}{c^2 + d^2} + \frac{(bc - ad)}{c^2 + d^2}i$
121. $0 \le x \le 2$ **123.** $-5 \le x \le \frac{3}{2}$
125. Yes

Study Guide: Review

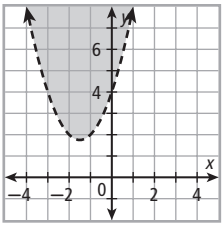
1. imaginary number; complex number **2.** zero of a function
3. vertex of a parabola
4. discriminant **5.** minimum value
6.



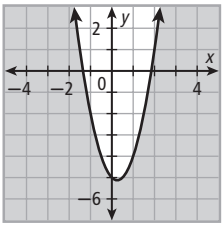
7.



8. g is f vertically stretched by a factor of 4 and translated 2 units right. **9.** g is f reflected across the x-axis, vertically stretched by a factor of 2, and translated 1 unit left. **10.** g is f vertically compressed by a factor of $\frac{1}{3}$ and translated 3 units down. **11.** g is f reflected across the x-axis and translated 2 units left and 6 units up.
12. Possible answer: $g(x) = -x^2 - 3$
13. Possible answer: $g(x) = 2(x - 4)^2$
14. Possible answer: $g(x) = \frac{1}{4}(x + 1)^2$
15. opens upward; $x = 2$; $(2, -1)$; 3
16. opens upward; $x = -1$; $(-1, 2)$; 3
17. opens upward; $x = 1.5$; $(1.5, -2.25)$; 0 **18.** opens upward; $x = 2$; $(2, 2)$; 4 **19.** minimum: 5
20. maximum: 4.5 **21.** minimum: -5.25 **22.** maximum: 18
23. maximum: 12 **24.** minimum: 7
25. $x = -1$ or $x = 8$ **26.** $x = 2$ or $x = 3$ **27.** $x = -12$ or $x = 12$
28. $x = 0$ or $x = 21$ **29.** $x = 2$
30. $x = -1$ or $x = -3$ **31.** $x = 2$ or $x = -16$ **32.** $x = -\frac{1}{3}$ **33.** Possible answer: $f(x) = x^2 + x - 6$
34. Possible answer: $f(x) = x^2 - 1$
35. Possible answer: $f(x) = x^2 - 9x + 20$ **36.** Possible answer: $f(x) = x^2 + 5x + 6$ **37.** Possible answer: $f(x) = x^2 + 10x + 25$
38. Possible answer: $f(x) = x^2 - 9x$
39. $x = 4$ or $x = 12$ **40.** $x = -14$ or $x = -6$ **41.** $x = -2$ or $x = 8$
42. $x = 7 \pm \sqrt{62}$ **43.** $f(x) = (x - 2)^2 + 5$; $(2, 5)$ **44.** $g(x) = (x + 1)^2 - 8$; $(-1, -8)$ **45.** $x = \pm 9i$ **46.** $x = \pm 5i$
47. $x = -3 \pm i$ **48.** $x = -6 \pm 3i$
49. $x = 7 \pm i\sqrt{26}$ **50.** $x = 11 \pm 2i\sqrt{3}$
51. $-5i - 4$ **52.** $3 - i\sqrt{5}$
53. $\frac{3 \pm \sqrt{41}}{2}$ **54.** $5 \pm 2i\sqrt{3}$
55. $\frac{5}{2} \pm \frac{i\sqrt{11}}{2}$ **56.** $-\frac{3}{2} \pm i\frac{\sqrt{3}}{2}$
57. $\frac{5}{2} \pm i\frac{\sqrt{15}}{2}$ **58.** 1 distinct real solution **59.** 2 real solutions
60. 2 nonreal complex solutions
61. 2 real solutions **62.** 2 nonreal complex solutions **63.** 2 nonreal complex solutions
64.



65.



66. $x \le -3$ or $x \ge 1$ **67.** $-4 < x < -1$
68. $x < 1$ or $x > 5$ **69.** $-3 \le x \le 3$
70. $-\sqrt{3} < x < \sqrt{3}$ **71.** $-2 \le x \le \frac{2}{3}$
72. $y = -x^2 - 3x + 6$ **73.** $y = -2x^2 + x$
74. $y \approx 0.000188x^2 - 0.0112x + 0.182$
75. ≈ 0.074 in. **76.** $y \approx 0.360x^2 - 11.9x + 105$ **77.** ≈ 37.8 ohms
78. 3 **79.** $2\sqrt{5}$ **80.** 20 **81.** 7
82. $7 + 4i$ **83.** $6 + 2i$ **84.** -6
85. $-20 - 15i$ **86.** $46 + 28i$
87. 13 **88.** $9 - 19i$
89. $-57 - 51i$ **90.** 1 **91.** $-5i$
92. $-\frac{9}{2} + i$ **93.** $\frac{7}{25} + \frac{26}{25}i$
94. $2 - 6i$ **95.** $4 + 5i$

Chapter 6

6-1

Check It Out! **1a.** 3 **b.** 0 **c.** 5 **d.** 9 **2a.** $-2x^2 + 4x + 2$; -2; 2; 3; quadratic trinomial
b. $x^3 - 18x^2 + 2x - 5$; 1; 3; 4; cubic polynomial with 4 terms
3a. $16x^3 - 30x^2 + 6x - 16$
b. $5x^3 - 9x^2 - 3x + 14$
4. $f(4) = 3.8398$; $f(17) = 1.6368$; the concentration of dye after 4 s; the concentration of dye after 17 s
5a. From left to right, the graph increases, decreases slightly, and then increases again. It crosses the x-axis 3 times, so there appear to be 3 real zeros.
b. From right to left, the graph decreases and then increases. It does not cross the x-axis, so there are no real zeros.
c. From left to right, the graph decreases and then increases. It crosses the x-axis twice, so there appear to be 2 real zeros.
d. From left to right, the graph alternately decreases and increases, changing direction 3 times. It crosses the x-axis 4 times, so there appear to be 4 real zeros.

Exercises **1.** The leading coefficient of a polynomial is the number being multiplied by the variable with the greatest degree. **3.** 5 **5.** 6 **7.** $3x^2 + 5x - 4$; 3; 2; 3; quadratic trinomial
9. $4x^4 + 8x^2 - 3x + 1$; 4; 4; 4; quartic with 4 terms
11. $3x^2 + 12x + 3$
13. $-5x^2 - 7x - 5$ **15.** From left to right, the graph increases. It crosses the x-axis once, so there appears to be 1 real zero. **17.** From left to right, the graph decreases. It crosses the x-axis once, so there appears to be 1 real zero. **19.** 8
21. 0 **23.** $2x^4 + 3x^3 + x^2 - 7x$; 2; 4; 4; quartic with 4 terms
25. $2x^3 + 10x - 9$; 2; 3; 3; cubic trinomial **27.** $x^3 + x^2$
29. $5y^3 - 3y^2 + 2y + 2$
31a. $d(1) = -3$; $d(2) = -28$
33. From left to right, the graph increases. There is 1 real zero.

35. The graph decreases, increases, and then decreases. There is 1 real zero. **41.** $S(x) = 4\pi x^2 + 8\pi x$ **43.** $S(x) = 5\pi x^2 + \frac{31}{2}\pi x + 12\pi$ **45a.** \$12.04 **b.** \$27.52 **47.** sometimes true **49.** sometimes true **51a.** The x-intercepts are -3, 1, and 4. **b.** The x-intercepts are -1, -2, 3, and 1. **c.** The x-intercepts are 0, -1, and 2. **d.** The x-intercepts are -2 and 3. **e.** The x-intercepts are $-\frac{1}{2}$, 0, and $\frac{1}{2}$. **53.** Yes **55.** J **57.** J **67.** vertical **69.** horizontal **71.** shift left 3 units and up 2 units

6-2

Check It Out!

1a. $12c^3d^3 - 18c^2d^3 + 42c^2d^4$
b. $6x^2y^4 + x^2y^3 - 28x^2y^2 + 30x^2y$
2a. $9b^3 - 9b^2c - 4bc^2 + 4c^3$
b. $x^4 + x^3 - 21x^2 + 13x - 2$
3. $T(x) = -0.00008x^4 - 0.0028x^3 + 0.028x^2 + 0.3x + 9$
4a. $x^4 + 16x^3 + 96x^2 + 256x + 256$
b. $8x^3 - 12x^2 + 6x - 1$
5a. $x^3 + 6x^2 + 12x + 8$
b. $x^5 - 20x^4 + 160x^3 - 640x^2 + 1280x - 1024$
c. $81x^4 + 108x^3 + 54x^2 + 12x + 1$

Exercises

1. $-20c^3d^5 - 12c^4d^4$
3. $5x^3y + 8x^2y - 7xy$
5. $x^3 + x^2y - 3xy^2 + y^3$
7. $3x^5 + 15x^4 + 16x^3 - 3x^2 + 6x - 2$
9. $-0.02x^4 - 0.3x^3 + 4.4x^2 - 14.2x + 20$
11. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
13. $x^3 - 9x^2y + 27xy^2 - 27y^3$
15. $16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$
17. $32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$
19. $6x^4 + 27x^3 - 18x^2$
21. $12r^5 + 28r^4 - 60r^3 + 28r^2$
23. $6x^3 + 7x^2y - 16xy^2 + 10y^3$
25. $12x^4 + 17x^3 + 8x^2 + x - 2$
27. $8x^3 - 24x^2 + 24x - 8$
29. $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$
31. $x^4 - 12x^3y + 54x^2y^2 - 108xy^3 + 81y^4$ **33.** $x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$
35. equivalent **37.** not equivalent
39. $T(x) = -0.0003x^4 - 0.0164x^3 + 2.572x^2 - 14.12x + 116.2$
41. $p^3 - 6p^2q + 12pq^2 - 8q^3$
43. $x^6 + x^4y^3 + x^3y^3 + xy^6$
45. $5x^3y + x^2y - 9xy + 10x^3 + 2x^2 - 18x$
47. $3x^4 - 24x^3 + 72x^2 - 96x + 48$
49. $-x^5 - 14x^3 - 45x^2 + 30x - 450$
51. $2x^6 - 3x^5 - 8x^4 + 12x^3 + 14x - 21$
53. No
55a. $f(n) = \frac{1}{4}n^4 + n^3 + \frac{5}{4}n^2 + \frac{1}{2}n$
b. 7098 **59.** J **61.** H
63. $x^{10} - 10x^9 + 45x^8 - 120x^7 + 210x^6 - 252x^5 + 210x^4 - 120x^3 + 45x^2 - 10x + 1$
65. $m^6 - 3m^4n^2 + 3m^2n^4 - n^6$
67. $B(x) = x - 3$
69. $B(x) = x^3 + 1$
71. $\begin{bmatrix} 8 & 1 \\ 4 & 13 \end{bmatrix}$
73. $\begin{bmatrix} 4 & -2 & 10 \\ -4 & 10 & 6 \\ -4 & -6 & 6 \end{bmatrix}$
75. $4x^4 - 6x^3 + 5x^2 + 3x$; 4; 4; 4; quartic with 4 terms
77. $3x^5 - 4x^2 - 2x + 9$; 3; 5; 4; quintic with 4 terms

6-3

Check It Out!

1a. $5x + 1 - \frac{13}{3x + 1}$
b. $x + 8 - \frac{4}{x - 3}$
2a. $6x - 23 + \frac{63}{x + 3}$ **b.** $x + 3$
3a. $P(-3) = 4$
b. $P\left(\frac{1}{5}\right) = 5$ **4.** $y - 5$

Exercises

3. $x + 2 + \frac{1}{x - 1}$
5. $7x - 2$ **7.** $x - 6$
9. $P(-8) = 42$ **11.** $P(-1) = 6$
13. $x + 4$ **15.** $x^2 - 1$
17. $\frac{1}{2}x^3 - 2x^2 - \frac{7}{2}$
19. $x + 4 + \frac{2}{x + 1}$
21. $x + 1 - \frac{2}{x + 8}$
23. $2x + 14 - \frac{1}{x - \frac{1}{2}}$
25. $P(4) = 9$
27. $P\left(-\frac{1}{3}\right) = \frac{5}{3}$
29. $I(t) = 0.5t^2 + 4t$
31. $a = 2$; $b = 8$; $c = 29$
33. $a = 3$; $b = 9$; $c = -4$
35. $x - 2$
37. $D(h) = \frac{1}{\pi} - \frac{4}{\pi h} + \frac{20}{\pi h^2}$
39. $y^2 + 5$ **41.** $x^2 - 5x - 12$
43. $t - 4$ **45.** $x^3 + 3x^2 - 10x - 1$
47. $x^3 - x^2 + 3x - 4 + \frac{1}{x - 6}$
49. Solution B is correct. **53.** B
55. D **57.** $P(-4) = -1{,}189{,}150$
59. $P(1) = 0$ **61.** $k = 18$
65. max = 1.25; D: $\mathbb{R}$; R: $\{y \mid y \le 1.25\}$
67. min = -2; D: $\mathbb{R}$; R: $\{y \mid y \ge -2\}$
69. $12x^3y^3 + 24x^3y + 20x^2y^4$
71. $4x^3 - 8x^2y + 8xy^2 - 4y^3$

6-4

Check It Out!

1a. no **b.** yes
2a. $(x + 3)(x - 3)(x - 2)$
b. $(x^2 + 4)(2x + 1)$
3a. $(2 + z^2)(4 - 2z^2 + z^4)$
b. $2x^2(x - 2)(x^2 + 2x + 4)$
4. $x = 1, 3, 4$; $V(x) = (x - 1)(x - 3)(x - 4)$

Exercises

1. yes **3.** yes
5. $(x + 2)(x - 2)(x + 5)$
7. $2(x + 2)(x - 2)(x - 1)$
9. $3(x - 2)(4x + 1)$
11. $2t^4(t + 3)(t^2 - 3t + 9)$
13. $(3 + x)(9 - 3x + x^2)$
15. $(y - 5)(y^2 + 5y + 25)$
17. no **19.** yes
21. $(b + 2)(b - 2)(4b + 3)$
23. $(x + 3)(x - 3)(3x + 1)$
25. $(x + 2)(x - 2)(5x - 1)$
27. $(s - 1)(s + 1)(s^2 + s + 1)(s^2 - s + 1)$
29. $6x(x - 3)(x^2 + 3x + 9)$
31. $y^2(y + 3)(y^2 - 3y + 9)$
33. $x^2(x^2 - 7)(x^2 - 7)$
35. $(x - 2)(x + 2)(4x + 1)$
37. $x(2x^2 - 1)(2x^2 - 3)(2x^2 + 3)$
39. $a = 3$; $d = 72$
41. $P(x) = (x - 2)(x^3 + 5x + 1)$
43. $P(x) = (x + 2)(2x^4 - 6x + 3)$
45a. $f(t) = -t(t - 8)(t - 18)$ $(t - 18)$ **b.** \$2,535,000
c. $f(15) = -945$
47. $B(x) = x^3 - 2x^2 + 4x - 8$ **51.** J
53. $[(x - 3) + 2][(x - 3)^2 - 2(x - 3) + 4]$; $(x - 1)(x^2 - 8x + 19)$

57. $(3x-5)(3x-5)$

59. $\left(x-\frac{22}{3}\right)\left(x+\frac{35}{6}\right)$ **61.** 20

63. $\frac{19}{26}+\frac{17}{26}i$ **65.** $P(5)=64$

67. $P(-1)=20$

6-5

Check It Out!

1a. $x=0,-1,6$ **b.** $x=-5,2,5$

2a. $x=2$ with multiplicity 4

b. $x=0$ with multiplicity 3; $x=-1$ with multiplicity 1; $x=6$ with multiplicity 2 **3.** 2 ft

4. $x=-\frac{1}{2}, 1\pm\sqrt{5}$

Exercises

3. $x=6,-6,1,-1$

5. $x=0,\frac{1}{3},-4$

7. $x=-5,-2,2,5$

9. $x=-2,2$ with multiplicity 3

11. $x=-6,\pm\sqrt{5}$ **13.** $x=-5,1,4$

15. $x=-3,3$ **17.** $x=-8,0,8$

19. $x=\frac{5}{2},\pm\sqrt{2}$ **21.** $x=0$ with multiplicity 2; $x=8$ with multiplicity 3 **23.** 3 in. by 3 in.

25. $x=-\frac{4}{3},\pm\sqrt{2}$ **27a.** $\pm1,\pm2,\pm4$

b. $x=-2,2$ **c.** 2 **d.** $x=-2.62,-0.38$ **29.** $x=3,2\pm\sqrt{2}$

31. $x=-5,-2,3,5$

33. $x=-1,0,1,2\pm\sqrt{5}$ **35a.** 126 ft

b. The coaster passes through 2 tunnels within the first 100 s.

c. Possible answer: $h(t)=3(t-3)(t-5)$ **41.** F **43.** H

45. $k=-18$ **47.** $k=6$

49. $x<-3$ or $x>2$ **51.** $-1<x<1$

53. $4(2x+1)(x+1)(x-1)$

6-6

Check It Out!

1a. $P(x)=x^3-4x^2-4x+16$

1b. $x^3-\frac{11}{3}x^2+2x$

2. $x=-5,1,2i,-2i$

3. $P(x)=x^5-5x^4+9x^3-17x^2+20x+12$ **4.** $r=9$ ft

Exercises

1. $P(x)=x^3-\frac{10}{3}x^2+3x-\frac{2}{3}$

3. $P(x)=x^3-\frac{1}{2}x^2-4x+2$

5. $x=2,\frac{2\pm\sqrt{2i}}{3}$

7. $P(x)=x^3-4x^2+6x-4$

9. $P(x)=x^5-2x^4+2x^3-4x^2-8x+16$

11. $P(x)=x^3-3x-2$

13. $P(x)=x^3+3x^2-6x-8$

15. $x=1,3$ **17.** $x=\frac{3}{2},2i,-2i$

19. $x=1,-1,\pm\sqrt{3}$

21. $P(x)=x^5+3x^4-13x^3-39x^2+40x+120$ **23.** $r=9$ ft

25. $x=2,-4,-1$

27. $x=-1$ **29.** $x=3,\frac{-1\pm i\sqrt{3}}{2}$

31. $x=0,7,3\pm2i$

33. $x=\pm3i,\pm i\sqrt{5}$ **35.** $x=-1,2,3$

37a. $x^3-6x^2-243=0$ **b.** 9 m

c. They are complex.

39. $P(x)=x^4+12x^2-64$

41. $P(x)=x^3-4x^2+5x-2$

43. $P(x)=x^4-6x^3+18x^2-54x+81$

45. never true **47.** Sometimes true

49. $x=0$ or $x\approx\pm0.537$

51. $x\approx-0.782,0.975,3.965$

53. $r=6$ **57.** B **59.** D **61.** D

63. $f(3i)=0; f\left(-\sqrt{3}\right)=-36-12\sqrt{3}$ **65.** $x=\pm3i$

67. $(a+bi)(a-bi)$

69. $(a+bi)(a-bi)\left(a^4-2a^2b^2+b^4\right)$

71. $x^4+2x^2+1=0;\pm i$

73. shift right 2 units, reflection across x-axis, and shift up 3 units

75. $(9,-3)$ **77.** $x=0,42,1$

79. $x=0,-8,2$

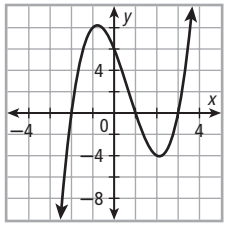
6-7

Check It Out!

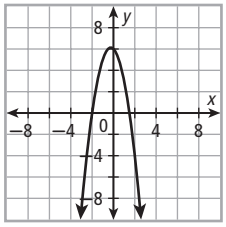
1a. 2; 5; as $x\to-\infty$, $P(x)\to-\infty$, as $x\to+\infty$, $P(x)\to\infty$ **b.** -3; 2; as $x\to-\infty$, $P(x)\to-\infty$ as $x\to+\infty$, $P(x)\to-\infty$

2a. odd; negative **b.** even; positive

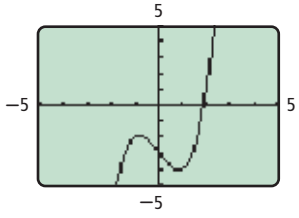
3a.



b.

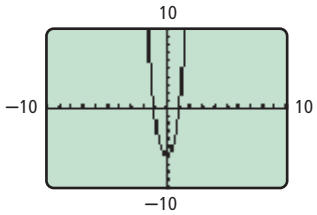


4a.



min $=-4.0887$; max $=-1.9113$

b.



min $=6$

5. 420.1 ft^3

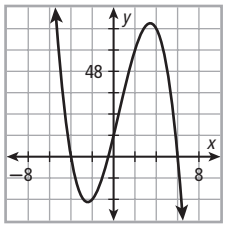
Exercises

1. A graph "turns around" at a turning point.

3. -2; 7; $x\to-\infty$ $Q\to+\infty$, $x\to+\infty$ $Q\to-\infty$ **5.** 3; 2; $x\to\infty$ $S\to+\infty$, $x\to+\infty$ $S\to+\infty$ **7.** even; positive

9. even; negative

11.



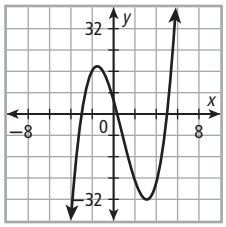
13. max $=-2.9098$; min $=-14.0902$

15. 2; 3; $x\to-\infty$ $P\to-\infty$, $x\to+\infty$ $P\to+\infty$

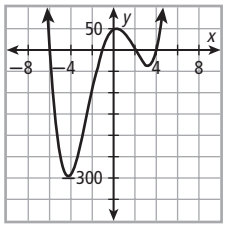
17. -1; 5; $x\to-\infty$ $R\to\infty$ $x\to+\infty$ $R\to+\infty$

19. even; negative **21.** odd; negative

23.



25.



27. min $=20$ **29.** max $=-1$

31. 3.425 L; 0.5 s **33.** B **35.** A

37. $+\infty$; $+\infty$ **39.** $+\infty$; $+\infty$

41. $-\infty$; $-\infty$

45a. $V(x)=-\frac{1}{3}x^3+\frac{10}{3}x^2$

b. 49.4 in^3 **c.** 6.7 in. $\times$ 3.3 in. $\times$ 6.7 in.

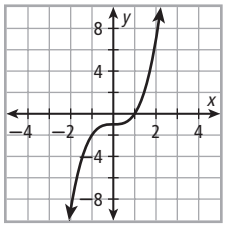
49. H

57. no **59.** $h(t)=-16(t-2)^2+70$; vertex $(2,70)$ **61.** $10x-2+\frac{8}{x+1}$

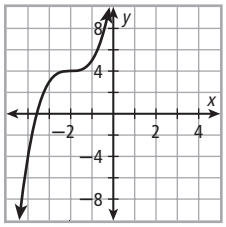
6-8

Check It Out!

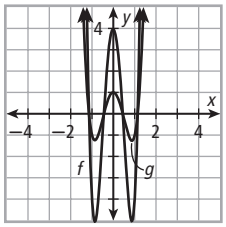
1a. $g(x) = x^3 - 1$



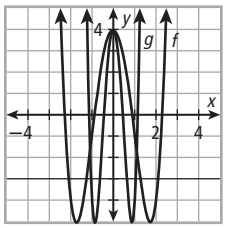
b. $g(x) = x^3 + 6x^2 + 12x + 12$



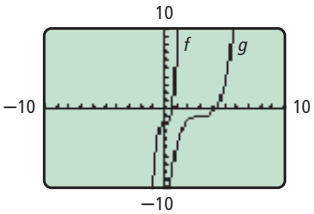
2a. $g(x) = -x^3 + 2x^2 + x - 2$
b. $g(x) = -x^3 - 2x^2 + x + 2$
3a. vertical compression



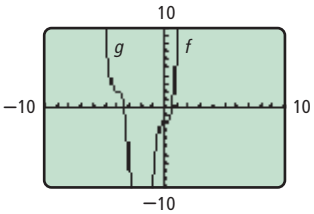
b. horizontal stretch



4a. $g(x) = 4(x-3)^3 - 1$
$= 4x^3 - 3x^2 + 108x - 109$



b. $g(x) = -8(x+3)^3 + 2$
$= -8x^3 - 96x^2 - 384x - 510$



5. $g(x) = 0.01x^3 + 0.55x^2 - 5.85x + 134.25$
Possible answer: The model represents the number of sales since March.

Exercises

1. $g(x) = x^4 - 4$ **3.** $j(x) = 81x^4 - 8$
5. $g(x) = x^3 + 3x^2 + 2x + 1$
7. horizontal stretch **9.** horizontal compression and vertical shift
11. $g(x) = -32x^3 + 2$
13. $c(x) = 4x^3 - 6x + 60$; the cost has doubled.
15. $h(x) = x^3 - 9x^2 + 27x - 31$
17. $g(x) = -x^3 + 2x^2 - 5x + 3$
19. vertical stretch
21. horizontal stretch
23. $g(x) = \frac{1}{3}x^4 - 1$
25. $V\left(\frac{2}{3}x\right) = \frac{8}{27}x^3 + \frac{4}{3}x^2 + \frac{2}{3}x + 8$
27a. $v > 2.04$ **b.** $G(v) = 0.24v^2 + 2.4v + 6$; a shift 5 units left
c. $v \geq 0$ **d.** a vertical stretch
31. B **33.** B **35.** shift right 2 units
37. shift right 3 units and up 8 units
39. $w(t) = 60t$; yes **41.** $3x^5 - 3x^4 + 5x^2 - 4x$ **43.** $x = -2, 1, \pm i\sqrt{5}$
45. $x = -4, -1, 2, \pm 2i\sqrt{2}$

6-9

Check It Out!

1. cubic **2.** $f(x) = 0.001x^3 - 0.113x^2 + 4.134x - 24.867$
3. $11,482.84

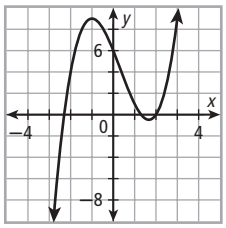
Exercises

1. linear **3.** quartic
5. 831 patients **7.** quartic
9. $f(x) = 0.821x^2 - 1.821x + 23.357$
13a. $f(x) = 0.019x^3 - 0.185x^2 + 0.95x + 12.056$; $R^2 = 0.9944$
b. $f(x) = 0.0075x^4 - 0.071x^3 + 0.143x^2 + 0.604x + 12.083$; $R^2 = 0.9967$ **c.** no **15.** yes **17.** C
19. $f(x) = x^3 - 5x + 4$
25. $4a - 25a^3$ **27.** cubic parent function; shift right 1 unit and up 2 units. **29.** 4; 5; $x \to -\infty$, $f(x) \to -\infty$; $x \to +\infty$, $f(x) \to +\infty$

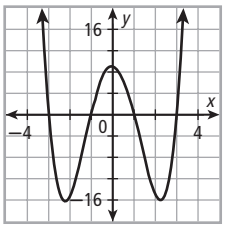
Study Guide: Review

1. monomial **2.** synthetic division
3. multiplicity **4.** end behavior
5. $-3x^3 + 4x^2 + 6x + 7$; -3; 3; 4: cubic polynomial with 4 terms
6. $-x^5 + 2x^4 + 5x^3 + 8x$; -1; 5; 4: quintic polynomial with 4 terms
7. $9x^2 - 11x + 1$; 9; 2; 3: quadratic trinomial **8.** $x^4 - 6x^2$; 1; 4; 2: quartic binomial **9.** $8x^3 + x^2 - 4x$
10. $-5x^3 + 6x^2 + 10x - 1$
11. $-6x^2 - x + 9$ **12.** $-4x^4 - x^3 - 3$
13. From left to right, it alternately increases and decreases, changing direction 3 times and crossing the x-axis 2 times. There appear to be 2 real zeros. **14.** From left to right, it increases, decreases slightly, and then increases again. It crosses the x-axis 1 time. There appears to be 1 real zero. **15.** From left to right, it alternately decreases and increases, changing direction 3 times. It crosses the x-axis 4 times. There appear to be 4 real zeros.
16. From left to right, it increases, decreases, and then increases again. It crosses the x-axis 3 times. There appear to be 3 real zeros.
17. $15x^3 - 10x^2$
18. $-6t^3 + 18t^2 - 3t$
19. $a^3b^2 - a^2b^2 + a^2b^3$
20. $x^3 - 4x^2 + x + 6$
21. $2x^4 + 3x^3 - 5x^2 + 2x + 5$
22. $x^3 - 9x^2 + 27x - 27$
23. $x^5 + 4x^4 - 3x^3 - 11x^2 + 4x$
24. $16x^4 + 32x^3 + 24x^2 + 8x + 1$
25. $4\pi x^4 - 4\pi x^3 - 12\pi x^2$
26. $x^2 - 7x + 16 - \frac{39}{x+2}$
27. $4x^3 + 2x^2 + 4x + 1 + \frac{5}{2x-1}$
28. $x^2 - x + \frac{2}{x-3}$
29. $x^2 + 2x + 6 + \frac{11}{x-2}$
30. $x^2 + 2x + 2$ in.; remainder 2 in. **31.** no **32.** yes **33.** yes
34. $(x-1)(x-4)(x+4)$
35. $(x-2)(2x-1)(2x+1)$
36. $3(x+3)(x^2 - 3x + 9)$
37. $2(2x-1)(4x^2 + 2x + 1)$
38. 1, 2 **39.** $-2, -2 \pm \sqrt{3}$
40. -1 **41.** $-3, 3, \pm\sqrt{3}$ **42.** $-1, \pm\sqrt{2}$
43. $1, 2 \pm 2\sqrt{2}$ **44.** 2 m
45. $P(x) = x^3 - 3x^2 - 10x + 24$
46. $P(x) = x^3 - \frac{1}{2}x^2 - \frac{13}{2}x - 3$
47. $P(x) = x^3 + x^2 - 2x - 2$
48. $P(x) = x^3 + 3x^2 + x + 3$
49. $P(x) = x^4 - 5x^2 + 6$
50. $P(x) = x^4 - 2x^3 + 2x^2 - 8x - 8$
51. $1, -2i, 2i$ **52.** $-i, i, -\sqrt{2}, \sqrt{2}$
53. $\pm 4, \pm\frac{1}{2}i$ **54.** $\pm\sqrt{5}, -3$
55. -2; 3; as $x \to -\infty$, $f(x) \to +\infty$; as $x \to +\infty$, $f(x) \to -\infty$

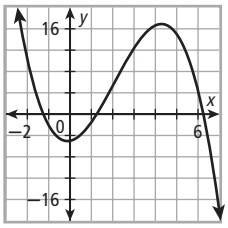
56. 1; 4; as $x \to \pm\infty$, $f(x) \to +\infty$
57. -3; 6; as $x \to \pm\infty$, $f(x) \to -\infty$
58. 7; 5; as $x \to -\infty$, $f(x) \to -\infty$, as $x \to +\infty$, $f(x) \to +\infty$
59.



60.



61.



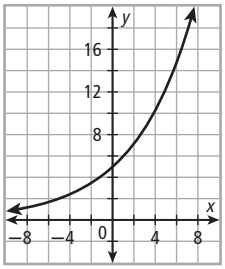
62. $g(x) = 2x^4 - 12x^2 + 1$
63. $g(x) = -x^4 + 6x^2 + 6$
64. $g(x) = (-x-3)^4 + 6(-x-3)^2 - 4$
65. $f(x) \approx -6\frac{2}{3}x^4 + 80x^3 - 328\frac{1}{3}x^2 + 575x - 72$
66. $f(x) \approx 80.5x^3 - 523.5x^2 + 1790x + 544$

Chapter 7

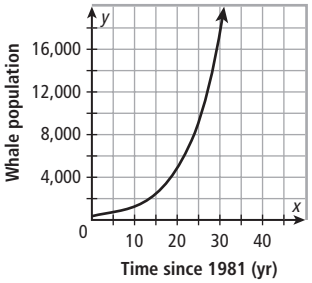
7-1

Check It Out!

1. growth

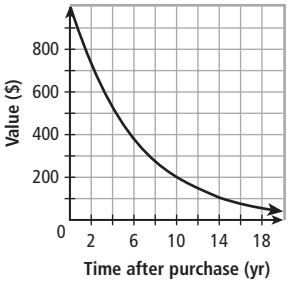


2. $P(t) = 350(1.14)^t$



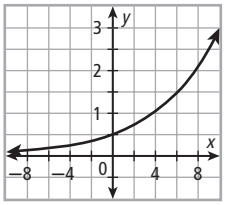
30.9 yr

3. $v(t) = 1000(0.85)^t$

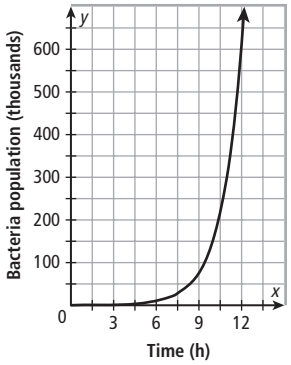


14.2 yr

Exercises **1.** exponential decay.
3. growth



5a. $f(x) = 150(2^x)$
b.

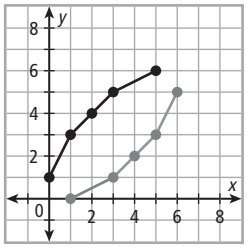


c. $\approx$ 600,000
7. decay **9.** growth
11a. $f(x) = 10(0.95)^x$ **c.** $\approx$ 6 units
d. 13.6 min. **13.** no
15. $\approx$ \$12,000,000 **17.** $\approx$ 5.8 yr
19. 15.63; 6.25; ...; 0.03; 0.01
21a. $\approx$ 3146 **b.** 12th month
25. (34.868, 100] **27a.** 17%
b. $A(t) = 500(0.83)^t$ **c.** $\approx$ 36.8 mg
29. 3^x **31.** B **37.** $x > 22.76$
39. 2; $(2, 4)$, $(\approx -0.767, \approx 0.588)$
43. D: $\mathbb{R}$; R: $\{y | y \leq 1\}$; $f(x) = x^2$ reflected across x-axis and shifted 1 unit up **47.** odd; positive **49.** even; negative

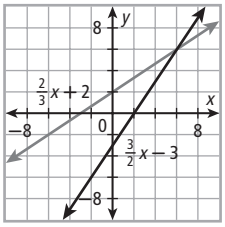
7-2

Check It Out!

1.



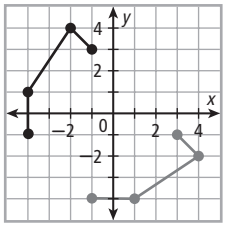
relation: D: $\{1 \leq x \leq 6\}$;
R: $\{0 \leq y \leq 5\}$
inverse: D: $\{0 \leq y \leq 5\}$;
R: $\{1 \leq x \leq 6\}$
2a. $f^{-1}(x) = 3x$ **b.** $f^{-1}(x) = x - \frac{2}{3}$
3. $f^{-1}(x) = \frac{x+7}{5}$
4. $f^{-1}(x) = \frac{3}{2}x - 3$



5. inverse: $z = 6t - 6$; 36 oz of water

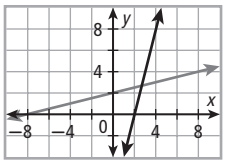
Exercises

1. relation
3.



relation: D: $\{-1 \leq x \leq 4\}$;
R: $\{-4 \leq y \leq -1\}$
inverse: D: $\{-4 \leq y \leq -1\}$;
R: $\{-1 \leq x \leq 4\}$
5. $f^{-1}(x) = \frac{1}{4}x$ **7.** $f^{-1}(x) = x + 2\frac{1}{2}$
9. $f^{-1}(x) = 2(x-3)$
11. $f^{-1}(x) = -\frac{2}{3}x + 1$
13. $f^{-1}(x) = \frac{2}{3}x + \frac{5}{3}$

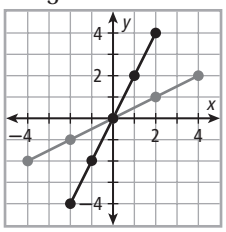
15.



$f^{-1}(x) = 4(x - 2)$

17. $F = \frac{9}{5}C + 32$; 61° F

19.



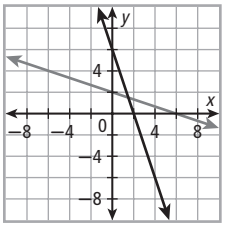
relation: D: $\{-4 \le x \le 4\}$; R: $\{-2 \le y \le 2\}$
inverse: D: $\{-2 \le y \le 2\}$; R: $\{-4 \le x \le 4\}$

21. $f^{-1}(x) = x + 1\frac{3}{4}$

23. $f^{-1}(x) = -\frac{1}{32}x + \frac{21}{32}$

25. $f^{-1}(x) = 5x - 60$

27. $f^{-1}(x) = -3x + 6$



29. 22 **31a.** $f^{-1}(x) = \frac{212 - x}{1.85}$
b. 6500 ft **c.** 27,946 ft

33. (4, 2), (2, 4), (−3, −1), (−1, −3)

35. $f(x) = \frac{10}{12.59}x$; $f^{-1}(x) = 1.259x$; 31.48 s **37.** B **39.** yes **41.** always

43. never **45.** always

47a. $P = \frac{147}{340}d + 14.7$

b. D: $\{d \mid d \ge 0\}$; R: $\{P \mid P \ge 14.7\}$

c. $d = \frac{340}{147}P - 34$; depth as a function of pressure **49.** F

51.

x	1	2	3	4	5
y	0	1	2	3	4

53. $y = -\frac{b}{a}x + \frac{c}{a}$

61. $2x^3 - 14x + 12 = 0$

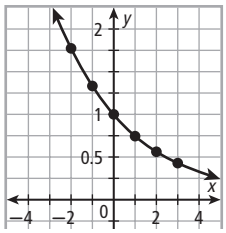
63. $2x^3 - 8x^2 + 12x - 8 = 0$

65. decay **67.** growth

7-3

Check It Out! **1a.** $\log_9 81 = 2$
b. $\log_3 27 = 3$ **c.** $\log_x 1 = 0$
2a. $10^1 = 10$ **b.** $12^2 = 144$
c. $\left(\frac{1}{2}\right)^{-3} = 8$ **3a.** −5 **b.** −1

4.



$f(x)$: D: $\mathbb{R}$, R: $\{y \mid y > 0\}$; $f^{-1}(x)$: D: $\{x \mid x > 0\}$; R: $\mathbb{R}$ **5.** 3.8

Exercises

1. x **3.** $\log_4 8 = 1.5$ **5.** $\log_3 243 = x$
7. $x^3 = -16$ **9.** $6^3 = x$ **11.** −2 **13.** 2
15. $f(x)$: D: $\mathbb{R}$, R: $\{y \mid y > 0\}$; $f^{-1}(x)$: D: $\{x \mid x > 0\}$; R: $\mathbb{R}$
17. $\log_x 32 = 2.5$ **19.** $\log_{1.2} 1 = 0$
21. $5^4 = 625$ **23.** $4.5^0 = 1$ **25.** 0 **27.** 3
29. $f(x)$: D: $\mathbb{R}$, R: $\{y \mid y > 0\}$; $f^{-1}(x)$: D: $\{x \mid x > 0\}$; R: $\mathbb{R}$ **31.** no **33.** 1
35. yes **37a.** orange **b.** lemon
c. grapefruit **39.** C **41.** A **43.** 6
47a. $2^{11} = 2048$ Hz, $\log_2 2048 = 11$
b. 3 octaves lower **49.** $\frac{2s}{t^2}$
51. $21a^{-1}b^4 + 28a^{-3}b^5$
53. 0.35; 0.59; 1; 1.7; 2.89
55. 11.11; 3.33; 1; 0.3; 0.09

7-4

Check It Out!
1a. $\log_5 (625 \cdot 25) = 6$
b. $\log_{\frac{1}{3}} 3 = -1$ **2.** $\log_7 7 = 1$
3a. $4 \log 10 = 4$ **b.** $4 \log_5 5 = 4$
c. $-5 \log_2 2 = -5$ **4a.** 0.9 **b.** $8x$
5a. 1.5 **b.** $1.\overline{3}$ **6.** ≈ 63

Exercises

1. $\log_5 3125 = 5$ **3.** $\log_3 81 = 4$
5. $\log 100 = 2$ **7.** 2 **9.** 6 **11.** $\frac{x}{2} + 5$
13. 5 **15.** −1.5 **17.** ≈ 1.43
19. 2 times as large **21.** $\log 10 = 1$
23. $\log 10 = 1$ **25.** $\log_{1.5} 3.375 = 3$
27. 0.2 **29.** $7 + x$ **31.** 4 **33.** 1.5
35. ≈ 3.16 times as intense
37. $\log_b m + \log_b n = \log_b mn$
39. $n \log_b b^m = mn$
41. 0 **43.** $-\frac{3}{2}$ **45.** 1
47. $10^{-7} - 10^{-7.6}$
49. $t = \log_{1.08}\left(\frac{50}{40}\right)$; 2.9

55a. ≈ 0.2 **b.** ≈ 2.6 **c.** ≈ 2.4
57. sometimes **59.** always
61. always **63.** sometimes **65.** B
67. H **71.** $\{x \mid x > 1\}$ **73.** $\{x \mid x > 0\}$
75. $\{x \mid -1 \le x < 0\}$ **77.** x **79.** ∅
81. 17 **83.** 7 **85.** $12i$ **87.** $8i\sqrt{2}$
89. $\log_5 125 = 3$ **91.** $\log_{36} 6 = 0.5$
93. 0 **95.** 0.5

7-5

Check It Out!
1a. 1.5 **b.** ≈ −1.565 **c.** ≈ 1.302
2. day 18 **3a.** 5 **b.** 2 **4a.** $x = 2$
b. $x < 2$ **c.** $x = 1000$

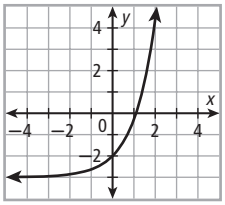
Exercises

1. exponential equation **3.** $x = -2$
5. $x \approx 1.661$ **7.** $x \approx 0.503$ **9.** $x = \frac{1}{8}$
11. $x = 108$ **13.** $x = \frac{9}{13}$ **15.** $x = 2$
17. $x = 3.5$ **19.** $x = 100$ **21.** $x = -5$
23. $x = 0.8$ **25.** ≈ 7.595
27. ≈ 41 min **29.** $x = 30$ **31.** ≈ 2.73
33. $x = 20$ **35.** $x < 1$ **37.** $x = 4$
39. $x = 4$ **41.** 24 keys below concert A **43.** 0 **47a.** 11 km; 25 km **b.** greater **49.** J **51.** no
53. $\{x \mid 0 < x < 12\}$ **55.** 26 **57.** 3
59. $f^{-1}(x) = \frac{1}{4}x - \frac{3}{4}$
61. $f^{-1}(x) = 3x - 27$

7-6

Check It Out!

1.

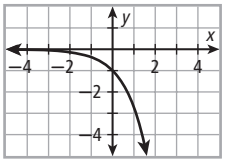


2a. 3.2 **b.** x^2 **c.** $x + 4y$
3. $132.31 **4.** ≈ 47.6 days

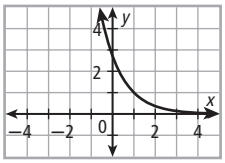
Exercises

1. $f(x) = \ln x$; natural logarithm

3.

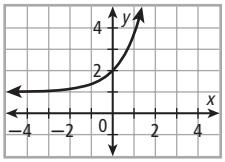


5.

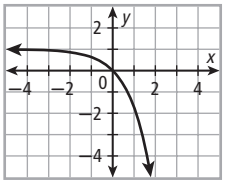


7. $x - y$ **9.** $2x$ **11.** \$9465.87

13.



15.



17. 0 **19.** $c + 2$ **21.** \$5553.55

23a. They are reciprocals.

25a. ≈ 2.4 min **b.** ≈ 2.8 min

c. room: ≈ 17.4 min

27. B **29.** C **31.** $x = \frac{e}{5} \approx 0.54$

33. $x = \frac{e^5}{\sqrt{10}} \approx 47$

35. $\{x|x > 0\}$ **37b.** 2

41. C **43.** A **45.** 4; yes

47a. $f(x) = \ln(-x)$ **b.** $f(x) = -\ln x$

c. $f(x) = -\ln(-x)$

d. one asymptote: $x = 0$

49. $g(x) = f(x) + 5 = -2x^2 + 3x + 1$

51. $g(x) = -f(x) = 2x^2 - 3x + 4$

53. $\log_2 4 = 2$

55. $\log_3\left(\frac{243}{2187}\right) = -2$

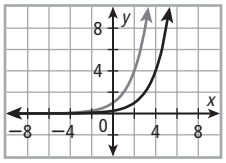
57. $\log_8 1 = 0$

7-7

Check It Out!

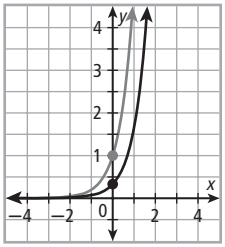
1.

x	-2	-1	0	1	2
$j(x)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1



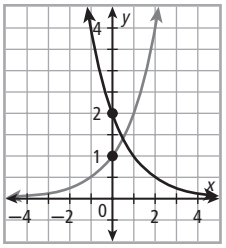
$y = 0$; $j(x) = 2^x$ translation 2 units right

2a.



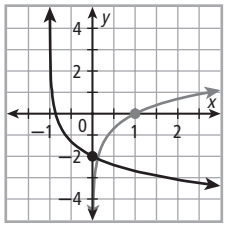
$\frac{1}{3}$; $y = 0$; $f(x) = 5^x$ vertical compression by a factor of 3

b.



2; $y = 0$; $j(x) = 2^x$ reflection across y-axis and vertical stretch by a factor of 2

3.



-2; $x = -1$; $f(x) = \ln x$ translation 1 unit left, reflection across the x-axis, and translation 2 units down; D: $\{x|x > -1\}$

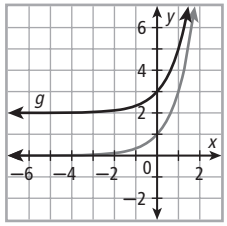
4. $g(x) = 2\log(x + 3)$

5. $t = 38{,}679$ yr; no

Exercises

1.

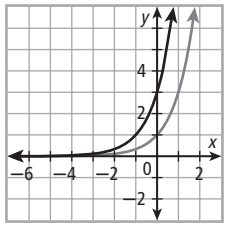
x	-2	-1	0	1	2
$g(x)$	2.1	2.3	3	5	11



$y = 2$; translation 2 units up; R: $\{y|y > 2\}$

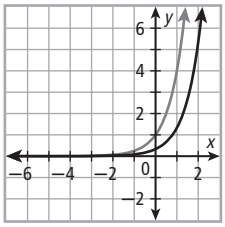
3.

x	-3	-2	-1	0	1
$j(x)$	0.11	0.33	1	3	9

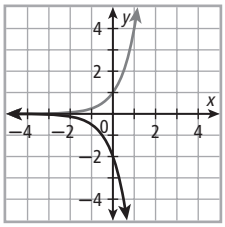


$y = 0$; translation 1 unit left

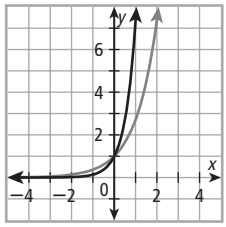
5. $\frac{1}{3}$; $y = 0$; vertical compression by a factor of $\frac{1}{3}$



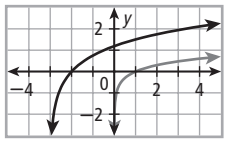
7. -2; $y = 0$; vertical stretch by a factor of 2 and reflection across the x-axis; R: $\{y|y < 0\}$



9. 1; $y = 0$; horizontal compression by a factor of $\frac{1}{2}$



11. $x = -3$; translation 3 units left and vertical stretch by a factor of 2.5; D: $\{x|x > -3\}$

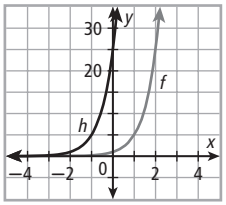


13. $g(x) = -0.7^{\left(\frac{x}{3} + 2\right)}$

15. translated 1 unit left, stretched vertically by a factor of 3, and translated 6 units up; D: $\{t|t \geq 0\}$; after about 39 years

17.

x	-2	-1	0	1	2
$h(x)$	1	5	25	125	625



$y = 0$; translation 2 units left

19. 4; $y = 0$; vertical stretch by a factor of 4 **21.** -0.25; $y = 0$; vertical compression by a factor of 0.25 and reflection across the x-axis; R: $\{y|y < 0\}$ **23.** 4; $y = 0$; vertical stretch by a factor of 4 and reflection across the y-axis **25.** $x = 5$; translation 5 units right; D: $\{x|x > 5\}$ **27.** $x = 0$; vertical stretch by a factor of 4 and reflection across the x-axis

29. $f(x) = \ln(4x + 3) - 0.5$

31. ≈ 47 yr **33.** A **35.** D **37.** F **39.** B and F **41.** always **43.** sometimes **45.** C **47.** B

51a. $N(t) = 419(0.99)^t$

b. $N(t) = 419(0.99)^{\frac{m}{12}}$

c. 413 **53.** H **59.** min: $\left(-\frac{1}{8}, -\frac{81}{16}\right)$; D: $\mathbb{R}$; R: $\{y|y \geq -5\}$

61. $f(x) \approx 0.032x^3 - 0.0076x^2 + 0.073x + 1.30$ **63.** $-5x$ **63.** $\frac{x}{4}$

7-8

Check It Out!

1a. yes; 1.5 **b.** no

2. $B(t) \approx 199(1.25)^t$; ≈10.3 min

3. $S(t) \approx 0.59 + 2.64 \ln t$; ≈16.6 min

Exercises

1. exponential regression **3.** yes; $\frac{2}{3}$

5. yes; $\frac{4}{3}$ **7.** $P(t) \approx 621.6 + 1221 \ln t$; ≈ 421 mo **9.** no **11.** yes; $\frac{1}{2}$

13. $T(t) \approx 4.45(1.165)^t$; ≈ 2011

15. yes; $f(x) = 1.55(7.54)^x$

19. $s(t) = 68.24(3.69)^t$; ≈ 46.5 million **23a.** 20.5 mi/h; 36.8 mi/h; 84.0 mi/h **b.** $s = 100(0.8^t)$

25a. exponential **b.** linear **27.** F

29. $f(x) = 7.68(2.5)^x$ **33.** $x = -4$

35. $x = \pm\frac{9}{4}$ **37.** $x = -8, 0$

39. $x = -12, 3$ **41.** $x = \frac{4}{3}$ **43.** $x = 0$

Study Guide: Review

1. natural logarithmic function

2. asymptote

3. inverse relation

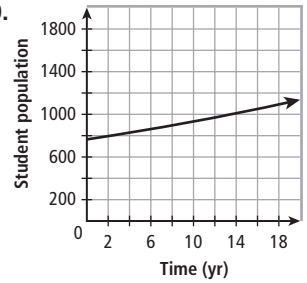
4. growth **5.** growth

6. decay **7.** growth

8. growth

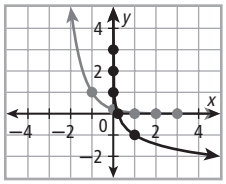
9. $P(t) = 765(1.02)^t$

10.



11. ≈ 845 **12.** ≈ 13.5 yr

13.



14. $P_T = P_L(1 - 0.03)$

15. $P_L = \frac{P_T}{0.97}$ **16.** $K = \frac{8}{5}M$; 40 km

17. $\log_3 243 = 5$ **18.** $\log_9 1 = 0$

19. $\log_{\frac{1}{3}} 27 = -3$ **20.** $2^4 = 16$

21. $10^1 = 10$ **22.** $0.6^2 = 0.36$

23. 2 **24.** 2 **25.** -1 **26.** -2 **27.** 0

28.

x	-2	-1	0	1	2
y	4	2	1	0.5	0.25

D: $\{x|x > 0\}$; R: $\mathbb{R}$

29. $\log_2 128 = 7$

30. $\log 1{,}000{,}000 = 6$

31. $\log_2 64 = 6$ **32.** $\log 100 = 2$

33. $\log_5 5^4 = 4$ **34.** $9 \log 10 = 9$

35. 10 times **36.** -1 **37.** $x \geq -6$

38. $x > 10$ **39.** 17.67 quarters, or 4.4 yr **40.** $k = 0.0346$

41. $g(x) = -3e^x - 2$ **42.** 0.6; $y = 0$; vertically compressed by a factor of $\frac{3}{5}$ and horizontally compressed by a factor of $\frac{1}{6}$ **43.** -0.5; $x = 0.5$; translated $\frac{1}{2}$ unit left and vertically stretched by a factor of 2

44. $V(t) = 5300(1 - 0.35)^t$

45. vertically stretched by a factor of 3500 **46.** $f(x) \approx 11.26(1.05)^x$

47. $f(x) \approx -97.8 + 56.4 \ln x$

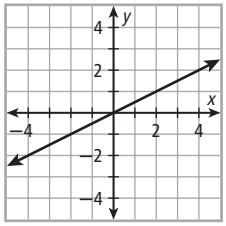
48. The exponential function; $r^2 \approx 0.94$ versus $r^2 \approx 0.60$ for the logarithmic function

Chapter 8

8-1

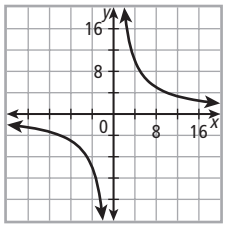
Check It Out!

1. $y = 0.5x$



2. 6.25 in. **3.** 1.6 m

4. $y = \frac{40}{x}$



5. $83\frac{1}{3}$ working hours

6a. inverse **b.** direct **7.** 20 L

Exercises

1. indirect variation

3. $y = -9x$ **5.** 12 ft **7.** 6 ft

9. $y = \frac{14}{x}$ **11.** $y = -\frac{5}{x}$ **13.** neither

15. direct **17.** $y = \frac{1}{2}x$ **19.** $y = -3x$

21. 88 Cal **23.** 0.2 kg **25.** $y = \frac{10.5}{x}$

27. 5 days **29.** neither

31. 1.375 atm **33.** sometimes

35. always **37a.** $s = \frac{6300}{t}$ **b.** 30 s

39a. $I = 0.02Pt$ **b.** True Federal Bank **c.** \$30 **41.** $x = 5$; $y = 4.4$; $z = 2$ **45.** D **47.** D **49.** 24 **51.** $y \approx 7$

53. $y = \frac{5}{4}x + \frac{1}{4}$ **55.** asymptote: $y = -2$; vertically compressed by a factor of $\frac{1}{2}$ and translated 2 units down

8-2

Check It Out!

1a. $2x^9$; $x \neq 0$ **b.** $\frac{1}{x-1}$; $x \neq -\frac{4}{3}$ and $x \neq 1$ **c.** $\frac{(2x+1)}{(2x-3)}$; $x \neq -\frac{2}{3}$ and $x \neq \frac{3}{2}$ **2a.** -2; $x \neq 5$ **b.** $\frac{-x}{2x-1}$; $x \neq 3$ and $x \neq \frac{1}{2}$ **3a.** $\frac{2x^3}{3}$ **b.** $\frac{2}{x-2}$

4a. $\frac{3y}{x^2}$ **b.** $\frac{4(x-4)}{x+3}$

5a. no solution **b.** $x = 4$

Exercises

3. $\frac{(2x+5)}{(2x-7)}$; $x \neq \frac{1}{3}$ and $x \neq \frac{7}{2}$

5. $\frac{-1}{x-5}$; $x \neq -4$ and $x \neq 5$

7. $6x$; defined for all real values of x

9. $\frac{2(x-2)}{(x+5)}$ **11.** $\frac{x^7y^4}{3}$

13. $\frac{(x+5)^2}{(2x-3)(x+2)}$ **15.** no solution

17. $x = -1$ **19.** $\frac{4}{x+5}$; $x \neq \frac{1}{2}$ and $x \neq -5$ **21.** $\frac{-3}{x-4}$; $x \neq -6$ and $x \neq 4$

23. -4; $x \neq -5$ **25.** $\frac{(x-4)(2x-1)}{(x-3)(x+4)}$

27. $\frac{3(2x-5)}{x}$ **29.** $\frac{x+1}{x-1}$ **31.** $\frac{x+3}{x+5}$

33. $x = 2$ **35.** $\frac{\pi r^2}{\pi (5r)^2}$; $\frac{1}{25}$

37. $\frac{4x-3}{2x-1}$ **39.** $2x^2y^2$ **41.** $\frac{3}{x+1}$

43a. square prism: $\frac{h}{1}$; cylinder: $\frac{h}{1}$

b. square prism: $\frac{2s+4h}{sh}$; cylinder: $\frac{2r+2h}{rh}$ **c.** The ratio would be reduced by a factor of $\frac{1}{2}$.

45. Student A **47.** D **49.** A

51. $\frac{2(x^2+5x+25)}{x^2-5x+25}$

53. $\frac{2(x+1)(x+2)}{x^2+1}$

55. $3x^3 + 8x^2 - 36x - 5$

57. $y \approx 56{,}800(1.39)^x$; about 800,000 births **59.** $y = -4x$

8-3

Check It Out!

1a. $\frac{9x+4}{x^2-3}$; $x \neq \pm\sqrt{3}$

b. $\frac{x^2+3x-3}{3x-1}$; $x \neq \frac{1}{3}$ **2a.** $12x^5y^7$

b. $(x+2)(x-2)(x+3)$

3a. $\frac{15x-4}{6(x-1)}$; $x \neq 1$ **b.** $\frac{x+2}{x+3}$; $x \neq -3$ **4a.** $\frac{15x^2-20x-6}{(2x+5)(5x-2)}$; $x \neq -\frac{5}{2}$ and $x \neq \frac{2}{5}$ **b.** $\frac{x+4}{x-8}$; $x \neq \pm 8$ **5a.** $\frac{1}{x}$ **b.** 10 **c.** $\frac{3(x-2)}{2x(x+4)}$

6. 42.4 mi/h

Exercises **3.** $\frac{-2x-7}{4x+5}$; $x \neq -\frac{5}{4}$

5. $16x^4y^3$ **7.** $\frac{2(4x^2+x-8)}{(x+6)(2x-1)}$; $x \neq -6$ and $x \neq \frac{1}{2}$

9. $\frac{2x^2-4x-1}{(x+3)(x-3)}$; $x \neq \pm 3$

11. $\frac{-1}{x-4}$; $x \neq \pm 4$

13. $\frac{(2x-3)(x+2)}{4x-3}$ **15.** $\frac{3(x+2)}{2x^2}$

17. $\frac{2(2x-3)}{4x-7}$; $x \neq \frac{7}{4}$

19. $\frac{x^2-2x+2}{2x+7}$; $x \neq -\frac{7}{2}$

21. $(4x-5)(4x+5)(x+1)$

23. $\frac{7(2x-3)}{3(x-2)}$; $x \neq 2$

25. $\frac{-(2x+3)(x-2)}{(x-3)(x+3)}$; $x \neq \pm 3$

27. $\frac{1}{x-2}$; $x \neq 2$ and $x \neq 4$

29. $\frac{(3x-2)(x+3)}{(5x+1)(x+2)}$ **31.** 0.6 °C/min

33. $\frac{x^2+6x-6}{(x+4)(x-3)}$; $x \neq -4$ and $x \neq 3$

35. $\frac{5x-9}{(x-5)(x+4)(x+3)}$; $x \neq -4$, $x \neq -3$, and $x \neq 5$

37. $\frac{2x^2-13x+9}{(x-1)(x-2)}$; $x \neq 1$ and $x \neq 2$

39. $\frac{2(4x^3-6x^2-3x-4)}{(3x+4)(2x-3)}$; $x \neq -\frac{4}{3}$ and $x \neq \frac{3}{2}$

41. $\frac{-9x^2-52x+7}{(x+7)(x+6)(x+1)}$; $x \neq -7$, $x \neq -6$, and $x \neq -1$ **43.** $\frac{24}{(x+2)^2}$

45. $\frac{7(x-3)}{6x(x-1)}$ **49.** D **51.** A

53. $\frac{-5x^2-15x+6}{(x+2)(x-2)}$

55. $\frac{-4}{(x+2)^2(x-2)}$

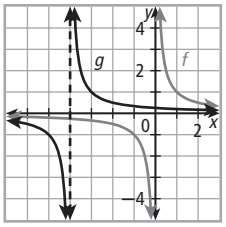
57. $6x^2 + 4x + 20$ **59.** $\frac{8}{5}$

61. The asymptote is $x = -4$. The transformation is a translation 4 units left.

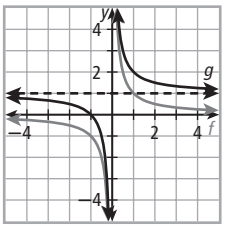
8-4

Check It Out!

1a. g is f translated 4 units left.

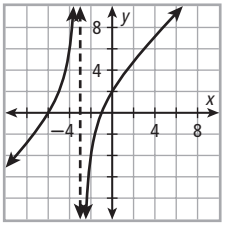


b. g is f translated 1 unit up.

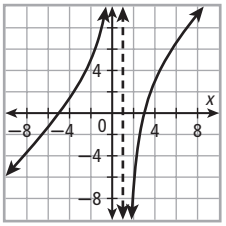


2. asymptotes: $x = 3$, $y = -5$; D: $\{x \mid x \neq 3\}$; R: $\{y \mid y \neq -5\}$

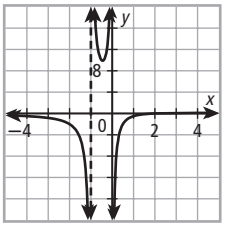
3. zeros: -6, -1; asymptote: $x = -3$



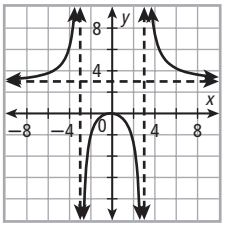
4a. zeros: -5, 3; asymptote: $x = 1$



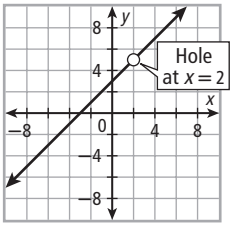
b. zero: 2; asymptotes: $x = -1$, $x = 0$, $y = 0$



c. zeros: $-\frac{1}{3}$, 0; asymptotes: $x = -3$, $x = 3$, $y = 3$



5. hole at $x = 2$



Exercises **1.** discontinuous
3. g is f translated 5 units left.
5. asymptotes: $x = 0$, $y = -1$; D: $\{x \mid x \neq 0\}$; R: $\{y \mid y \neq -1\}$
7. asymptotes: $x = 2$, $y = -8$; D: $\{x \mid x \neq 2\}$; R: $\{y \mid y \neq -8\}$
9. zeros: 0, 5; vertical asymptote: $x = 2$ **11.** zeros: -2, -1; asymptote: $x = 3$
13. zero: $-\frac{2}{5}$; asymptotes: $x = -1$, $y = 5$ **15.** hole at $x = 2$
17. g is f translated 5 units down.
19. g is f vertically stretched by a factor of 2. **21.** asymptotes: $x = 0$, $y = 5$; D: $\{x \mid x \neq 0\}$; R: $\{y \mid y \neq 5\}$
23. zeros: -2, 5; vertical asymptote: $x = 2$ **25.** zeros: -2, 2; vertical asymptote: $x = -3$
27. zero: 3; asymptotes: $x = -2$, $x = 2$, $y = 0$ **29.** hole at $x = 0$
31. hole at $x = 7$ **33.** zero: -1; asymptotes: $x = 0$, $y = 1$; hole at $x = 3$ **35.** zero: $\frac{5}{6}$; asymptotes: $x = \frac{2}{3}$, $y = -2$
37. zero: 0; asymptotes: $x = 3$, $x = -3$, $y = 0$ **43b.** ≈ 17 g
47b. $t = 12$; the number of seconds the driver spent at the pit stop
c. 57 s **51.** F **53.** holes at $x = 1$, $x = 2$, $x = 3$ **59.** $x = \frac{10}{3}$ **61.** $x = \frac{1}{3}$
63. $\frac{8x - 13}{2x + 1}$; $x \neq -\frac{1}{2}$

8-5

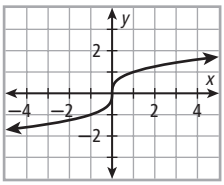
Check It Out! **1a.** $x = 3$ **b.** $x = -2$
c. $x = -3$, $x = 2$ **2a.** no solution
b. $x = -6$ **3.** 1.5 mi/h
4. about 24 min **5a.** $3 < x \leq 4$
b. $x = -5$
6a. $x \leq \frac{1}{2}$ or $x > 2$
b. $x < -3$ or $x > -\frac{3}{2}$
Exercises **3.** $w = \frac{1}{11}$
5. $x = -1$, $x = 6$ **7.** $k = 1$
9. $x = 0$, $x = 7$ **11.** 2.4 mi/h
13. $-5 < x < 0$ **15.** $x < 0$ or $x > 1$
17. $x < 4$ or $x \geq 8$ **19.** $x = 1$
21. $a = \frac{22}{3}$ **23.** $z = 2$, $z = 7$
25. $x = -8$ **27.** $x = -2$ **29.** about 6 h
31. $x = -3$
33. $x < 0$ or $x > \frac{1}{6}$
35. $-10 < x < -7$ **37a.** 2003
b. 18 hits **c.** 170 hits **39.** $z = 0$
41. $x = -5$ **43.** $a = 2$ **45.** $-1 < x < 1$
47. $x = \pm 0.45$ **49.** $x = 0$, $x = 2$
51a. 2001 winner: $\frac{500}{s}$; 2002 winner: $\frac{500}{s + 25}$ **b.** 141 mi/h
55. G **57b.** about 13 h **59.** all real numbers except -3, 0, and 3
61. $x < -21$ or $3 < x < 4$
63. $4(x + 4) - \frac{1}{2}(4x) = 2x + 16$
65. $\frac{5\sqrt{7}}{28}$ **67.** $y = \frac{3}{x}$

8-6

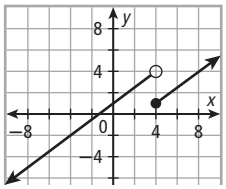
Check It Out! **1a.** no real roots
b. ± 1 **c.** 5 **2a.** $2x$ **b.** $\frac{\sqrt[4]{27}}{3}x^2$ **c.** x^3
3a. 4 **b.** 32 **c.** 125
4a. $81^{\frac{3}{4}}$ **b.** 1000 **c.** $5^{\frac{1}{2}}$ **5a.** 6 **b.** $-\frac{1}{2}$
c. 25 **6.** 32 cm from the bridge
Exercises **1.** 3 **3.** ± 5 **5.** $2x$
7. $\frac{5x^2\sqrt[3]{36}}{6}$ **9.** $x^3\sqrt[3]{x}$ **11.** $-2x\sqrt{10}$
13. 216 **15.** -3 **17.** $9^{\frac{10}{5}} = 9^2 = 81$
19. $5^{\frac{1}{2}}$ **21.** 169 **23.** 2 **25.** $\frac{1}{5}$ **27.** $-\frac{1}{5}$
29. 44 in. **31.** 2 **33.** $3x$
35. $\frac{x^2\sqrt[3]{4}}{10}$ **37.** $2x^3\sqrt[3]{7}$
39. $x^2\sqrt[5]{x^3}$ **41.** 8 **43.** 10,000
45. $14^{\frac{3}{3}} = 14^1 = 14$ **47.** $144^{\frac{1}{2}} = 12$
49. 64 **51.** $\frac{2}{3}$ **53.** $\frac{9}{7}$ **55.** $5^{\frac{1}{9}}$, or $\sqrt[9]{5}$
57. \$1189 **59a.** about 18%
b. about 12.6 g **61a.** $\frac{2\pi\sqrt{Lg}}{g}$
b. 1.2 s **63.** $(5x)^{\frac{7}{2}}$ **65.** $11^{\frac{3}{2}}x^{12}$
67. $5\sqrt[4]{125x^3}$ **69.** $b\sqrt[3]{a^2b}$
71. $b\sqrt[4]{4a^3b^2}$ **73.** always
75. never **77.** 2 and 3; about 2.62
79. -5 and -4; about -4.31
81. A is incorrect. **85.** A **87.** A
89. $20^{\frac{1}{8}}$ **91.** $a < -1$ or $0 < a < 1$
93. $A + D = \begin{bmatrix} 7 & 7 & -2 \\ 9 & 5 & 5 \end{bmatrix}$
95. $B + C = \begin{bmatrix} 6 & 6 \\ 0 & -6 \end{bmatrix}$
97. $h(x) = -x^2 + 3$ **99.** zero: -3; asymptotes: $x = -5$, $x = -1$, $y = 0$

8-7

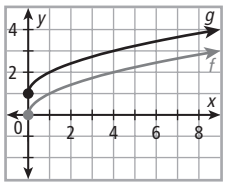
Check It Out! **1a.** D: $\{x \mid x \in \mathbb{R}\}$; R: $\{y \mid y \in \mathbb{R}\}$



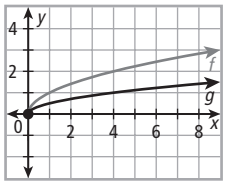
b. D: $\{x \mid x \geq -1\}$; R: $\{y \mid y \geq 0\}$



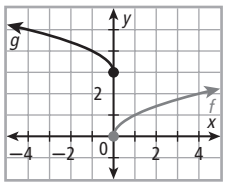
2a. g is f translated 1 unit up.



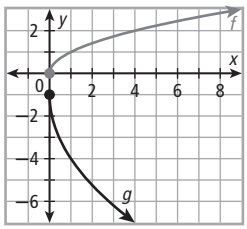
b. g is f vertically compressed by a factor of $\frac{1}{2}$.



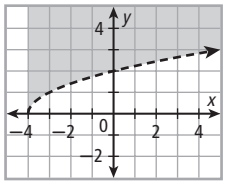
3a. g is f reflected across the y-axis and translated 3 units up.



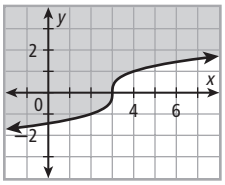
b. g is f vertically stretched by a factor of 3, reflected across the x-axis, and translated 1 unit down.



4. $g(x) = -2\sqrt{x} + 1$
5. $h(x) = \sqrt{\frac{256}{25}x}$; about 23 ft/s
6a.



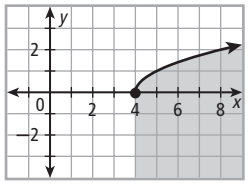
b.



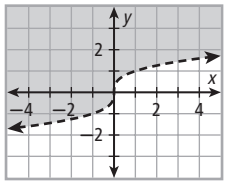
Exercises **3.** D: $\{x|x \geq 0\}$; R: $\{y|y \geq -1\}$ **5.** D: $\{x|x \in \mathbb{R}\}$; R: $\{y|y \in \mathbb{R}\}$ **7.** D: $\{x|x \in \mathbb{R}\}$; R: $\{y|y \in \mathbb{R}\}$ **9.** h is f vertically stretched by a factor of 3. **11.** g is f compressed vertically by a factor of $\frac{1}{2}$ and translated 1 unit down. **13.** j is f reflected across the y-axis and then translated 3 units right. **15.** h is f reflected across the y-axis, horizontally compressed by a factor of $\frac{1}{2}$, and then translated 2 units left. **17.** $g(x) = 4\sqrt{(x+5)} - 2$

19. $g(x) = \frac{6}{5}\sqrt{\frac{5}{9}x}$; about 2.2 mi

21.



23.

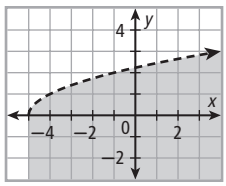


25. D: $\{x|x \geq 0\}$; R: $\{y|y \leq 0\}$
27. D: $\{x|x \in \mathbb{R}\}$; R: $\{y|y \in \mathbb{R}\}$
29. D: $\{x|x \in \mathbb{R}\}$; R: $\{y|y \in \mathbb{R}\}$
31. h is f translated 4 units right. **33.** g is f horizontally compressed by a factor of $\frac{1}{3}$ and then translated 5 units left. **35.** j is f translated 4 units left and 1 unit down. **37.** h is f reflected across the y-axis, vertically stretched by a factor of 3, and then translated 2 units up.

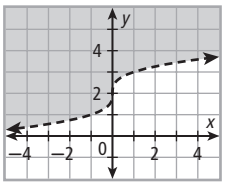
39. $g(x) = \frac{1}{3}\sqrt{x+3}$

41. $g(x) = -\sqrt{x+1} - 4$

43.



45.



47a. $\approx$ 762 beats/min **b.** $\approx$ 58 beats/min **49.** a vertical stretch by a factor of 3 followed by a translation 1 unit right and 9 units down **51.** D **53.** A **55a.** about 373 km **b.** It will appear to decrease by about 76 km. **57.** yes **59c.** by a factor of 4 **61.** sometimes **63.** never **65.** yes **67b.** about 346 m/s **c.** −273.15°C **69.** 1.4 s **73.** D **75.** A

79. $f(x) = -2\sqrt{\frac{1}{5}(x+3)} + 4$

81. $x \leq 8$ **83.** $x = 6, y = 2$

85. $x = -\frac{40}{7}, y = -\frac{68}{7}$ **87.** $x = 4$

8-8

Check It Out! **1a.** $x = 2$ **b.** $x = 4$ **c.** $x = 39$ **2a.** $x = 6$ **b.** $x = 2$ **3a.** $x = 1$ **b.** $x = -4, x = 3$ **4a.** $x = 22$ **b.** $x = 5$ **c.** $x = 3$ **5a.** $3 \leq x \leq 12$ **b.** $x \geq -1$ **6.** If the car were traveling 30 mi/h, its skid marks would have measured about 43 ft. Because the actual skid marks measure less than 43 ft, the car was not speeding.

Exercises **1.** No; the expression under the radical does not contain a variable. **3.** $x = 12$ **5.** $x = 5$ **7.** $x = 3$ **9.** $x = 10$ **11.** $x = 8$ **13.** $x = 4, x = 5$ **15.** $x = -1$ **17.** $x = 14$ **19.** $x = 5$ **21.** $x = -\frac{3}{4}$ **23.** $-5 \leq x \leq 20$ **25.** $\frac{-5}{2} \leq x < 10$ **27.** $x = 93$ **29.** $x = 18$ **31.** $x = 38$ **33.** $x = 6$ **35.** $x = \frac{7}{3}$ **37.** $x = 4$ **39.** $x = 25$ **41.** $x = 7$ **43.** $3 \leq x \leq 19$ **45.** about 1.5 tons **47.** $A = \pi r^2$ **49.** $E = \frac{1}{2}mv^2$ **51a.** 5 m **b.** 25.6 m/s^2 **53a.** $r \leq \sqrt{\frac{A}{\pi}}$ **b.** no **55.** $x \approx 5.84$ **57.** $x \approx 2.35$ **59.** 995 g **63.** H **65.** F **67.** always true **69.** always true **71.** $x = 9$ **73.** $x = 10$ **75a.** $D(n) = 2.00n + 5.00$ **c.** vertical translation 5 units down

77. $f^{-1}(x) = -\frac{1}{3}x - \frac{1}{3}$ **79.** $4x^3$

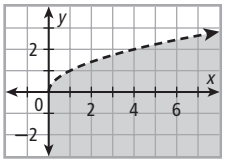
81. $\frac{\sqrt[3]{18x}}{x}$

Study Guide: Review

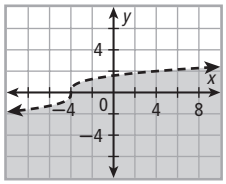
1. rational function **2.** direct variation; constant of variation **3.** $y = \frac{1}{3}x$ **4.** $y = 4x$ **5.** 306 tiles **6.** \$2000 **7.** $y = \frac{6}{x}$ **8.** $y = \frac{4}{x}$ **9.** 24 ohms **10.** inverse variation **11.** $\frac{8}{3x^2}; x \neq 0$ **12.** $\frac{2x^3}{x+4}; x \neq -4$ **13.** $\frac{x-3}{x+1}; x \neq -4, x \neq -1$ **14.** $\frac{3}{x-5}$ **15.** $\frac{-x}{(x-4)(x+3)}$ **16.** $\frac{x-1}{x+1}$ **17.** $\frac{3x-1}{x-3}$ **18.** $\frac{2x}{y}$ **19.** $\frac{2(x+5)}{x+3}$ **20.** 1 **21.** $\frac{x+3}{3(x+4)}$ **22.** $\frac{x^2+12}{x^2+4}$ **23.** $\frac{2x}{(x+3)(x-3)}; x \neq \pm 3$ **24.** $\frac{2(x+1)}{(x+2)(x-2)}; x \neq \pm 2$ **25.** $\frac{8x^2+4x+45}{(3x+7)(4x-1)}; x \neq -\frac{7}{3}, x \neq \frac{1}{4}$ **26.** $(x-3)^2(x+3)$ **27.** $(x-5)(x+2)(x+7)$ **28.** $\frac{2x-3}{x+4}; x \neq -4$ **29.** $\frac{x^2-10x-25}{(x+5)(x-5)}; x \neq \pm 5$ **30.** $\frac{-(x^2-3x-1)}{(x-3)(x+2)}; x \neq -2, x \neq 3$ **31.** $\frac{6x^2-16x-7}{(2x+1)(3x-1)}; x \neq -\frac{1}{2}, x \neq \frac{1}{3}$ **32.** $\frac{8(x-6)}{5(x+2)}$ **33.** $\frac{2x-3}{x(x-3)}$ **34.** $\frac{(x-2)^2}{4x}$ **35.** $\approx$ 548 mi/h **36.** g is f translated 4 units right. **37.** g is f translated 2 units right and 3 units up. **38.** asymptotes: $x = 1, y = -3$; D: $\{x|x \neq 1\}$; R: $\{y|y \neq -3\}$ **39.** asymptotes: $x = -2, y = 1$; D: $\{x|x \neq -2\}$; R: $\{y|y \neq 1\}$ **40.** zeros: 0, 3; asymptote: $x = -4$ **41.** zero: 3; asymptotes: $x = -5, x = -1, y = 0$ **42.** zero: 2; asymptotes: $x = -3, y = 2$ **43.** zeros: −3, 3; asymptote: $x = 2$ **44.** hole at $x = -3$ **45.** $x = -2$ or $x = 3$ **46.** no solution **47.** $x = 2$ **48.** $x = 0$ **49.** $x < -\frac{4}{3}$ or $x > 0$ **50.** $x < 3$ or $x > \frac{7}{2}$ **51.** $3x^2$ **52.** $3x^3$ **53.** $2x\frac{\sqrt[3]{9}}{3}$ **54.** $(-27)^{\frac{2}{3}}$ **55.** $16^{\frac{3}{4}}$

56. $9^{\frac{3}{2}}$ **57.** 17 **58.** 81 **59.** $\frac{1}{2}$
60. D: $\{x \mid x \geq 0\}$; R: $\{y \mid y \geq 5\}$
61. D: $\mathbb{R}$; R: $\mathbb{R}$ **62.** g is f reflected across the x-axis and translated 1 unit up. **63.** h is f compressed horizontally by a factor of $\frac{1}{4}$.
64. j is f reflected across the y-axis and translated 8 units right.
65. k is f reflected across the x-axis, compressed vertically by a factor of $\frac{1}{2}$, and translated 1 unit up.
66. $g(x) = 3\sqrt{x+4}$

67.



68.



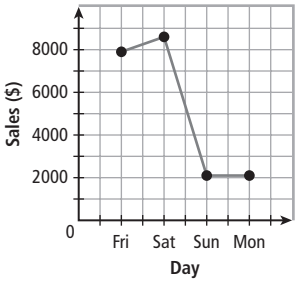
69. $x = 19$ **70.** $x = 109$ **71.** $x = 9$
72. $x = 2$ **73.** $x = 2$ or $x = 8$
74. $x = 8$ **75.** $x = 0.5$ **76.** $x = 85$
77. $x = 7$ **78.** $x = -219$
79. $4 \leq x \leq 13$ **80.** $x > 9$
81. $0 \leq x < 12$ **82.** $x > -7$
83. ≈ 1.6 m **84.** 60.3 m^3

Chapter 9

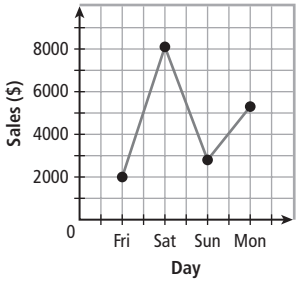
9-1

Check It Out!

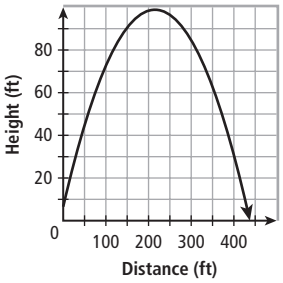
1.



b.

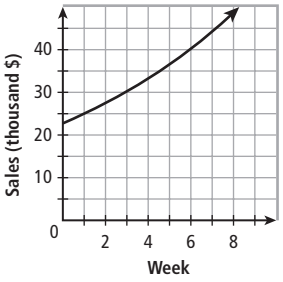


2.



$h(x) = -0.002x^2 + 0.86x + 6.55$

3.



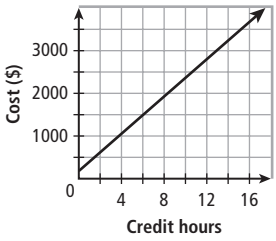
$f(x) \approx 22{,}727.15(1.1)^x$; 8 weeks

Exercises **1.** graph D **3.** graph B

5.

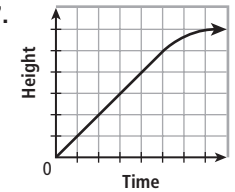
Enrollment Costs	
Credit Hours	**Cost ($)**
1	397.75
2	616.15
3	834.55
4	1053.00
5	1271.40

$C = 179.35 + 218.4x$



7. graph C **9.** graph D **11.** $T(t) = -0.02375t^2 + 0.525t + 101.1$
13a. $W(t) = 3t + 4$; The whale weighs 4 tons at birth and gains 3 tons per month. **15.** $14

17.



19a. $C(t) = -3t^2 + 21t + 24$
b. ≈ 61 **c.** 8 h after opening
21b. $h(t) \approx 0.0071t^3 - 0.1714t^2 + 2.1t + 2.071$ **c.** during year 14
25. C **29.** $C(p) = 1.065(0.8p - 10)$
31. $(-3, -4)$ **33.** 40×20 ft
35. D: $\{x \mid x \geq 1\}$; R: $\{y \mid y \geq 0\}$

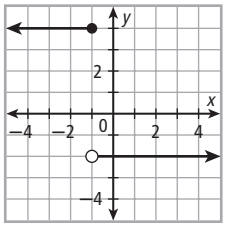
9-2

Check It Out!

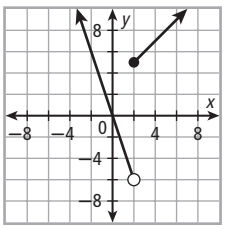
1.

Time Range (h)	Green Fee ($)
[8 A.M. – noon)	28
[noon – 4 P.M.)	24
[4 P.M. – 9 P.M.)	12

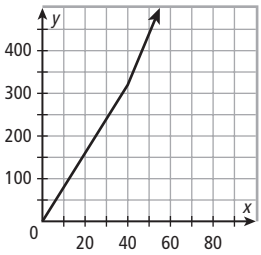
The green fee is $28 from 8 A.M. up to noon, $24 from noon up to 4 P.M., and $12 from 4 P.M. up to 9 P.M.
2a. 15; 15 **b.** 4; 13
3a.



b.



4.



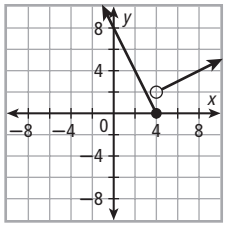
$$f(h) = \begin{cases} 8h & \text{if } 0 \leq h \leq 40 \\ 12(h-40) + 320 & \text{if } h > 40 \end{cases}$$

Exercises **1.** Step functions are a subset of piecewise functions. A step function is a piecewise function that is constant over each interval in its domain. **3.** The price per yard is $10 for less than 5 yd^3, $7 for 5 yd^3 up to 25 yd^3, and $4 for 25 yd^3 or more.

Topsoil Prices	
Price per Cubic Yard ($)	**Volume (yd^3)**
10	$0 \leq x < 5$
7	$5 \leq x < 25$
4	$x \geq 25$

5. −39; −5

7.

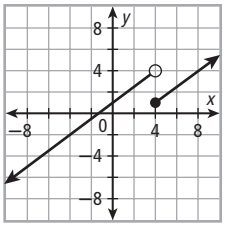


9.

Buffet Prices	
Price ($)	Age (yr)
0	$0 < x < 3$
2	$3 \le x < 8$
5	$8 \le x < 18$
8	$18 \le x$

The buffet is free for children under 3, $2 for children from 3 up to 8, $5 for children from 8 up to 18, and $8 for adults. **11.** 1; 5; 5

13.



15. $f(x) = \begin{cases} 30 & \text{if } 0 < x \le 15 \\ 50 & \text{if } 15 < x \le 50 \\ 75 & \text{if } x > 50 \end{cases}$

17. $f(x) = \begin{cases} \frac{6}{5}x - 3 & \text{if } x < 5 \\ \frac{2}{5}x + 2 & \text{if } x \ge 5 \end{cases}$

19. $f(x) = \begin{cases} 6 & \text{if } x \le 4 \\ 6 + 3(x - 4) & \text{if } x > +4 \end{cases}$

21. $f(x) = \begin{cases} +x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$

23. $h(x) = \begin{cases} 2x - 4 & \text{if } x \ge 0 \\ -2x - 4 & \text{if } x < 0 \end{cases}$

27a. $d(t) = \begin{cases} 18t & \text{if } 0 \le t \le 10 \\ 16.5(t-10) + 180 & \text{if } 10 < t \le 20 \end{cases}$

b. First half; the slope is steeper.
29. D: ℝ; R: $\{y | y \ge -4\}$ **33.** C **35.** B
37. $f(x) = 4 + 1.5(\lceil x \rceil - 1)$; $11.50
39. vertical: $x = 1$; horizontal: $y = -3$; D: $\{x | x \ne 1\}$; R: $\{y | y \ne -3\}$
41. vertical: $x = 3$; horizontal: $y = 1$; D: $\{x | x \ne 3\}$; R:$\{y | y \ne 1\}$ **43.** A
45. B

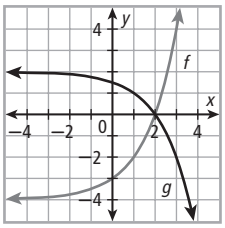
9-3

Check It Out!

1. $g(x) = \begin{cases} \left(\frac{x}{2}\right)^2 & \text{if } x \le 0 \\ \frac{x}{2} - 3 & \text{if } x > 0 \end{cases}$

2a. $f(x)$: x-int. = −6, y-int. = 4; $g(x)$: x-int. = −6, y-int. = −4
b. $f(x)$: x-int. = ±3, y-int. = −9; $g(x)$: x-int. = ±3, y-int. = −3

3.



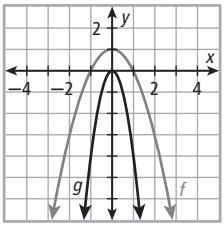
4. $f(x) = \begin{cases} 6.50 & \text{if } x < 12 \\ 9.50 & \text{if } x \ge 12 \end{cases}$

Exercises

1. $g(x) = \begin{cases} x + 3 & \text{if } x \le -6 \\ 4(x + 6) & \text{if } x > -6 \end{cases}$

3. $f(x)$: x-int. = −3, y-int. = 12; $g(x)$: x-int. = −3, y-int. = 2

5.

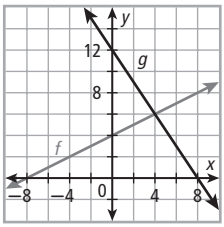


7. $T(x) = \begin{cases} 0.024x + 100 & \text{if } 0 < x \le 10{,}000 \\ 0.060x + 100 & \text{if } x > 10{,}000 \end{cases}$

9. $h(x) = \begin{cases} \left(\frac{x}{2}\right)^2 & \text{if } x < 2 \\ 2x & \text{if } x \ge 2 \end{cases}$

11. $f(x)$: x-int. = 6, y-int. = 9; $g(x)$: x-int. = 6, y-int. = 6
13. $f(x)$: x-int. = 5, y-int. = 2; $g(x)$: x-int. = 2.5, y-int. = 2
15. $f(x)$: x-int. = 0, y-int. = 0; $g(x)$: x-int. = 1, y-int. = −4

17.



19a. $f(n) = \begin{cases} 16.2n & \text{if } n \le 50 \\ 360 + 9n & \text{if } n > 50 \end{cases}$

b. $f(n) = \begin{cases} 14.2n & \text{if } n \le 50 \\ 360 + 7n & \text{if } n > 50 \end{cases}$

21a. n x-intercepts

23a. $T(n) = \begin{cases} 2.8n & \text{if } n \le 8 \\ 3.6n - 6.4 & \text{if } n > 8 \end{cases}$

25. $f(x) - 7 = \begin{cases} 2^x - 8 & \text{if } x \le -3 \\ -5x - 4 & \text{if } x > -3 \end{cases}$

27b. $C(x) = \begin{cases} 1.29x & \text{if } 0 < x < 4 \\ 0.85(1.29x) & \text{if } 4 \le x < 7 \\ 0.7(1.29x) & \text{if } x \ge 7 \end{cases}$

c. horizontal stretch by a factor of 2
29. x-int: 2; y-int: 3 **33.** J **35a.** 28
b. 56 **37.** about 418 **39.** −6; D: ℝ; R: $\{y | y \le -6\}$ **41.** 13; 4; 9

9-4

Check It Out! **1a.** $(f + g)(x) = x^2$
b. $(f - g)(x) = -x^2 + 10x - 12$
2a. $(fg)(x) = x^3 + 2x^2 - 4x - 8$
b. $\left(\frac{g}{f}\right)(x) = x - 2, x \ne -2$ **3a.** 15
b. 9 **4a.** $f(g(x)) = 3\sqrt{x} + 2, x \ge 0$
b. $g(f(x)) = \sqrt{3x - 4} + 2, x \ge \frac{4}{3}$
5a. $f(c) = 0.68c$ **b.** $168.64

Exercises

3. $-x^2 + 13x + 13$
5. $2x^3 + 4x^2 + 2x$ **7.** $\frac{1}{2x}, x \ne 0$ or -1
9. −68 **11.** $4x^2 - 12x + 9$; ℝ
13. $x + 1; x \ge -1$ **15.** $3x^2 + 5x - 2$
17. $2x^2 + 2x - 4$
19. $2x^4 + 10x^3 + 4x^2 - 40x - 48$
21. $\frac{1}{x - 2}, x \ne -2$ or 2
23. $\frac{x + 3}{2}, x \ne -2$ **25.** −11 **27.** −17
29. −59 **31.** $\frac{4x + 3}{4x + 6}; x \ne -\frac{3}{2}$
33a. $C(x) = 4\left(\frac{x}{9}\right) + 100$ **b.** 630 ft²
35a. $f(p) = p - 10$ **b.** $g(p) = 0.85p$
c. $f(g(p)) = 0.85p - 10$; $g(f(p)) = 0.85p - 8.5$ **d.** 15%
e. $31.65 **37a.** $D(t) = 704 \cdot 1.05^t$
b. about 3043 **c.** about 2020 **39.** 4
41. 2 **43.** no **45.** B **47.** B
49. $g(x) = \frac{3}{2}x^2 + 5$ **51a.** 12 ft **b.** 8 ft
53. $f(x) = 1.25(2^t)$
55. $g(x) = \begin{cases} 8(x + 5) & x \ge -5 \\ x - 4 & x < -5 \end{cases}$

9-5

Check It Out! **1.** function
2. $f^{-1}(x) = \sqrt[3]{x+2}$; function; D: $\mathbb{R}$; R: $\mathbb{R}$ **3a.** yes **b.** no

Exercises **1.** function **3.** function
5. $y = \pm\sqrt{x+9}$; not a function; D: $\{x \mid x \geq -9\}$; R: $\mathbb{R}$ **7.** no **9.** not a function **11.** function
13. $f^{-1}(x) = \frac{\sqrt[3]{x}}{2}$; function; D: $\mathbb{R}$; R: $\mathbb{R}$ **15.** $f^{-1}(x) = \frac{6}{5}x - \frac{9}{5}$; function; D: $\mathbb{R}$; R: $\mathbb{R}$
17. $f^{-1}(x) = (x-5)^2 - 8$; function; D: $\{x \mid x \geq 5\}$; R: $\{y \mid y \geq -8\}$ **19.** no
21. yes **23a.** $d(t) = \frac{t-20}{2.5}$
b. within 4 mi **25.** $y = \frac{5}{x} - 4$; D: $\{x \mid x \neq 0\}$; R: $\{y \mid y \neq -4\}$
27. $y = x^3 + 12$; D: $\mathbb{R}$; R: $\mathbb{R}$
29. $y = \log_7 x$; D: $\{x \mid x > 0\}$; R: $\mathbb{R}$
31. $y = \ln\left(\frac{x}{3}\right) - 5$; D: $\{x \mid x > 0\}$; R: $\mathbb{R}$
33. g and h **35.** f and h
37a. $a(h) = \left(\frac{h-19}{3}\right)^2$
b. about 20.25 mo
39a. $t(d) = \frac{\sqrt{d^2 - 1600}}{3}$
b. $\approx$ 1833.28 s (about 31 min)
41a. $h(s) = \frac{s - 18\pi}{6\pi}$ **b.** 23.53 cm
43a. $s = \sqrt{A}$ **c.** $\approx$ 894 ft **47.** J
49. G **53.** $y = e^{\frac{3x-3}{x+1}}$
55. $f^{-1}\left(g^{-1}(x)\right) \neq \left(f(g(x))\right)^{-1}$
57. $2\pi x^2 + 2\pi x$ **59.** $x^3 + 27$
61. $x^2 + 8x - 6$ **63.** $x^2 - 10x + 4$

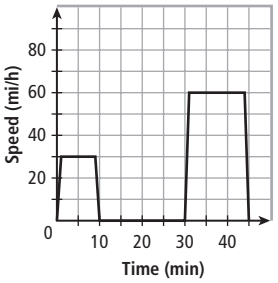
9-6

Check It Out! **1a.** square root
b. exponential
2. $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x + 8$
3. $f(x) \approx -0.2x^2 + 23.99x + 5.28$

Exercises **1.** linear **3.** exponential
5a. $V(t) \approx 0.08t^2 - 2.04t + 60.86$
b. about \$51.68 **7.** quadratic
9. $f(x) \approx -0.009x^2 + 2.28x - 55.31$
11a. $y \approx 34.37x + 85{,}851.76$
b. about 2594 ft^2
15a. $V(t) \approx 6126.9(1.016)^t$
b. about 34,611 ft^3
17a. $f(x) \approx 75.95(1.055^x)$
b. about 5.5%/yr **c.** The model predicts \$160.65, which is about \$5 more than the actual FCI.
d. about 2008 **21.** B **23.** D
25. $f(x) \approx x^{0.5}$; $f(x) = \sqrt{x}$
29. a. -2, 4, and 10 **31.** yes

Study Guide: Review

1. one-to-one function
2. step function
3. composition of functions
4.



5.

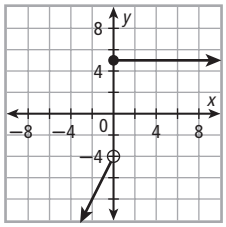
Guests	10	20	30	40	50
Appetizers	160	200	240	280	320

$y = 4x + 120$

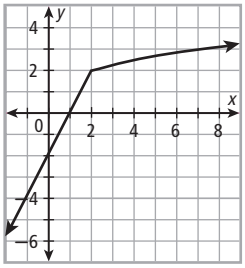
6a.

Radius (in.)	1.5	2	2.5	3	4
Time (s)	3	5	7.5	10.5	18

$y = x^2 + \frac{1}{2}x$ **b.** 52.5 s **7.** 51; 7
8.



9.



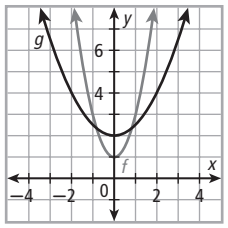
10. $f(x) = \begin{cases} \frac{5}{2}x - 4 & \text{if } x < 4 \\ -\frac{3}{2}x + 8 & \text{if } x \geq 4 \end{cases}$

11. $f(x) = \begin{cases} 6 & 0 < x \leq 8 \\ 6 + 1.5(x-8) & 8 < x \leq 48 \end{cases}$

12. $h(x) = \begin{cases} 2x & \text{if } x \leq 3 \\ -4x + 18 & \text{if } x > 3 \end{cases}$

13. $g(x) = \begin{cases} 3(x-7) + 2 & \text{if } x \leq 7 \\ (x-7)^2 & \text{if } x > 7 \end{cases}$

14.

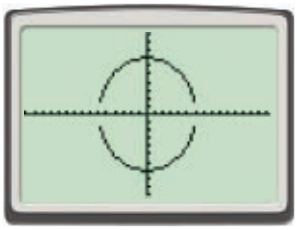


15. $x^2 - 4x - 21$ **16.** $x^2 - 6x - 7$
17. $-x^2 + 6x + 7$
18. $x^3 - 12x^2 + 21x + 98$
19. $x + 2$, $x \neq 7$ **20.** $\frac{1}{x+2}$, $x \neq 7$ or -2 **21.** -10; $-\frac{8}{3}$ **22.** 2; undefined
23. $g(f(x)) = \frac{8}{x-1}$; D: $\{x \mid x \neq 1\}$
24. $f(g(x)) = \frac{8}{x+1} - 2$; D: $\{x \mid x \neq -1\}$
25. $P(x) = 1.09(x + 30)$ **26.** function
27. $f^{-1}(x) = \frac{-x+5}{8}$; function; D: $\mathbb{R}$; R: $\mathbb{R}$ **28.** $y = \pm 3\sqrt{x} - 6$; not a function; D: $\{x \mid x \geq 0\}$; R: $\mathbb{R}$
29. $f^{-1}(x) = \frac{5}{2x} - 4$; function; D: $\{x \mid x \neq 0\}$; R: $\{y \mid y \neq -4\}$
30. $f^{-1}(x) = (x-3)^2 + 5$; function; D: $\{x \mid x \geq 3\}$; R: $\{y \mid y \geq 5\}$ **31.** no
32. yes **33.** $r = \sqrt{\frac{A}{4\pi}}$; r is the radius for a sphere with a given surface area.
34a. $f(x) = 23.96(1.02)^x$
b. $\approx$ 129.0 million gal **c.** $\approx$ 37°F

Chapter 10

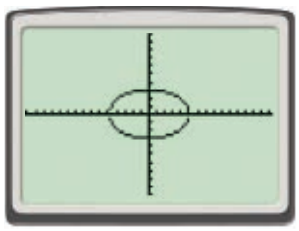
10-1

Check It Out!
1a.



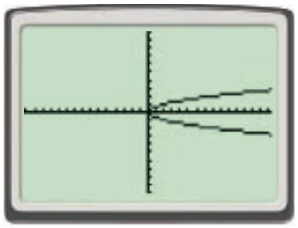
circle; center: (0, 0); intercepts: (0, $\pm$7), ($\pm$7, 0)

b.



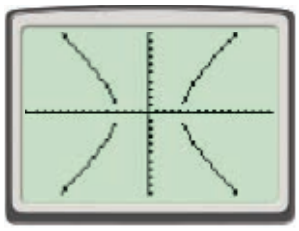
ellipse; center: (0, 0); intercepts: $(\pm 5, 0)$, $(0, \pm 3)$

2a.



parabola; vertex: (0, 0); opens right

b.



hyperbola; vertices: $(\pm 4, 0)$; opens horizontally **3.** Center: (8, 14); $r = 10$

Exercises **1.** circles, ellipses, hyperbolas, and parabolas **3.** ellipse; center: (0, 0); intercepts: $(0, \pm 3)$, $(\pm 4, 0)$ **5.** parabola; vertex: (0, 0); opens right **7.** hyperbola; vertices: $(0, \pm 5)$; opens vertically **9.** hyperbola; vertices: $(\pm\sqrt{2}, 0)$; opens horizontally **11.** center: (8, 18); $r = 13$ **13.** center: $(-1, 15)$; $r = 25$ **15.** circle; center: (0, 0); intercepts: $(0, \pm 3)$, $(\pm 3, 0)$ **17.** ellipse; center: (0, 0); intercepts: $(0, \pm 5)$, $(\pm 2, 0)$ **19.** ellipse; center: (0, 0); intercepts: $\left(0, \pm\frac{15}{2}\right)$, $\left(\pm\frac{5}{2}, 0\right)$ **21.** circle; center: (0, 0); intercepts: $\left(0, \pm\frac{9}{2}\right)$, $\left(\pm\frac{9}{2}, 0\right)$ **23.** parabola; vertex: (0, 0); opens upward **25.** parabola; vertex: (0, 0); opens left **27.** hyperbola; vertices: $(0, \pm 6)$; opens vertically **29.** parabola; vertex: $(-3, 0)$; opens right **31.** hyperbola; vertices: $(\pm 4, 0)$; opens horizontally **33.** center: $\left(\frac{7}{2}, \frac{11}{2}\right)$; $r = \sqrt{10}$ **35a.** $C = 68\pi$; $A = 1156\pi$ **b.** $(-37, 26)$ **37.** D **39.** A **41a.** $AB = 10$; $AD = 10$; $BC = 10$; $CD = 10$ **b.** rhombus **c.** 80 square units **43.** C **45a.** 13 units **b.** 6.5 units **c.** $\frac{12}{5}$; $\frac{12}{5}$ **47.** Sometimes true **51.** J **53.** J **55.** $a = -32$ or 40 **57. a.** (9, 2, −11) **b.** $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$ **c.** $d = \sqrt{101}$ **d.** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ **59.** $x = -6, 8$ **61.** $x = 4, 7$ **63.** $x = 1.5, 11$ **65.** y-int.: 2.5; asymptote: $y = 3$; reflection across the x-axis, vertical compression by a factor of $\frac{1}{2}$, shift 3 units up **67.** y-int.: 5; asymptote: $y = -1$; vertical stretch by a factor of 6, shift 1 unit down

10-2

Check It Out!

1. $(x - 4)^2 + (y - 2)^2 = 49$

2. $(x + 3)^2 + (y - 5)^2 = 169$

3. C, E **4.** $y = \frac{4}{3}x - \frac{35}{3}$

Exercises

3. $(x + 11)^2 + (y - 3)^2 = 81$ **5.** $(x - 3)^2 + y^2 = 36$ **7.** $(x + 2)^2 + (y + 5)^2 = 289$ **9.** K, H, G **11.** $y = -\frac{3}{4}x - \frac{59}{4}$ **13.** $(x - 5)^2 + (y-1)^2 = 100$ **15.** $(x + 4)^2 + (y - 2)^2 = 64$ **17.** $(x + 6)^2 + (y + 4)^2 = 25$ **19.** E **21.** $x = -15$ **23.** $\{x | -6 \le x \le 6\}$; $\{y | -6 \le y \le 6\}$ **25.** $\{x | -5 \le x \le 1\}$; $\{y | -3 \le y \le 3\}$ **27.** $(x + 4)^2 + y^2 = 64$ **29a.** $(x + 5)^2 + (y - 20)^2 = 4489$ **b.** 67 million mi **c.** 134π million mi **31.** No **33.** C **35.** $(x + 4)^2 + (y - 8)^2 = 81$ **37a.** $(2, -5)$ **b.** $(x - 2)^2 + (y + 5)^2 = 625$ **41.** $y = \frac{1}{2}x + 2$

43a. $f(x) = \begin{cases} \frac{1}{2}x + 15 & 0 \le x \le 20 \\ x + 20 & x > 20 \end{cases}$

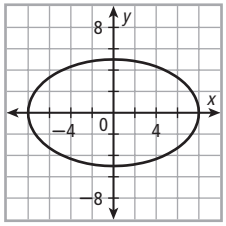
c. 25 min **45.** parabola; vertex: (0, 0); opens left

10-3

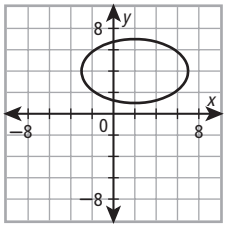
Check It Out! **1.** 20

2a. $\frac{x^2}{81} + \frac{y^2}{25} = 1$ **b.** $\frac{y^2}{25} + \frac{x^2}{16} = 1$

3a.



b.

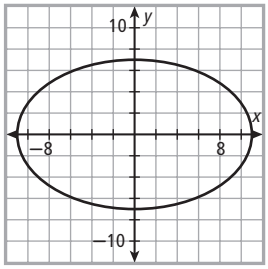


4a. width: 32 ft; height: 18 ft

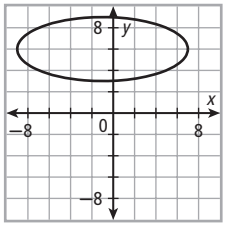
b. $\frac{x^2}{256} + \frac{y^2}{324} = 1$

Exercises **1.** The major axis of an ellipse is always longer than the minor axis of an ellipse. **3.** 30 **5.** $\frac{y^2}{625} + \frac{x^2}{225} = 1$ **7.** $\frac{x^2}{49} + \frac{y^2}{36} = 1$

9.



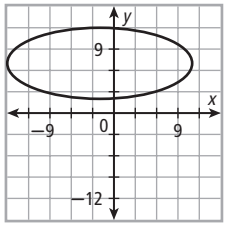
11.

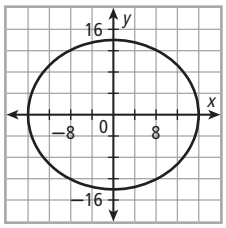


13. 42 **15.** $\frac{x^2}{25} + \frac{y^2}{4} = 1$

17. $\frac{y^2}{25} + \frac{x^2}{16} = 1$

19.



21. 

23. $\frac{x^2}{279{,}312.25} + \frac{y^2}{193{,}600} = 1$

25. $\frac{(y-7)^2}{100} + \frac{(x+4)^2}{51} = 1$

27. $\frac{x^2}{49} + \frac{y^2}{25} = 1$; D: $\{x \mid -7 \le x \le 7\}$; R: $\{y \mid -5 \le y \le 5\}$

29. $\frac{(y-4)^2}{36} + \frac{(x+6)^2}{9} = 1$; $\{x \mid -9 \le x \le -3\}$, $\{y \mid -2 \le y \le 10\}$

31a. $\frac{9x^2}{5041} + \frac{16y^2}{729} = 1$

b. $(\pm 22.68, 0)$; ~45.36 ft **33.** center: $(-9, -4)$; vertices: $(0, -4)$, $(-18, -4)$; co-vertices: $(-9, -7)$, $(-9, -1)$; foci: $(-9 \pm 6\sqrt{2}, -4)$; D: $\{x \mid -18 \le x \le 0\}$; R: $\{y \mid -7 \le y \le -1\}$ **35a.** Instead of r^2, the formula for the area of an ellipse uses the values of a and b because an ellipse can be defined by a and b rather than a radius. **b.** 65π **37.** The length of an ellipse's major axis is equal to the distance $PF_1 + PF_2$. **39.** H

41a. $\frac{21}{29}$ **b.** $\frac{x^2}{169} + \frac{y^2}{144} = 1$

c. $0 < e < 1$ **43.** $\frac{x^2}{25} + \frac{y^2}{16} = 1$

45. 24 **47.** 56

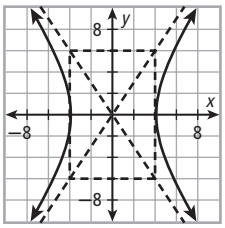
49. $x^2 + (y+1)^2 = 100$

10-4

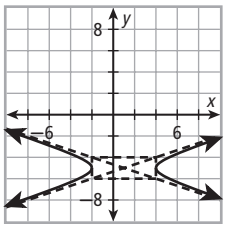
Check It Out! 1. 12

2a. $\frac{y^2}{81} - \frac{x^2}{49} = 1$ **b.** $\frac{x^2}{64} - \frac{y^2}{36} = 1$

3a. vertices: $(\pm 4, 0)$; co-vertices: $(0, \pm 6)$; asymptotes: $y = \pm\frac{3}{2}x$



b. vertices: $(1, -4)$, $(1, -6)$; co-vertices: $(4, -5)$, $(-2, -5)$; asymptotes: $y = \pm\frac{1}{3}(x-1) - 5$



Exercises 1. transverse axis

3. 30 **5.** $\frac{x^2}{81} - \frac{y^2}{49} = 1$

7. $\frac{y^2}{100} - \frac{x^2}{64} = 1$

9. vertices: $(\pm 5, 0)$; co-vertices: $(0, \pm 8)$; asymptotes: $y = \pm\frac{8}{5}x$

11. vertices: $(0, \pm 10)$; co-vertices: $(\pm 9, 0)$; asymptotes: $y = \pm\frac{10}{9}x$

13. vertices: $(8, -6)$, $(0, -6)$; co-vertices: $(4, 1)$, $(4, -13)$; asymptotes: $y = \pm\frac{7}{4}(x-4) - 6$

15. vertices: $(0, -5)$, $(0, -9)$; co-vertices: $(\pm 5, -7)$; asymptotes: $y = \pm\frac{2}{5}x - 7$ **17.** 42

19. $\frac{x^2}{64} - \frac{y^2}{225} = 1$

21. $\frac{(x-3)^2}{49} - \frac{(y-3)^2}{9} = 1$

23. vertices: $(0, \pm 5)$; co-vertices: $(\pm 9, 0)$; asymptotes: $y = \pm\frac{5}{9}x$

25. vertices: $(\pm 2, 0)$; co-vertices: $(0, \pm 11)$; asymptotes: $y = \pm\frac{11}{2}x$

27. vertices: $(0, 3)$, $(-10, 3)$; co-vertices: $(-5, 7)$, $(-5, -1)$; asymptotes: $y = \pm\frac{4}{5}(x+5) + 3$

29. vertices: $(9, 2)$, $(3, 2)$; co-vertices: $(6, 6)$, $(6, -2)$; asymptotes: $y = \pm\frac{4}{3}(x-6) + 2$

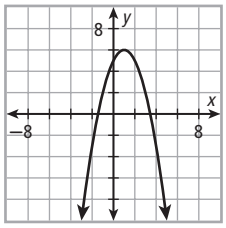
33b. Yes **35.** $c^2 = a^2 + b^2$, so c always has the greatest value. There is not enough information given to determine whether a or b has the least value. **37a.** (0, 214)

b. 184 million mi **c.** $y \approx \pm 0.142x$

39. G **41.** J

43. $\frac{(x-7)^2}{400} - \frac{(y+9)^2}{144} = 1$

45. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

47. 

49a. $y = 30{,}000 + 3{,}000x$ **b.** 10 yr

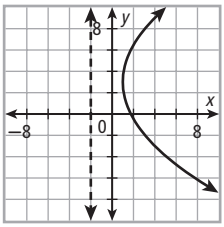
51. $\frac{y^2}{4} + \frac{x^2}{2} = 1$

10-5

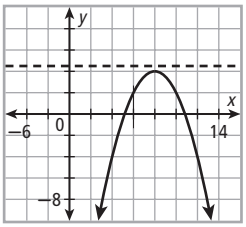
Check It Out! 1. $y = \frac{1}{16}x^2$

2. a. $x = -\frac{1}{5}y^2$ **b.** $y = -\frac{1}{28}x^2$

3a. vertex: $(1, 3)$; $p = 3$; axis of symmetry: $y = 3$; focus: $(4, 3)$; directrix: $x = -2$



b. vertex: $(8, 4)$; $p = -\frac{1}{2}$; axis of symmetry: $x = 8$; focus: $(8, 3.5)$; directrix: $y = 4.5$



4. 11 in.

Exercises 3. $x = \frac{1}{28}y^2$

5. $y = -\frac{1}{16}x^2$ **7.** $y = \frac{1}{2}x^2$

9. $x = -\frac{1}{32}y^2$ **11.** vertex: $(0, 4)$; $p = 6$; axis of symmetry: $y = 4$; focus: $(6, 4)$; directrix: $x = -6$

13. 9.5 in **15.** $x - 3 = -\frac{1}{20}y^2$

17. $y = \frac{1}{12}(x+3)^2$ **19.** $x = \frac{1}{4}y^2$

21. $y = -\frac{1}{24}x^2$ **23.** vertex: $(1, 0)$; $p = \frac{1}{8}$; axis of symmetry: $y = 0$; focus: $\left(\frac{9}{8}, 0\right)$; directrix: $x = \frac{7}{8}$

27. $y + 6 = -\frac{1}{12}(x-2)^2$; D: $\{x \mid x \in \mathbb{R}\}$; R: $\{y \mid y \le -6\}$

29. $x + 7 = \frac{1}{36}(y+3)^2$; D: $\{x \mid x \ge -7\}$; R: $\{y \mid y \in \mathbb{R}\}$

31. $y - 5 = -\frac{1}{20}x^2$; D: $\{x \mid x \in \mathbb{R}\}$; R: $\{y \mid y \le 5\}$ **33.** $x - 8 = -\frac{1}{16}(y+5)^2$; D: $\{x \mid x \le 8\}$; R: $\{y \mid y \in \mathbb{R}\}$ **35a.** $y = \frac{1}{20}x^2$

b. $y + 4 = \frac{1}{16}x^2$ **c.** 7.2 in

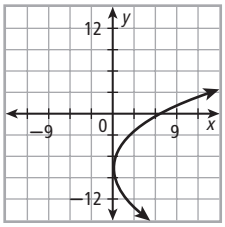
37a. $(-96, 41)$ **b.** 133 million km

c. $(-96, 174)$ **39.** vertex: $(-4, 5)$;

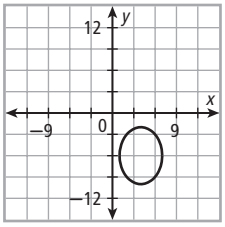
$p = -\frac{1}{8}$; axis of symmetry: $x = -4$; focus: $\left(-4, 4\frac{7}{8}\right)$; directrix: $y = 5\frac{1}{8}$
41. vertex: $(-3, 2)$; $p = 2$; axis of symmetry: $y = 2$; focus: $(-1, 2)$; directrix: $x = -5$
45. G **47.** $(7, 0)$ **49.** $y - 7 = -\frac{1}{8}(x-6)^2$ or $y - 3 = \frac{1}{8}(x-6)^2$
51. $4p$ **53.** $f^{-1}(x) = \frac{x - 22}{4}$; D: $\{x \mid x \in \mathbb{R}\}$; R: $\{y \mid y \in \mathbb{R}\}$; function
55. $f^{-1}(x) = 3x + 2$; D: $\{x \mid x \in \mathbb{R}\}$; R: $\{y \mid y \in \mathbb{R}\}$; function **57.** vertices: $(\pm 9, 0)$; co-vertices: $(0, \pm 5)$; asymptotes: $y = \pm\frac{5}{9}x$ **59.** vertices: $(0, \pm 8)$; co-vertices: $(\pm 2, 0)$; asymptotes: $y = \pm 4x$

10-6

Check It Out! 1a. circle
b. hyperbola **2a.** circle **b.** parabola
3a. $x = \frac{1}{9}(y + 8)^2$; parabola



b. $\frac{(x-4)^2}{9} + \frac{(y+6)^2}{16} = 1$; ellipse



4. 200 ft

Exercises 1. ellipse **3.** parabola
5. ellipse **7.** ellipse
9. $(x-8)^2 + (y+5)^2 = 36$; circle
11. $\frac{x^2}{9} + \frac{(y+4)^2}{25} = 1$; ellipse
13. 4 in **15.** parabola **17.** ellipse
19. parabola **21.** ellipse
23. $x^2 + (y-4)^2 = 49$; circle
25. $\frac{x^2}{4} - \frac{(y+9)^2}{25} = 1$; hyperbola
27. $(x+5)^2 + (y+2)^2 = 20$; circle
29. $\frac{(x+1)^2}{4} - \frac{(y-7)^2}{9} = 1$; hyperbola
31. $(x-2.5)^2 + (y+4.5)^2 = 16$; circle
33a. ellipse **b.** 40 m **c.** 100 m
35. $36x^2 + 25y^2 - 360x + 400y + 1600 = 0$ **37.** outside **39.** inside
41. 36.4 cm **43a.** $(x-40)^2 + (y-30)^2 = 40{,}000$ **b.** $40{,}000\pi$
c. inside **45a.** $y - 84 = -\frac{1}{500}(x-200)^2$ **b.** 84 ft **c.** 4 ft
47. A **49.** D
53. It rotates the graph.
55. no **57.** no **59.** $(4, 1)$
61a. $f(x) = 1.65(1 + 0.05)^t$ **b.** $5.32

10-7

Check It Out! 1. $(0, 4.5)$
2a. $(-4, 3)$, $(3, -4)$
b. $(0, 5)$, $(\pm 3, -4)$ **3.** no solution
4. yes, at $(\pm 4, -3.6)$

Exercises 3. $(4, -2)$, $(8, 2)$
5. $(12, 5)$, $(5, 12)$ **7.** no solution
9. $(0, -6)$, $\left(\pm 3\sqrt{3}, 3\right)$
11. $(\pm 4, 2)$, $(\pm 4, -2)$
13. no solution
15. $(0, 5)$, $(-4, 3)$
17. $(2.5, -6.5)$, $(-4.5, 7.5)$
19. $(0, 4)$, $\left(\frac{8}{3}, \frac{20}{3}\right)$ **21.** $(5, 1)$, $(7, 4)$
23. $(\pm 2, -2)$, $\left(\pm\sqrt{7}, 1\right)$ **25.** no solution **27.** $(0, -7)$, $(1, -6)$
29. $\left(2\sqrt{2}, \pm 1\right)$, $\left(-2\sqrt{2}, \pm 1\right)$
31. $(0, 0)$, $(1, 1)$ **33.** $(6, 1)$, $(-26, -3)$
35. $(0, \pm 3)$ **37.** $(2, \pm 4)$
39. no solution **41.** $(10, \pm 4)$, $(-10, \pm 4)$ **43.** 25 s **45.** $(2.8, -2.6)$, $(3.3, 2.9)$, $(-7.9, -1.3)$, $(-8.1, 1.5)$
47a. hyperbola **b.** yes; $(6.69, 2.68)$, $(1.31, 2.68)$ **49.** $(-2, 0)$ **53.** G
55. $(4, \pm 3)$ **57.** $(3, 4)$, $(4, 3)$
61. about $100 **63.** cylinder **65.** 4
67. $-\frac{2}{5}$ **69.** $f(x) = 5x - 2$

Study Guide: Review

1. transverse axis **2.** tangent line
3. focus; directrix **4.** conic section **5.** circle with center $(0, 0)$ and radius $r = 9$ **6.** hyperbola with center $(0, 0)$ and intercepts $(5, 0)$ and $(-5, 0)$ **7.** parabola with vertex $(0, -1)$, opening in the positive x-direction **8.** ellipse with center $(0, 0)$, and intercepts $(\pm 3.5, 0)$, and $(0, \approx \pm 1.98)$ **9.** B
10. center: $(3, -3)$; $r = 12$
11. center: $(8, -2.5)$; $r = 12.5$
12. center: $(6, 0)$; $r = 19$
13. center: $(-12, 4)$; $r = \sqrt{15}$
14. $(x-8)^2 + (y+7)^2 = 196$
15. $(x-3)^2 + (y-6)^2 = 80$
16. $(x+3)^2 + (y-8)^2 = 34$
17. $y - 5 = -\frac{3}{5}(x-3)$
18. $y = 4$ **19.** $y + 2 = -\frac{4}{5}(x-6)$
20. $y + 7 = \frac{5}{8}(x-1)$
21. center: $(0, 0)$; vertices: $(0, \pm 6)$; co-vertices: $(\pm 3, 0)$; foci: $\left(0, \pm 3\sqrt{3}\right)$
22. center: $(0, 0)$; vertices: $(\pm 8, 0)$; co-vertices: $(0, \pm 5)$; foci: $\left(\pm\sqrt{39}, 0\right)$
23. center: $(3, -2)$; vertices: $(3, 6)$, $(3, -10)$; co-vertices: $(10, -2)$, $(-4, -2)$; foci: $\left(3, -2 \pm \sqrt{15}\right)$
24. $\frac{(x-4)^2}{36} + \frac{(y+5)^2}{9} = 1$
25. $\frac{x^2}{144} + \frac{y^2}{225} = 1$
26. $\frac{(x+2)^2}{36} + \frac{(y-3)^2}{27} = 1$
27. center: $(0, 0)$; vertices: $(\pm 5, 0)$; co-vertices: $(0, \pm 7)$; foci: $\left(\pm\sqrt{74}, 0\right)$; asymptotes: $y = \pm\frac{7}{5}x$
28. center: $(0, 0)$; vertices: $(0, \pm 6)$; co-vertices: $(\pm 8, 0)$; foci: $(0, \pm 10)$; asymptotes: $y = \pm\frac{3}{4}x$ **29.** center: $(3, -6)$; vertices: $(5, -6)$, $(1, -6)$; co-vertices: $(3, 1)$, $(3, -13)$; foci: $\left(3 \pm \sqrt{53}, -6\right)$; asymptotes: $y + 6 = \pm\frac{7}{2}(x-3)$ **30.** $\frac{x^2}{25} - \frac{y^2}{36} = 1$
31. $\frac{x^2}{121} - \frac{y^2}{16} = 1$ **32.** $\frac{y^2}{25} - \frac{x^2}{36} = 1$
33. $\frac{(y-5)^2}{25} - \frac{(x+7)^2}{144} = 1$
34. vertex: $(0, 0)$; $p = -3$; axis of symmetry: $x = 0$; focus: $(0, -3)$; directrix: $y = 3$ **35.** vertex: $(0, 0)$; $p = \frac{1}{8}$; axis of symmetry: $y = 0$; focus: $\left(\frac{1}{8}, 0\right)$; directrix: $x = -\frac{1}{8}$
36. vertex: $(-4, 5)$; $p = \frac{1}{4}$; axis of symmetry: $x = -4$; focus: $\left(-4, 5\frac{1}{4}\right)$; directrix: $y = 4\frac{3}{4}$ **37.** vertex: $(4, -2)$; $p = -1.5$; axis of symmetry: $y = -2$; focus: $(2.5, -2)$; directrix: $x = 5.5$
38. $y = -\frac{1}{20}(x-3)^2$
39. $x - 4 = -\frac{1}{10}(y-6)^2$
40. $x - 9 = \frac{1}{12}(y+4)^2$ **41.** ellipse
42. hyperbola **43.** parabola
44. circle **45.** ellipse
46. $x + 3 = \frac{1}{4}(y+6)^2$; parabola

47. $\frac{(x+4)^2}{6} + \frac{y^2}{2} = 1$; ellipse
48. $(x+5)^2 + (y-4)^2 = 36$; circle
49. $\frac{(x+1)^2}{8} - \frac{(y+3)^2}{4} = 1$; hyperbola **50.** $(2, -6), (-2, 2)$
51. $(4, 0), (0, -5)$
52. $(8, \pm 6), (-8, \pm 6)$
53. $(3, 2), (5, 6)$ **54.** $(0, 2), (0, -2)$
55. $(6, 4), (6, -4), (-6, 4), (-6, -4)$
56. $(2, 6), (-7, 3)$ **57.** no solution

Chapter 11

11-1

Check It Out! 1a. 120
b. 73,116,160 **2a.** 336 **b.** 20 **3.** 28

Exercises 1. important; permutation **3.** 225 **5.** 1320
7. 5985 **9.** 12 **11.** 72 **13.** 20
15. 71,916,768 **17.** 1 **19.** 6 **21.** 72
23. 6700 **25.** 35 **27.** > **29.** <
33a.

President	A	A	A	A	A	A	A	A	A	A	A	A
Vice President	B	B	B	C	C	C	D	D	D	E	E	E
Secretary	C	D	E	B	D	E	B	C	E	B	C	D

b.

President	B	B	B	B	B	B	B	B	B	B	B	B
Vice President	A	A	A	C	C	C	D	D	D	E	E	E
Secretary	C	D	E	A	D	E	A	C	E	A	C	D

60 ways

c. 60 **d.** 10; 60; 10 **37.** A **39.** D
41. 1365 **43.** $({}_{30}C_{12})({}_{18}C_2)$
45. $n = 119$ **47.** $n = 13.0625$
49. hyperbola

11-2

Check It Out! 1a. $\frac{5}{36}$ **b.** 0
c. $\frac{5}{12}$ **2.** $\frac{16}{25}$ **3.** $\frac{1}{28}$ **4.** $\frac{16}{225}$
5a. $\frac{9}{26}$ **b.** $\frac{19}{26}$

Exercises 1. theoretical probability **3.** $\frac{1}{4}$ **5.** $\frac{1}{4}$ **7.** $\frac{303}{365}$
9. $\frac{1}{220}$ **11.** $\frac{1}{9}$ **13.** $\frac{3}{5}$ **15.** $\frac{4}{5}$ **17.** $\frac{1}{56}$
19. $\approx\frac{1}{42}$ **21.** never **23a.** $\frac{\pi}{4}$
25a. 0.68; 0.84; 0.76; 0.64
b. 0.73 **27.** $\frac{2}{5}$ **29.** June; ≈ 0.13
31. no; yes **33.** $\frac{1}{2}$ **37.** G **39.** H
45. max.: 16 **47.** $y = -\frac{1}{20}x^2$

11-3

Check It Out! 1a. $\frac{1}{36}$ **b.** $\frac{1}{8}$
2. $\frac{5}{36}$ **3a.** ≈ 0.014 **b.** ≈ 0.186
4a. independent; $\frac{3}{20}$
b. dependent; $\frac{1}{6}$ **c.** dependent; $\frac{1}{12}$

Exercises 1. independent
3. $\frac{1}{8}$ **5.** The probability that the yellow cube shows a multiple of 3 increases from $\frac{1}{3}$ if the product is 6; $\frac{1}{2}$ **7.** $\frac{1}{100}$ **9.** dependent; $\frac{9}{38}$
11. $\frac{1}{12}$ **13.** The probability that the product is 8 increases from $\frac{1}{18}$ if the blue cube is less than 3; $\frac{1}{36}$
15. ≈ 0.72 **17.** dependent; $\frac{1}{6}$
19. independent
21. independent **23a.** ≈ 0.61
b. ≈ 0.05 **25a.** $\frac{625}{1296}$ **b.** $\frac{1}{36}$
c. $\frac{1}{6}$ **27.** ≈ 0.6 **29.** 40 **33.** F
35. 7 **37.** $\frac{11}{18}$; no
39a. $P(d) = 18.3\text{g}$; $P(j) = 32.5\text{g}$
c. vertical stretch by a factor of ≈ 1.78 **41.** $x \approx \pm 2.6$; $y \approx \pm 2.2$
43. $\frac{1}{36}$ **45.** $\frac{3}{4}$

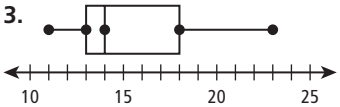
11-4

Check It Out! 1a. Each student can vote only once. **b.** 75%
2a. $\frac{4}{13}$ **b.** $\frac{8}{13}$ **3.** $\frac{31}{40}$ **4.** ≈ 0.1524

Exercises 1. inclusive events
3. $\frac{3}{5}$ **5.** $\frac{4}{5}$ **7.** $\frac{7}{9}$ **9.** $\frac{54}{65}$
11. ≈ 0.92 **13.** $\frac{1}{2}$ **15.** $\frac{1}{4}$ **17.** $\frac{32}{49}$
19. $1 - 0.75^{13} \approx 0.976$ **21.** 0.37; experimental **23.** 87%; 100%
25b. 4.16%; 52.24% **27.** 0.49
29a. 0.42 **b.** 0.02 **c.** 0.44; it is the sum of the probabilities. **31.** D
33. D **35.** ≈ 0.12 **37.** $\frac{13}{18}$ **39.** 0.9
41. 0.2 **43.** $y = -1.5x^3 - 6x^2 + 16.5x + 45$ **47.** $\frac{1}{16}$

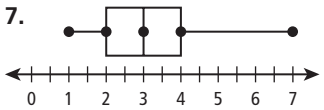
11-5

Check It Out! 1a. 6.5; 7; no mode **b.** 4.2; 5; 2 and 6 **2.** 0.37
3.

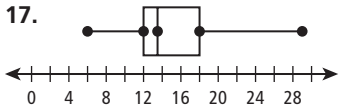


IQR = 5
4. 1.4; ≈ 1.6 **5.** 19; the mean increases from ≈ 4.3 to ≈ 5.4, and the standard deviation increases from ≈ 2.2 to ≈ 4.5.

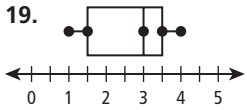
Exercises 1. variance **3.** 5.375; 6; 6 **5.** $0.36
7.



IQR = 2
9. 0.8; 0.89 **11.** 142.92; 11.95
13. $23.1\overline{6}$; 20.5; no mode
15. 15; 15; no mode
17.



IQR = 6
19.



IQR = 2
21. 343.71; 18.54 **23.** 58; the mean increases from ≈ 19.8 to ≈ 22.8, and the standard deviation increases from ≈ 5.6 to ≈ 11.5.
25. the mean; 37° is an outlier and affects the mean greatly.
27. 15; $Q_1 - 1.5(IQR) = 79 - 1.5(90-79) = 62.5$; $15 < 62.5$
29. < 0.3 min or > 6.9 min; none **31.** Ruth **35.** −$0.499
37. B **39a.** 12.25 **b.** $\frac{13}{36}$ **c.** $\frac{23}{36}$
d. no **41.** D **43.** C **45.** 1
47. $-2x^4 - 5x^3 + 7x^2 + 10x - 6$
49. $\frac{2}{3}$ **51.** $\frac{1}{2}$

11-6

Check It Out! 1a. $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$
b. $a^3 + 6a^2b + 12ab^2 + 8b^3$
2a. $\frac{2}{9} \approx 0.22$ **b.** $\frac{47}{128} \approx 0.37$
3a. ≈ 0.98 **b.** ≈ 0.09

Exercises 1. 2 **3.** $27x^3 + 135x^2 + 225x + 125$
5. $x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$
7. ≈ 0.026 ≈ 0.181
9. $y^4 + 20y^3 + 150y^2 + 500y + 625$
11. $1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$
13. ≈ 0.86 **15.** $\frac{3}{8}$; $\frac{1}{8}$

17. $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$ **19.** $256k^4 - 256k^3 + 96k^2 - 16k + 1$ **21.** 0.384 **23.** $\frac{8}{27}$

25. $\frac{1}{16}$; $\frac{5}{16}$ **27.** ≈ 0.989

29. ≈ 0.94 **33a.** $\frac{82}{365}$ **b.** ≈ 0.14

c. ≈ 0.19 **35.** ≈ 0.03 **37.** ≈ 0.59

39. ≈ 0.3 **41.** B **43.** B **45.** ≈ 0.29

47a. ≈ 0.67 **b.** ≈ 0.62 **51.** -19; -4; -4 **53.** no **55.** 13.4; 15; 18

57. 25; 24; 24

Extension

Check it Out! **1.** $\approx 97.7\%$

Exercises **1.** $\approx 95.4\%$ **3.** $\approx 68.2\%$

5. $\approx 47.7\%$ **7.** $\approx 15.9\%$

Study Guide: Review

1. dependent events

2. expected value

3. permutation **4.** 7,000,000

5. 792 **6.** 2,162,160 **7.** 604,800

8. 20 **9.** $\frac{5}{36}$ **10.** $\frac{5}{18}$ **11.** $\frac{1}{2}$

12. $\frac{11}{12}$ **13.** $\frac{1}{210}$ **14.** $\frac{1}{10{,}000}$

15. $\frac{5}{24}$ **16.** ≈ 0.21 **17.** $\frac{1}{5}$

18. $\frac{4}{5}$ **19.** $\frac{7}{25}$ **20.** $\frac{13}{25}$ **21.** $\frac{1}{4}$

22. $\frac{3}{4}$ **23.** $\frac{1}{4}$ **24.** $\frac{1}{2}$ **25.** $\frac{1}{216}$

26. $\frac{6}{25}$ **27.** $\frac{11}{21}$ **28.** $\frac{1}{13}$

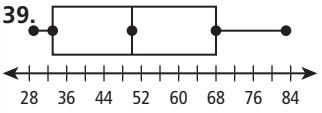
29. $\frac{13}{31}$ **30.** $\frac{14}{99}$ **31.** Each coupon offers only 1 discount. **32.** $\frac{5}{6}$

33. $\frac{7}{13}$ **34.** $\frac{1}{2}$ **35.** $\frac{7}{10}$

36. mean: 5.4; median: 6; mode: 8 **37.** mean: $13.\overline{3}$; median: 13; modes: 12, 13, and 15

38. 0.51

39.



IQR = 35

40. [5.4, 9.6] **41.** yes **42.** The mean decreases from 75.5 to 69.3, and the standard deviation increases from ≈ 21.5 to ≈ 25.1.

43. $125 + 150x + 60x^2 + 8x^3$

44. $x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$ **45.** 48.75; ≈ 4.13

46. ≈ 0.10; ≈ 0.40

Chapter 12

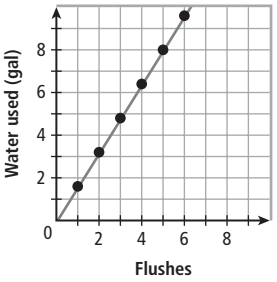
12-1

Check It Out! **1a.** $-5, -13, -21, -29, -37$ **b.** $2, -6, 18, -54, 162$

2a. $-1, 0, 3, 8, 15$ **b.** $-2, 1, 4, 7, 10$

3a. $a_n = 9 - 2n$ **b.** $a_n = \frac{1}{n}$

4.



The graph shows the points lie on a line with positive slope; 16 gal. **5.** 8, 16

Exercises **1.** recursive **3.** 3, 14, 25, 36, 47 **5.** $-12, 0, 12, 24, 36$

7. $-3, -12, -27, -48, -75$ **9.** 1, 4, 16, 64, 256 **11.** $a_n = 3 + 3n$

13. $a_n = 35 - 10n$ **15.** 16, 32

17. $-2, 5, -16, 47, -142$ **19.** 9, 10, 12, 16, 24 **21.** $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}$

23. $a_n = 13 - 4n$ **25.** linear with a slope of 4; 36 **27.** 12, 8, 6, 5, $4\frac{1}{2}$ **29.** 10, 20, -10, 20, -10

31. 7.9, 7.8, 7.7, 7.6, 7.5 **33.** B is incorrect. The formula is explicit, not recursive. **35.** $a_n = \frac{16}{9} - \frac{1}{9}n$; $\frac{2}{3}$ **37.** $a_n = \frac{(-1)^{n+1}}{n}$; $-\frac{1}{10}$

39. $a_n = 25 - n^2$; -75 **41.** 15, 21

43a. 15, 21 **b.** $a_n = \frac{1}{2}n^2 - \frac{1}{2}n$

c.

Players	1	2	3	4	5
Games	0	2	6	12	20

The sequence is twice the previous sequence. The function is a vertical stretch by a factor of 2.

45a. $a_n = 180(n - 2)$ for $n \geq 3$; 1800°

c. $a_n = \frac{180(n-2)}{n}$ for $n \geq 3$

47a. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$;

$a_1 = 1$, $a_n = \frac{1}{2}a_{n-1}$; $a_n = \left(\frac{1}{2}\right)^{n-1}$

b. $4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$;

$a_1 = 4$, $a_n = \frac{1}{2}a_{n-1}$; $a_n = 4\left(\frac{1}{2}\right)^{n-1}$

51. H **53.** H **55.** $a_n = \frac{n^3}{3} - 1$; $\frac{997}{3}$

57. $a_n = -0.05n^2 + 0.05n + 0.9$; -3.6 **59.** $\frac{x-3}{x+2}$ **61.** $\frac{1}{x+5}$

63. $\frac{2(3x^2 - 5x - 3)}{(x+1)(x-1)}$

65. $\frac{x(x^2 - 7)}{(2x+7)(x+2)}$

12-2

Check It Out! **1a.** $\sum_{k=1}^{5} \frac{2}{(k+1)^2}$

b. $\sum_{k=1}^{6} (-1)^k (2k)$

2a. $1 + 3 + 5 + 7 = 16$

b. $-5 - 10 - 20 - 40 - 80 = -155$

3a. 240 **b.** 120 **c.** 385 **4.** 294 in., or $24\frac{1}{2}$ ft

Exercises **1.** $\sum_{k=1}^{n} k$

3. $\sum_{k=1}^{5} (-1)^k (3k)$

5. $\sum_{k=1}^{5} [100 - 5(k-1)]$

7. $12 - 3 + \frac{4}{3} - \frac{3}{4} = 9\frac{7}{12}$

9. 231 **11.** 126 **13.** $\sum_{k=1}^{5} 1.1k$

15. $\sum_{k=1}^{6} (-1)^{k+1} (k + 10)$

17. $16 + 24 + 32 + 40 + 48 = 160$

19. $0 + \frac{1}{3} + \frac{2}{4} + \frac{3}{5} = \frac{43}{30}$ **21.** 195

23. 210 cans **25.** $\sum_{k=1}^{25} 26 - k$

27. $\sum_{k=1}^{5} -800 \left(\frac{1}{10}\right)^{k-1}$

29. $\sum_{k=1}^{6} (-1)^{k+1} (k + 2)^2$

31. $\sum_{k=1}^{5} 3.4k - 3.4$, or $\sum_{k=1}^{5} 3.4(k-1)$

33. $\sum_{k=1}^{5} \frac{1000}{10^{k-1}}$ **35b.** $\sum_{k=1}^{5} 3^k =$ $3 + 9 + 27 + 81 + 243 = 363$

c. 3542

37. $-5 + 10 - 15 + 20 - 25 + 30 = 15$

39. $1 + 4 + 7 + 10 + 13 + 16 = 51$

41. $\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + 1 = 3$

43. 420 **45.** -2550

47a. Both equal 165; $\sum_{k=1}^{n} ca_k = c\sum_{k=1}^{n} a_k$.

b. Both equal 75; $\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$. **49. a.** $a_n = 4n$

b. $\sum_{k=1}^{6} 4k$; 84 toothpicks **53.** H

55. J **57.** $\sum_{k=1}^{1000} k$; 500, 500

59. $\sum_{k=1}^{n} ca_k = ca_1 + ca_2 + \cdots + ca_n$
$= c(a_1 + a_2 + \cdots + a_n)$
$= c\sum_{k=1}^{n} a_k$

63. x-int.: 3; y-int.: -6 **65.** 10 ft
67. 2, 3, 8, 63, 3968

12-3

Check It Out! 1a. arithmetic; $d = -0.7$; -1.6 **b.** not arithmetic **2a.** -23 **b.** 8.7 **3.** $\frac{3}{2}$, 1, $\frac{1}{2}$ **4a.** -25 **b.** 8.5 **5a.** -408 **b.** -1650 **6a.** 37 seats **b.** 336 total seats

Exercises 1. arithmetic series **3.** not arithmetic **5.** 38 **7.** -4.6 **9.** 16, 23, 30 **11.** -13 **13.** -17 **15.** -35 **17.** 495 **19.** 11.7 **21.** not arithmetic **23.** arithmetic; -0.09; 0.63 **25.** $\frac{12}{5}$ **27.** 66, 55, 44 **29.** 2.1, 1.9, 1.7 **31.** 94 **33.** -60 **35.** 143.5 **37a.** 78; 156 **b.** adds 1 to each term of the sequence; adds 24 to the total number per day **39.** 0 **41.** 60 **43a.** $\sum_{k=1}^{n} 4k$ **b.** 684 **c.** 673 **45.** 45 minutes; after 2 years, her exercise routine would be over 8 h long, which is not realistic. **47a.** 61 **b.** 650 **49a.** 6th, 11th, 16th, 21st, and 26th Streets **b.** 0.25 mi **53.** J **55.** G **57.** 6 **63.** growth **65.** 92 dB **67.** $\sum_{k=1}^{5} \frac{4k}{5}$ **69.** $\sum_{k=1}^{5} -\frac{k}{3}$

12-4

Check It Out! 1a. geometric; $r = \frac{1}{3}$ **b.** arithmetic; $d = -0.4$ **c.** neither **2a.** $\frac{3}{1024}$ **b.** 100,000 **3a.** -1000 **b.** $\frac{3}{4}$ or $-\frac{3}{4}$ **4.** 20 **5a.** $\frac{63}{16}$ **b.** -189 **6.** \$616,218.04

Exercises 1. geometric mean **3.** neither **5.** 39,366 **7.** 64 **9.** 324 **11.** $\frac{3}{2}$ **13.** 48 **15.** 61 **17.** 511 **19.** neither **21.** arithmetic; $d = 5$ **23.** 768 **25.** 52,488 **27.** 30.375 **29.** 1 **31.** 3 **33.** 11.111111 **35.** 8,888,888 **37.** $a_n = \frac{1}{16}(2)^{n-1}$; $a_{10} = 32$; $S_{10} = \frac{1023}{16} \approx 63.94$ **39.** $a_n = 8(2)^{n-1}$; $a_{10} = 4096$; $S_{10} = 8184$ **41.** $a_n = 162\left(-\frac{1}{3}\right)^{n-1}$; $a_{10} = -\frac{2}{243}$; $S_{10} = 121\frac{121}{243} \approx 121.5$ **43a.** \$34.98; \$61.18 **45.** 2,441,406 **47a.** 12.8 mm **b.** 27 folds **49a.** \$24 million **b.** 60% **c.** week 6 **d.** about \$99.93 million **53a.** about 261.6 Hz **b.** $a_n \approx 16.3(2)^n$ **c.** C11 **55a.** 30.198 **b.** 31.899 **c.** 31.994 **d.** 32.000 **e.** Yes, the series appears to be approaching 32. **59.** G **61.** G **63.** $a_{18} = 1{,}310{,}720$ **65.** $a_{17} \approx 1{,}208{,}925.82$ **67a.** 89, 144, 233, 377, 610 **b.** Their sum is the next term. **69.** zero: -5; vertical asymptotes: $x = -2$ and $x = 3$; horizontal asymptote: $y = 0$ **71a.** $f(x) = 0.9(0.8x) = 0.72x$ **b.** \$198 **73.** 52.1 **75.** 104.6

12-5

Check It Out! 1a. diverges **b.** converges **2a.** $\frac{125}{6}$ **b.** $\frac{2}{3}$ **3.** $\frac{1}{9}$

4. Step 1: $\sum_{k=1}^{1} (2k-1) = 1$; $1^2 = 1$
Step 2: $1 + 3 + \cdots + (2k-1) = k^2$
Step 3: $1 + 3 + \cdots + (2k-1) + [2(k+1)-1]$
$= k^2 + [2(k+1)-1]$
$= k^2 + 2k + 1$
$= (k+1)^2$

5. $a = 5$: $\frac{5^2}{2} \stackrel{?}{\leq} 2(5) + 1$
$12.5 \not\leq 11$

Exercises 1. converge **3.** diverges **5.** $\frac{9}{4}$ **7.** $1066\frac{2}{3}$ **9.** $\frac{56}{99}$

11. Step 1: $2 \cdot 1 = n(n+1)$
$= 1(1+1) = 2$
Step 2: $2 + 4 + \cdots + 2(k)$
$= (k)(k+1)$
Step 3: $2 + 4 + \cdots + 2k + 2(k+1) = k(k+1) + 2(k+1)$
$= k^2 + k + 2k + 2$
$= k^2 + 3k + 2$
$= (k+1)(k+2)$

13. $n = -1$ **15.** converges **17.** diverges **19.** $\frac{16}{15}$ **21.** $\frac{2}{3}$ **23.** $\frac{541}{999}$ **25.** $a = 0$ **27.** $a = 0$ **29.** 320 in., or $26\frac{2}{3}$ ft **31.** 2500 **33.** $-\frac{40}{7}$ **35.** No sum exists. **37.** 500 **39.** $\frac{4}{9}$ **41.** $\frac{41}{333}$ **43.** $\frac{5}{9}$ **45a.** about 415.0 million **b.** about 6.2 billion **c.** about 11 billion **53.** $x = \frac{1}{2}$ **55.** $x = -2$ **57.** $x = 0$ **59.** For $a_1 > 0$, $S > S_n$, and both sums are positive. For $a_1 < 0$, $S_n > S$, and both sums are negative. **61.** B **63.** A **67.** $\frac{5}{12}$ **69.** No; the partial sums will approach infinity if $d > 0$ and negative infinity if $d < 0$. **71.** 73.728 % **73.** geometric; $r = \frac{1}{3}$ **75.** geometric; $r = 0.25$

Extension

Check It Out! 1. 53 square units **2.** 155.25 square units

Exercises 1. 1236 square units **3.** 304 square units **5.** 220 square units **7.** 15,875 square units **9a.** about 3056 square units

Study Guide: Review

1. arithmetic; geometric **2.** diverges; converges **3.** explicit formula; recursive formula **4.** infinite sequence; finite sequence **5.** iteration **6.** $-8, -7, -6, -5, -4$ **7.** $\frac{1}{2}$, 2, $\frac{9}{2}$, 8, $\frac{25}{2}$ **8.** 1, $-\frac{3}{2}$, $\frac{9}{4}$, $-\frac{27}{8}$, $\frac{81}{16}$ **9.** 55, 53, 51, 49, 47 **10.** 200, 40, 8, $\frac{8}{5}$, $\frac{8}{25}$ **11.** -3, 10, -29, 88, -263 **12.** $a_n = -4n$ **13.** $a_n = 5(4)^{n-1}$ **14.** $a_n = 5n - 29$ **15.** $a_n = 27\left(\frac{2}{3}\right)^{n-1}$ **16.** 0.72 ft, or 8.6 in.; 0.12 ft, or 1.5 in. **17.** $-1 + 4 - 9 + 16 = 10$ **18.** $4.5 + 5.0 + 5.5 + 6.0 + 6.5 = 27.5$ **19.** $1 - 3 + 5 - 7 + 9 = 5$ **20.** $5 + \frac{5}{2} + \frac{5}{3} + \frac{5}{4} = \frac{125}{12}$ **21.** -40 **22.** 385 **23.** 78 **24.** \$27,600; \$207,000 **25.** 17 **26.** $\frac{21}{5}$ **27.** -1.2 **28.** 29.5 **29.** -18 **30.** 23 **31.** -630 **32.** -7 **33.** 150 **34.** 330 **35.** 50, 58, 66, 74,...; no, because he will have a total savings of only \$458 **36.** 0.000004 **37.** $\frac{243}{2}$ **38.** $-\frac{1}{8}$ **39.** 768 **40.** 98,304 **41.** $\frac{512}{3}$ **42.** ± 32 **43.** 62,500 **44.** 5 **45.** 2 **46.** $\frac{\sqrt{3}}{24}$ **47.** $\frac{25}{36}$ **48.** $\frac{121}{81}$ **49.** 72,727.2 **50.** 21,845 **51.** $-39{,}062$ **52.** $\frac{315}{8} = 39.375$ **53.** $\frac{279}{8} = 34.875$ **54.** \$1044.26 **55.** \$9847.32; \$43,969.32 **56.** -2025 **57.** $-\frac{4}{3}$ or $-1.\overline{3}$

58. -343 **59.** 5 **60.** 4.5 **61.** $-\frac{21}{2}$
62. $\frac{1}{9}$ **63.** No sum exists.
64. Step 1: $2^1 = 2^{1+1} - 2 = 2$
Step 2: $2 + \cdots + 2^k = 2^{k+1} - 2$
Step 3: $2 + \cdots + 2^k + 2^{k+1}$
$= 2^{k+1} - 2 + 2^{k+1}$
$= 2(2^{k+1}) - 2$
$= 2^{k+2} - 2 = 2^{(k+1)+1} - 2$
65. Step 1: $5^{1-1} = \frac{5^1 - 1}{4} = 1$
Step 2: $1 + \cdots + 5^{k-1} = \frac{5^k - 1}{4}$
Step 3: $1 + \cdots + 5^{k-1} + 5^k$
$= \frac{5^k - 1}{4} + 5^k$
$= \frac{5^k - 1}{4} + \frac{4(5^k)}{4}$
$= \frac{5^k + 4(5^k) - 1}{4}$
$= \frac{5(5^k) - 1}{4} = \frac{5^{k+1} - 1}{4}$
66. Step 1: $\frac{1}{4(1^2) - 1}$
$= \frac{1}{2(1) + 1}$
$= \frac{1}{3}$
Step 2: $\frac{1}{3} + \cdots + \frac{1}{4k^2 - 1}$
$= \frac{k}{2k + 1}$
Step 3: $\frac{1}{3} + \cdots + \frac{1}{4k^2 - 1} + \frac{1}{4(k+1)^2 - 1}$
$= \frac{k}{2k + 1} + \frac{1}{4(k+1)^2 - 1}$
$= \frac{k}{2k + 1} + \frac{1}{4k^2 + 8k + 3}$
$= \frac{k}{2k + 1} + \frac{1}{(2k + 1)(2k + 3)}$
$= \frac{k(2k + 3)}{(2k + 1)(2k + 3)} + \frac{1}{(2k + 1)(2k + 3)}$
$= \frac{k(2k + 3) + 1}{(2k + 1)(2k + 3)}$
$= \frac{2k^2 + 3k + 1}{(2k + 1)(2k + 3)}$
$= \frac{\cancel{(2k+1)}(k + 1)}{\cancel{(2k+1)}(2k + 3)}$
$= \frac{k + 1}{2k + 2 + 1}$
$= \frac{k + 1}{2(k + 1) + 1}$
67a. $\sum_{k=1}^{\infty} 9(0.85)^{k-1}$ **b.** 60 ft

Chapter 13

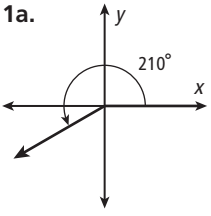
13-1

Check It Out! **1.** $\sin\theta = \frac{15}{17}$; $\cos\theta = \frac{8}{17}$; $\tan\theta = \frac{15}{8}$ **2.** $x = 10\sqrt{2}$ **3.** 41 in. **4.** 220 ft **5.** $\sin\theta = \frac{40}{41}$; $\cos\theta = \frac{9}{41}$; $\tan\theta = \frac{40}{9}$; $\csc\theta = \frac{41}{40}$; $\sec\theta = \frac{41}{9}$; $\cot\theta = \frac{9}{40}$

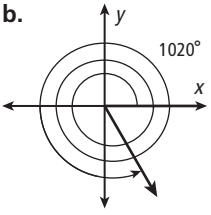
Exercises **1.** tangent
3. $\sin\theta = \frac{3\sqrt{13}}{13}$; $\cos\theta = \frac{2\sqrt{13}}{13}$; $\tan\theta = \frac{3}{2}$ **5.** $x = \frac{100\sqrt{3}}{3}$
7. $x = \frac{250\sqrt{3}}{3}$ **9.** 241 m
11. $\sin\theta = \frac{3\sqrt{10}}{10}$; $\cos\theta = \frac{\sqrt{10}}{10}$; $\tan\theta = 3$; $\csc\theta = \frac{\sqrt{10}}{3}$; $\sec\theta = \sqrt{10}$; $\cot\theta = \frac{1}{3}$ **13.** $\sin\theta = \frac{1}{3}$; $\cos\theta = \frac{2\sqrt{2}}{3}$; $\tan\theta = \frac{\sqrt{2}}{4}$ **15.** $\sin\theta = \frac{5\sqrt{41}}{41}$; $\cos\theta = \frac{4\sqrt{41}}{41}$; $\tan\theta = \frac{5}{4}$
17. $x = 140$ **19a.** 147 m **b.** 187 m
21. $\sin\theta = \frac{4}{5}$; $\cos\theta = \frac{3}{5}$; $\tan\theta = \frac{4}{3}$; $\csc\theta = \frac{5}{4}$; $\sec\theta = \frac{5}{3}$; $\cot\theta = \frac{3}{4}$
23. $\sin\theta = \frac{\sqrt{2}}{2}$; $\cos\theta = \frac{\sqrt{2}}{2}$; $\tan\theta = 1$; $\csc\theta = \sqrt{2}$; $\sec\theta = \sqrt{2}$; $\cot\theta = 1$ **25a.** 8022 ft **b.** 32 s
27. 135 ft **31.** F **35a.** 3 ft
b. $13.5\sqrt{3}$ ft^2 **37.** $x = 32$
39. $x = 38{,}416$ **41.** $\frac{1}{2}$ **43.** $16\frac{2}{3}$

13-2

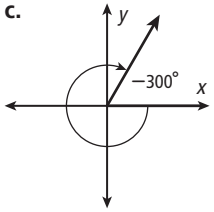
Check It Out!
1a.



b.

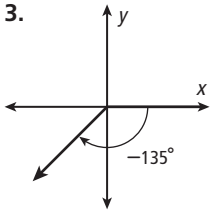


c.

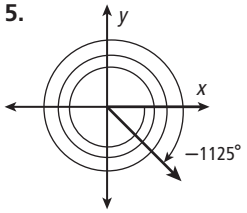


2a. Possible answer: 448°; −272°
b. Possible answer: 860°; −220°
c. Possible answer: 240°; −480°
3a. 75° **b.** 65° **c.** 50°
4. $\sin\theta = \frac{2\sqrt{5}}{5}$; $\cos\theta = -\frac{\sqrt{5}}{5}$; $\tan\theta = -2$; $\csc\theta = \frac{\sqrt{5}}{2}$; $\sec\theta = -\sqrt{5}$; $\cot\theta = -\frac{1}{2}$

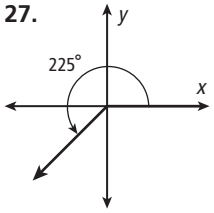
Exercises **1.** terminal
3.



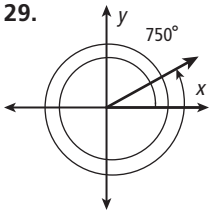
5.



11. 70° **13.** 20° **15.** 50° **17.** 40°
19. $\sin\theta = -\frac{\sqrt{5}}{5}$; $\cos\theta = \frac{2\sqrt{5}}{5}$; $\tan\theta = -\frac{1}{2}$; $\csc\theta = -\sqrt{5}$; $\sec\theta = \frac{\sqrt{5}}{2}$; $\cot\theta = -2$
21. $\sin\theta = -\frac{4}{5}$; $\cos\theta = -\frac{3}{5}$; $\tan\theta = \frac{4}{3}$; $\csc\theta = -\frac{5}{4}$; $\sec\theta = -\frac{5}{3}$; $\cot\theta = \frac{3}{4}$
23. $\sin\theta = \frac{6\sqrt{37}}{37}$; $\cos\theta = \frac{\sqrt{37}}{37}$; $\tan\theta = 6$; $\csc\theta = \frac{\sqrt{37}}{6}$; $\sec\theta = \sqrt{37}$; $\cot\theta = \frac{1}{6}$
25. $\sin\theta = \frac{2\sqrt{5}}{5}$; $\cos\theta = -\frac{\sqrt{5}}{5}$; $\tan\theta = -2$; $\csc\theta = \frac{\sqrt{5}}{2}$; $\sec\theta = -\sqrt{5}$; $\cot\theta = -\frac{1}{2}$
27.



29.



35. 50° **37.** 20° **39.** 85° **41.** 35°

43. $\sin\theta = -\frac{2\sqrt{29}}{29}$; $\cos\theta = \frac{5\sqrt{29}}{29}$; $\tan\theta = -\frac{2}{5}$; $\csc\theta = -\frac{\sqrt{29}}{2}$; $\sec\theta = \frac{\sqrt{29}}{5}$; $\cot\theta = -\frac{5}{2}$

45. $\sin\theta = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$; $\tan\theta = \frac{3}{4}$; $\csc\theta = \frac{5}{3}$; $\sec\theta = \frac{5}{4}$; $\cot\theta = \frac{4}{3}$

47. $\sin\theta = -\frac{2\sqrt{5}}{5}$; $\cos\theta = \frac{\sqrt{5}}{5}$; $\tan\theta = -2$; $\csc\theta = -\frac{\sqrt{5}}{2}$; $\sec\theta = \sqrt{5}$; $\cot\theta = -\frac{1}{2}$

49. $\sin\theta = \frac{4\sqrt{41}}{41}$; $\cos\theta = \frac{5\sqrt{41}}{41}$; $\tan\theta = \frac{4}{5}$; $\csc\theta = \frac{\sqrt{41}}{4}$; $\sec\theta = \frac{\sqrt{41}}{5}$; $\cot\theta = \frac{5}{4}$ **51.** 1364°/s **53.** $\left(-2, 2\sqrt{3}\right)$

55a. 402 ft **b.** 5 s **c.** 215 ft **d.** 29 ft

57a. 7.5 min **b.** 68 rotations

59. −0.643 **61.** 30°, 150°, 210°, 330°

63. 82°, 98°, 262°, 278° **67.** F

69. $\sin\theta = \frac{b\sqrt{a^2+b^2}}{a^2+b^2}$; $\cos\theta = \frac{a\sqrt{a^2+b^2}}{a^2+b^2}$; $\tan\theta = \frac{b}{a}$

71. $\sin\theta = \frac{b\sqrt{a^2+b^2}}{a^2+b^2}$; $\cos\theta = \frac{a\sqrt{a^2+b^2}}{a^2+b^2}$; $\tan\theta = \frac{b}{a}$

73. sine and cosine: none; tangent and secant: for $\theta = 90°$ for $\theta = 270°$ and all angles coterminal with these angles; cosecant and cotangent: for $\theta = 0°$ for $\theta = 180°$ and all angles coterminal with these angles **75.** 3 **77.** $g\left(f(4)\right) = 37$

79. $\sin\theta = \frac{5}{13}$; $\cos\theta = \frac{12}{13}$; $\tan\theta = \frac{5}{12}$

13-3

Check It Out! 1a. $\frac{4\pi}{9}$ radians **b.** 40° **c.** $-\frac{\pi}{5}$ radians **d.** 720°

2a. $-\frac{\sqrt{2}}{2}$ **b.** 0 **c.** $-\frac{1}{2}$

3a. $\sin 270° = -1$; $\cos 270° = 0$; $\tan 270°$: undefined

b. $\sin\frac{11\pi}{6} = -\frac{1}{2}$; $\cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$; $\tan\frac{11\pi}{6} = -\frac{\sqrt{3}}{3}$

c. $\sin(-30°) = -\frac{1}{2}$; $\cos(-30°) = \frac{\sqrt{3}}{2}$; $\tan(-30°) = -\frac{\sqrt{3}}{3}$ **4.** 1.5 ft

Exercises 1. 1 unit; 2π units

3. $-\frac{5\pi}{12}$ radians **5.** $\frac{3\pi}{4}$ radians

7. −112.5° **9.** 80° **11.** −1 **13.** $-\frac{1}{2}$

15. $\sin 120° = \frac{\sqrt{3}}{2}$; $\cos 120° = -\frac{1}{2}$; $\tan 120° = -\sqrt{3}$

17. $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$; $\cos\frac{\pi}{3} = \frac{1}{2}$; $\tan\frac{\pi}{3} = \sqrt{3}$ **19.** $\frac{4\pi}{3}$ radians

21. $-\frac{5\pi}{36}$ radians **23.** −20°

25. 630° **27.** $-\sqrt{3}$ **29.** $-\frac{\sqrt{3}}{2}$

31. $\sin 225° = -\frac{\sqrt{2}}{2}$; $\cos 225° = -\frac{\sqrt{2}}{2}$; $\tan 225° = 1$

33. $\sin\frac{11\pi}{6} = -\frac{1}{2}$; $\cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$; $\tan\frac{11\pi}{6} = -\frac{\sqrt{3}}{3}$ **35.** about 2793 mi

37. reference angle: $\frac{\pi}{4}$

39. 600 revolutions/min

41a. 45° **b.** 28 ft **51.** C

53. $\sin\theta = -\frac{\sqrt{3}}{2}$; $\csc\theta = -\frac{2\sqrt{3}}{3}$; $\sec\theta = 2$; $\cot\theta = -\frac{\sqrt{3}}{3}$

55. $\left(-5\sqrt{3}\,, -5\right)$ **59.** D: $\{x \mid x \geq -4\}$; R: $\{y \mid y \geq 0\}$ **61.** D: $\{x \mid x \geq 0\}$; R: $\{y \mid y \leq 0\}$ **63.** 25,165,824

65. 45° **67.** 5°

13-4

Check It Out! 1. $\frac{\pi}{4} + (2\pi)n$ or $\frac{5\pi}{4} + (2\pi)n$, where n is an integer

2a. $-\frac{\pi}{4}$ or −45° **b.** $\frac{\pi}{2}$ or 90°

3. 37° north of east **4a.** $\theta = -63.4°$

b. $\theta = 116.6°$

Exercises 3. $\frac{\pi}{6} + (2\pi)n$ and $\frac{7\pi}{6} + (2\pi)n$, where n is an integer

5. $\frac{\pi}{6}$; 30° **7.** undefined **9.** $\frac{\pi}{4}$; 45°

11. 5° **13.** $\theta = 234.5°$ **15.** $\theta = 255.5°$

17. $\frac{\pi}{3} + (2\pi)n$ and $\frac{2\pi}{3} + (2\pi)n$, where n is an integer **19.** $\frac{\pi}{3}$; 60°

21. $-\frac{\pi}{6}$; −30° **23.** $\frac{\pi}{3}$; 60° **25.** 75°

27. $\theta = 228.6°$ **29.** $\theta = 275.7°$

31a. style A: 7.5°; style B: 9.1°; style C: 5.1° **b.** style B **c.** 9.5°

33a. 84.0° **b.** 121 ft **35.** 0.7

39. A **41.** C **43.** $\frac{\pi}{3} \leq \theta \leq \frac{5\pi}{3}$

45. $\frac{\pi}{8} \leq \theta < \frac{\pi}{4}$ or $\frac{5\pi}{8} \leq \theta < \frac{3\pi}{4}$ or $\frac{9\pi}{8} \leq \theta < \frac{5\pi}{4}$ or $\frac{13\pi}{8} \leq \theta < \frac{7\pi}{4}$

47. linear; translation 5 units up (or 5 units left) **49.** $f^{-1}(x) = 4x - 4$; function; D: ℝ; R: ℝ **51.** $\frac{4\pi}{3}$ radians

53. $\frac{7\pi}{3}$ radians

13-5

Check It Out! 1. 47.9 ft²

2a. $m\angle K = 31°$; $k \approx 6.5$; $h \approx 8.4$

b. $m\angle N = 18°$; $m \approx 4.7$; $p \approx 4.0$

3. 1 triangle; $m\angle B \approx 35.4°$; $m\angle C \approx 39.6°$; $c \approx 6.6$ cm

Exercises 1. 4.9 cm² **3.** 6900.5 m²

5. $m\angle Z = 40°$; $x \approx 36.1$; $y \approx 18.3$

7. $m\angle C = 65°$; $a \approx 2.0$; $b \approx 2.9$

9. $m\angle R = 55°$; $s \approx 38.8$; $t \approx 18.3$

11. 1 triangle; $m\angle B \approx 20.3°$; $m\angle C \approx 39.7°$; $c \approx 7.4$ m

13. 1 triangle; $m\angle B \approx 37.3°$; $m\angle C \approx 97.7°$; $c \approx 9.8$ m

15. 1376.6 yd² **17.** $m\angle D = 61°$; $c \approx 9.9$; $d \approx 8.7$ **19.** $m\angle K = 38°$; $\ell \approx 9.4$; $m \approx 7.6$ **21.** 0 triangles

23. 1 triangle; $m\angle B \approx 22.5°$; $m\angle C \approx 27.5°$; $c \approx 4.2$ in.

25. $m\angle C = 64°$; $b \approx 15.3$; $c \approx 15.6$

27. $m\angle A = 59°$; $a \approx 21.8$; $c \approx 16.7$

29. 21 ft **31.** 1 triangle; $m\angle A \approx 16.9°$; $m\angle C \approx 28.1°$; $a \approx 4.9$ **33.** 1 triangle; $m\angle A = 90°$; $m\angle C = 60°$; $c \approx 5.2$

35a. distance from tower 1 to tower 2: 4.2 mi; distance from tower 2 to tower 3: 4.9 mi **b.** 8.9 mi²

37. 16.7 cm **39.** B is incorrect.

43. B **45b.** no **47.** $0° < m\angle A < 60°$

49. y-intercept: 5; x-intercept: 5

51. y-intercept: 2; x-intercept: 6

53. $x = \frac{1}{3}$ **55.** 135°; $\frac{3\pi}{4}$ radians

57. 30°; $\frac{\pi}{6}$ radians

13-6

Check It Out! 1a. $a \approx 40.9$; $m\angle B \approx 3.9°$; $m\angle C \approx 3.1°$

b. $m\angle A \approx 43.4°$; $m\angle B \approx 55.6°$; $m\angle C \approx 81.0°$ **2.** 34 mi **3.** 367 m²

Selected Answers

Exercises **1.** $q \approx 9.1$; $m\angle P \approx 40.5°$; $m\angle R \approx 59.5°$ **3.** $r \approx 11.6$; $m\angle P \approx 40.3°$; $m\angle R \approx 50.7°$ **5.** $m\angle P \approx 43.2°$; $m\angle Q \approx 86.5°$; $m\angle R \approx 50.3°$ **7.** 9 min **9.** $f \approx 55.5$; $m\angle G \approx 53.1°$; $m\angle H \approx 61.9°$ **11.** $f \approx 21.2$; $m\angle G \approx 59.2°$; $m\angle H \approx 40.8°$ **13.** $m\angle F \approx 54°$; $m\angle G \approx 59.6°$; $m\angle H \approx 66.4°$ **15.** 3.8 mi **17.** $m\angle B \approx 26.3°$; $m\angle C \approx 33.7°$; $a \approx 31.2$ **19.** $m\angle A \approx 51.3°$; $m\angle B \approx 32.7°$; $c \approx 16.6$ **21.** $m\angle A \approx 38.6°$; $m\angle B \approx 92.9°$; $m\angle C \approx 48.5°$ **23.** 74°, 46°, and 60° **25a.** 89 mi **b.** 38° **27a.** $m\angle A = 43°$; $m\angle B = 44°$ **b.** $m\angle A = 28°$; $m\angle B = 41°$ **29.** 524.6 cm^2 **31.** 7.3 ft^2 **33.** 1.2 km **39.** H **41.** No, Abby did not make an error. A triangle cannot be formed from sides that measure 2 units, 3 units, and 5 units. **43.** $x = 9.9$ **45.** $x = \pm 4i$ **47.** x-intercept of f: 4; y-intercept of f: -8; x-intercept of g: 4; y-intercept of g: -4 **49.** x-intercept of f: -12; y-intercept of f: 6; x-intercept of g: -4; y-intercept of g: 6 **51.** $m\angle C = 48°$; $b \approx 8.5$; $c \approx 13.8$

Study Guide: Review

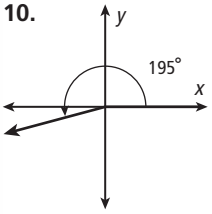
1. radian **2.** cosecant **3.** standard position

4. $\sin\theta = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$; $\tan\theta = \frac{3}{4}$; $\csc\theta = \frac{5}{3}$; $\sec\theta = \frac{5}{4}$; $\cot\theta = \frac{4}{3}$

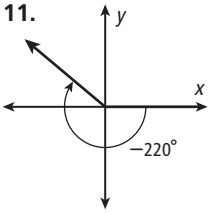
5. $\sin\theta = \frac{2}{3}$; $\cos\theta = \frac{\sqrt{5}}{3}$; $\tan\theta = \frac{2\sqrt{5}}{5}$; $\csc\theta = \frac{3}{2}$; $\sec\theta = \frac{3\sqrt{5}}{5}$; $\cot\theta = \frac{\sqrt{5}}{2}$

6. $x = 12\sqrt{3}$ **7.** $x = \frac{9\sqrt{2}}{2}$ **8.** 18 ft **9.** 178 m

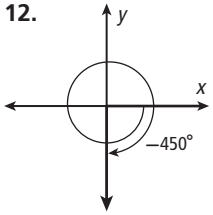
10.



11.



12.



13. Possible answer: 475°; −245°
14. Possible answer: 22°; −338°
15. Possible answer: 225°; −495°
16. 84° **17.** 53° **18.** 75°

19. $\sin\theta = \frac{3}{5}$; $\cos\theta = -\frac{4}{5}$; $\tan\theta = -\frac{3}{4}$; $\csc\theta = \frac{5}{3}$; $\sec\theta = -\frac{5}{4}$; $\cot\theta = -\frac{4}{3}$

20. $\sin\theta = \frac{12}{13}$; $\cos\theta = \frac{5}{13}$; $\tan\theta = \frac{12}{5}$; $\csc\theta = \frac{13}{12}$; $\sec\theta = \frac{13}{5}$; $\cot\theta = \frac{5}{12}$

21. $\sin\theta = -\frac{8}{17}$; $\cos\theta = -\frac{15}{17}$; $\tan\theta = \frac{8}{15}$; $\csc\theta = -\frac{17}{8}$; $\sec\theta = -\frac{17}{15}$; $\cot\theta = \frac{15}{8}$

22. $\sin\theta = -\frac{3\sqrt{73}}{73}$; $\cos\theta = \frac{8\sqrt{73}}{73}$; $\tan\theta = -\frac{3}{8}$; $\csc\theta = -\frac{\sqrt{73}}{3}$; $\sec\theta = \frac{\sqrt{73}}{8}$; $\cot\theta = -\frac{8}{3}$

23. $\sin\theta = -\frac{\sqrt{82}}{82}$; $\cos\theta = -\frac{9\sqrt{82}}{82}$; $\tan\theta = \frac{1}{9}$; $\csc\theta = -\sqrt{82}$; $\sec\theta = -\frac{\sqrt{82}}{9}$; $\cot\theta = 9$

24. $\sin\theta = \frac{2\sqrt{5}}{5}$; $\cos\theta = -\frac{\sqrt{5}}{5}$; $\tan\theta = -2$; $\csc\theta = \frac{\sqrt{5}}{2}$; $\sec\theta = -\sqrt{5}$; $\cot\theta = -\frac{1}{2}$

25. $\frac{3\pi}{2}$ radians **26.** $-\frac{2\pi}{3}$ radians **27.** $\frac{20\pi}{9}$ radians **28.** 30° **29.** −20° **30.** 405° **31.** $-\frac{1}{2}$ **32.** −1 **33.** 2

34. $\sin\frac{7\pi}{6} = -\frac{1}{2}$; $\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$; $\tan\frac{7\pi}{6} = \frac{\sqrt{3}}{3}$ **35.** $\sin 300° = -\frac{\sqrt{3}}{2}$; $\cos 300° = \frac{1}{2}$; $\tan 300° = -\sqrt{3}$

36. $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$; $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$; $\tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}$

37. 22 in. **38a.** $\frac{\pi}{3}$ radians **b.** 1.6 m

39. $\frac{\pi}{3} + (2\pi)n$ and $\frac{4\pi}{3} + (2\pi)n$, where n is an integer

40. $\frac{5\pi}{6} + (2\pi)n$ and $\frac{7\pi}{6} + (2\pi)n$, where n is an integer

41. $\frac{5\pi}{4} + (2\pi)n$ and $\frac{7\pi}{4} + (2\pi)n$, where n is an integer

42. $\frac{5\pi}{6} + (2\pi)n$ and $\frac{11\pi}{6} + (2\pi)n$, where n is an integer

43. −30°; $-\frac{\pi}{6}$ radians **44.** 30°; $\frac{\pi}{6}$ radians **45.** 180°; π radians **46.** 45°; $\frac{\pi}{4}$ radians **47.** 34°

48. 41° **49.** 17.5° **50.** 162.5°
51. 65.6° **52.** 245.6° **53.** 5.4 m^2
54. 4953.1 ft^2 **55.** 24.0 $in.^2$
56. 112.5 cm^2 **57.** $m\angle F = 97°$; $d \approx 26.3$; $e \approx 37.0$ **58.** $m\angle B = 75°$; $b = 15$; $c \approx 7.8$ **59.** $m\angle P = 113°$; $p \approx 10.0$; $q \approx 4.9$ **60.** $m\angle Y = 64°$; $w = 4.8$; $x = 8.0$ **61.** 2 triangles; $m\angle B_1 \approx 69.4°$; $m\angle C_1 \approx 55.6°$; $c_1 \approx 14.1$ cm; $m\angle B_2 \approx 110.6°$; $m\angle C_2 \approx 14.4°$; $c_2 \approx 4.3$ cm
62. $m\angle A \approx 20.9°$; $m\angle B \approx 130.1°$; $c \approx 19.0°$ **63.** $m\angle B \approx 43.0°$; $m\angle C \approx 27.0°$; $a \approx 24.8$
64. $m\angle A \approx 125.7°$; $m\angle B \approx 11.7°$; $m\angle C \approx 42.6°$ **65.** $m\angle A \approx 39.4°$; $m\angle B \approx 54.7°$; $m\angle C \approx 85.9°$
66a. 40.0 km **b.** 1.4 h **67.** 60 ft^2
68. 95 $in.^2$

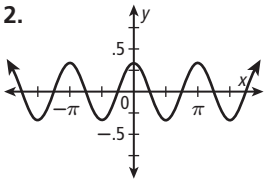
Chapter 14

14-1

Check It Out! **1a.** not periodic

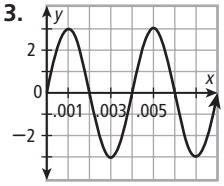
b. periodic; 3

2.



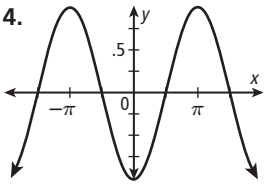
amplitude: $\frac{1}{3}$; period: π

3.



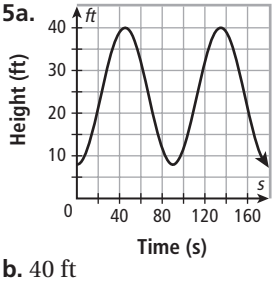
frequency: 250 Hz

4.



x-intercepts: $\frac{\pi}{2} + n\pi$; phase shift: π right

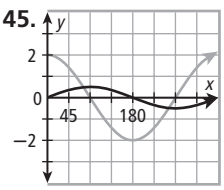
5a.



b. 40 ft

Exercises **1.** periods **3.** not periodic **5.** amplitude: $\frac{1}{4}$; period: 2π **7.** frequency: 100 Hz **9.** x-intercepts: πn; phase shift: $\frac{\pi}{2}$right **11.** 4 ft **13.** periodic; 2π **15.** amplitude: $\frac{3}{2}$; period: 2π **17.** amplitude: 6; period: 6π **19.** x-intercepts: πn; phase shift: π left **21.** x-intercepts: $\frac{\pi}{4} + \pi n$; phase shift: $\frac{3\pi}{4}$ left **23.** max.: 24.5 ft; min.: 21.5 ft **25.** amplitude: 1; period: 2π; phase shift $\frac{\pi}{4}$ left and vertical shift 1 down **27.** amplitude: 1; period: 1; horizontal compression and vertical shift 2 down **29.** ≈0.3 **31.** ≈0.25 **33.** $f(x) = 6 \sin 2x$; $f(x) = 6 \cos 2x$ **35.** $f(x) = -4 \sin 2x$; $g(x) = 4 \cos 2\left(x + \frac{\pi}{4}\right)$ **37a.** period: 12.2; amplitude: 1.5; max.: 3; min.: 0 **b.** $h(0) = 3$; $h(6.1) = 0$ **c.** $h(t) = 1.5 \cos \frac{2\pi}{12.2}t + 1.5$ **39.** The period decreases for $b > 1$ and increases for $b < 1$ because the period is given by $\frac{2\pi}{b}$. **41.** H **43.** phase shift π right, horizontal compression, vertical stretch, and reflection across the x-axis amplitude: 4; period: π; x-intercepts 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π; max.: 4, min.: -4

45.



47. $76° < \theta < 256°$ **49.** $(-\infty, -2]$ or $[1, 13)$ **51a.** $6r + 2l + 4c = 100$

b.

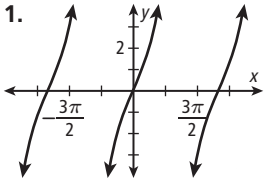
Roses	6	4	3	7
Lilies	10	8	5	3
Carnations	11	15	18	13

53. $m\angle A = 10°$; $m\angle B = 12.4°$; $m\angle C = 157.6°$

14-2

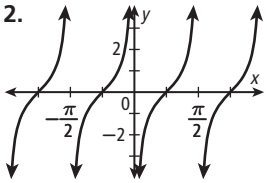
Check It Out!

1.



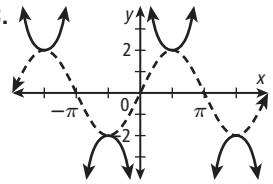
period: 2π; x-intercepts: $2\pi n$; asymptotes: $\pi + 2\pi n$

2.



period: $\frac{\pi}{2}$; x-intercepts: $\frac{\pi}{4} + \frac{\pi}{2}n$; asymptotes: $\frac{\pi}{2}n$

3.



period: 2π; asymptotes: πn

Exercises **1.** period: $\frac{\pi}{3}$; x-intercepts: $\frac{\pi}{3}n$; asymptotes: $\frac{\pi}{6} + \frac{\pi}{3}n$ **3.** period: $\frac{1}{2}$; x-intercepts: $\frac{1}{2}n$; asymptotes: $\frac{1}{4} + \frac{1}{2}n$ **5.** period: $\frac{\pi}{2}$; x-intercepts: $\frac{\pi}{4} + \frac{\pi}{2}n$; asymptotes: $\frac{\pi}{2}n$ **7.** period: 2π; asymptotes: $\frac{\pi}{2} + \pi n$ **9.** period: 2π; asymptotes: πn **11.** period: π; x-intercepts: $\frac{3\pi}{4} + \pi n$; asymptotes: $\frac{\pi}{4} + \pi n$ **13.** period: 2; x-intercepts: $2n$; asymptotes: $1 + 2n$ **15.** period: 4π; x-intercepts: $2\pi + 4\pi n$; asymptotes: $4\pi n$ **17.** period: 2π; asymptotes: $\frac{\pi}{2} + \pi n$ **19.** period: 2π; asymptotes: πn **21.** $\frac{\pi}{2}$; $\frac{3\pi}{2}$; $-\frac{\pi}{2}$; $\frac{5\pi}{2}$ **23.** $\frac{\pi}{2}$; $\frac{3\pi}{2}$; $-\frac{\pi}{2}$; $\frac{5\pi}{2}$ **25a.** 3 s **c.** $t = \frac{3}{4}$ and $t = \frac{9}{4}$ **27.** increasing; decreasing; decreasing; increasing **29.** decreasing; decreasing; increasing; increasing **31.** increasing; increasing; increasing; increasing **37.** G **39.** H **41.** period: 2; local maximum: 1; local minimum: 7; phase shift: 1 right **47.** D: $\{x \mid x \le -1 \text{ or } x \ge 1\}$; R: $\left\{y \mid 0 \le y \le \pi \text{ and } y \ne \frac{\pi}{2}\right\}$ **49.** D: $\{x \mid x \le -1 \text{ or } x \ge 1\}$; R: $\{y \mid -\frac{\pi}{2} \le y \le \frac{\pi}{2} \text{ and } y \ne 0\}$ **51.** $\frac{1}{10}$; -10 **53.** $3\sqrt{5}$; $-\frac{\sqrt{5}}{15}$ **55.** 11 pages **57.** 135° **59.** $-60°$

14-3

Check It Out!

1a. $\sin\theta \cot\theta = \sin\theta\left(\frac{\cos\theta}{\sin\theta}\right)$
$= \cos\theta$

b. $1 - \sec(-\theta) = 1 - \frac{1}{\cos(-\theta)}$
$= 1 - \frac{1}{\cos\theta}$
$= 1 - \sec\theta$

2a. $1 + \sin\theta$ **b.** $\frac{1}{\sin^2\theta} - 1$ **3.** $\theta \approx 22°$

Exercises

1. $\sin\theta \sec\theta = \sin\theta\left(\frac{1}{\cos\theta}\right)$
$= \frac{\sin\theta}{\cos\theta}$
$= \tan\theta$

3. $\cos^2\theta(\sec^2\theta - 1) = \cos^2\theta(\tan^2\theta)$
$= \cos^2\theta\left(\frac{\sin\theta}{\cos\theta}\right)^2$
$= \cos^2\theta\left(\frac{\sin^2\theta}{\cos^2\theta}\right)$
$= \sin^2\theta$

5. $1 + \cos^2\theta$ **7.** $\theta \approx 43°$

9. $\frac{\sin\theta - \cos\theta}{\sin\theta} = \frac{\sin\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$
$= 1 - \cot\theta$

11. $\sec^2\theta(1 - \cos^2\theta) = \left(\frac{1}{\cos^2\theta}\right)(\sin^2\theta)$
$= \frac{\sin^2\theta}{\cos^2\theta}$
$= \tan^2\theta$

13. $\frac{\sin^2\theta}{1 - \sin^2\theta}$ **15.** $\sin^2\theta$ **17.** 1 **19.** $\sec\theta$ **21.** $\cot\theta$ **23.** $\sin\theta$ **25.** $\csc\theta$ **27.** 1 **29.** $\sec^2\theta$ **31.** $\tan\theta$

33. $\sin^2\theta(\csc^2\theta - 1) = \sin^2\theta \cot^2\theta$
$= \sin^2\theta\left(\frac{\cos^2\theta}{\sin^2\theta}\right)$
$= \cos^2\theta$

35. $\frac{\cos\theta}{1-\sin^2\theta} = \frac{\cos\theta}{\cos^2\theta} = \frac{1}{\cos\theta} = \sec\theta$

39. $\cot\theta = \frac{x}{y} = \frac{r\cos\theta}{r\sin\theta} = \frac{\cos\theta}{\sin\theta}$

43. $x^2 + y^2 = r^2$

$\frac{x^2}{x^2} + \frac{y^2}{x^2} = \frac{r^2}{x^2}$

$1 + \left(\frac{y}{x}\right)^2 = \left(\frac{r}{x}\right)^2$

$1 + \tan^2\theta = \sec^2\theta$

45. no **47.** yes **49.** no

51a. $r = \ell\sin\theta$ **b.** $\ell = \frac{g}{\omega^2}\sec\theta$

55. an infinite number of equivalent forms **57.** D **59.** A

63. $\frac{1}{\sin\theta\cos\theta}$ **65.** $\frac{1}{1-\cos^2\theta}$

67. $\sin\theta + \cos\theta$ **69.** $\frac{\sin\theta}{\sin\theta+1}$

71. $\frac{1}{36}$ **73.** $\frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}$

75. $0, \pi, -\pi, 2\pi$

14-4

Check It Out! **1a.** $-2-\sqrt{3}$

b. $\frac{\sqrt{2}-\sqrt{6}}{4}$

2. $\cos\left(\frac{\pi}{2}+x\right) = \cos\left(\frac{\pi}{2}\right)\cos x - \sin\left(\frac{\pi}{2}\right)\sin x = (0)\cos x - (1)\sin x = -\sin x$

3. $\frac{24}{25}$ **4.** $A'\left(-\sqrt{3},1\right), B'\left(-2\sqrt{3},2\right), C'(0,2), D'\left(-\sqrt{3},-1\right)$

Exercises **1.** A rotation matrix assumes a counterclockwise rotation about the origin.

3. $\frac{\sqrt{6}-\sqrt{2}}{4}$ **5.** $\frac{\sqrt{6}-\sqrt{2}}{4}$

7. $\tan(\pi+x) = \frac{\tan\pi + \tan x}{1-\tan\pi\tan x} = \frac{0+\tan x}{1-0} = \tan x$

9. $\frac{16}{65}$ **11.** $\frac{16}{63}$ **13.** $A'(-1.73,-1), B'(-0.87,0.5), C'(-1.5,2.60)$

15. $\sqrt{3}-2$ **17.** $\frac{-\sqrt{2}-\sqrt{6}}{4}$

19. $\sin\left(\frac{3\pi}{2}+x\right) = \sin\frac{3\pi}{2}\cos x + \cos\frac{3\pi}{2}\sin x = (-1)\cos x + (0)\sin x = -\cos x$

21. $\frac{63}{65}$ **23.** $-\frac{16}{65}$ **25.** $A'(-1.41,1.41), B'(-0.71,2.12), C'(-0.71,0.71)$

27. $2+\sqrt{3}$ **29.** $\frac{\sqrt{2}-\sqrt{6}}{4}$

31. $2+\sqrt{3}$ **33.** $2-\sqrt{3}$ **35.** $\theta = 90°$

37. $\theta = 30°$ or $150°$

39. $\frac{204}{253}; -\frac{253}{325}; \frac{36}{325}$

41a. $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}; \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix};$

b. $P'(0,0), Q'(-1,1), R'(0,4), S'(1,1); P''(0,0), Q''(-1,-1), R''(-4,0), S''(-1,1); P'''(0,0), Q'''(1,-1), R'''(0,-4), S'''(-1,-1)$

43a. 4.2; 3 **b.** $y(t) = -4.2\cos\frac{2\pi}{3}t$ **c.** 2.1 **45.** $A'(-2.60,1.5), B'(-2.96,2.87), C'(-1.60,3.23), D'(1,1.73)$ **47.** $A'(1.50,2.60), B'(2.87,2.96), C'(3.23,1.60), D'(1.73,-1)$ **49.** A **51.** A

57. 45° **59.** 30° **61.** $\frac{x-1}{x+2}$

63. parabola **65.** $\frac{1}{\cos\theta - \cos^3\theta}$

67. $\sin^2\theta$

14-5

Check It Out! **1.** $\frac{4\sqrt{2}}{7}; -\frac{7}{9}$

2a. Possible answer:

$\cos^4\theta - \sin^4\theta = (\cos^2\theta+\sin^2\theta)(\cos^2\theta-\sin^2\theta) = (1)(\cos 2\theta) = \cos 2\theta$

b. Possible answer:

$\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\left(\frac{\sin\theta}{\cos\theta}\right)}{\sec^2\theta} = \frac{2\left(\frac{\sin\theta}{\cos\theta}\right)}{\frac{1}{\cos^2\theta}} \cdot \frac{\left(\frac{\cos^2\theta}{1}\right)}{\left(\frac{\cos^2\theta}{1}\right)} = 2\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\cos^2\theta}{1}\right) = 2\sin\theta\cos\theta = \sin 2\theta$

3a. $\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}}$ **b.** $-\frac{\sqrt{2-\sqrt{2}}}{2}$

4. $\frac{\sqrt{5}}{5}; \frac{2\sqrt{5}}{5}$

Exercises **1.** $-\frac{120}{169}; -\frac{119}{169}; \frac{120}{119}$

3. $2\cos 2\theta = 2(2\cos^2\theta - 1) = 4\cos^2\theta - 2$

5. $\frac{1+\cos 2\theta}{\sin 2\theta} = \frac{1+(2\cos^2\theta-1)}{(2\sin\theta\cos\theta)} = \frac{2\cos^2\theta}{2\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta$

7. $\frac{\sqrt{2-\sqrt{2}}}{2}$ **9.** $\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$

11. $\frac{4}{5}; -\frac{3}{5}; -\frac{4}{3}$

13. $-\frac{336}{625}; -\frac{527}{625}; \frac{336}{527}$

15. $\frac{\sin 2\theta}{\sin\theta} = \frac{(2\sin\theta\cos\theta)}{\sin\theta} = 2\cos\theta$

17. $\frac{1-\cos 2\theta}{\sin 2\theta} = \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta} = \frac{2\sin^2\theta}{2\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

19. $\frac{\sqrt{2+\sqrt{3}}}{2}$ **21.** $\frac{\sqrt{2-\sqrt{2}}}{2}$

23. $\frac{\sqrt{37}}{37}; -\frac{6\sqrt{37}}{37}; -\frac{1}{6}$

25. $3\sin\theta\cos^2\theta - \sin^3\theta$

27. $\cos\theta\left(1-4\sin^2\theta\right)$ **29.** 1

31. $2\tan\theta$ **33.** $\sin\theta$

35a. $y(t) = 6.2\sin t\cos t$

b. about 0.66 s **c.** about 3.00 m

37. $\frac{4\sqrt{5}}{9}; \frac{1}{9}; 4\sqrt{5}; \frac{\sqrt{18+6\sqrt{5}}}{6}; -\frac{\sqrt{18-6\sqrt{5}}}{6}; -\frac{\sqrt{3+\sqrt{5}}}{\sqrt{3-\sqrt{5}}}$

39. $-\frac{4}{5}; \frac{3}{5}; -\frac{4}{3}; \sqrt{\frac{5-2\sqrt{5}}{10}}; -\sqrt{\frac{5+2\sqrt{5}}{10}}; -\sqrt{\frac{5-2\sqrt{5}}{5+2\sqrt{5}}}$

41. $\frac{\sqrt{2-\sqrt{3}}}{2}$ **43.** $-\frac{\sqrt{2-\sqrt{3}}}{2}$

49a. $d(\theta) = \frac{{v_0}^2\sin 2\theta}{32}$

b. 100 ft; ≈ 173 ft; 200 ft; ≈ 173 ft; 100 ft **c.** 45° **d.** $30.52° < \theta < 59.48°$

53. F **55.** G

59. $\sqrt{\frac{2-\sqrt{2+\sqrt{3}}}{2+\sqrt{2+\sqrt{3}}}}$

61. $\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{3}}}$

65. no **67.** $\frac{5x+12}{x+7}; x \neq 7$

69. $\frac{2x^2-30x-20}{(x+1)(x-3)}; x \neq 1, 3$

71. $\frac{\sqrt{2}-\sqrt{6}}{4}$ **73.** $\frac{\sqrt{2}-\sqrt{6}}{4}$

14-6

Check It Out! **1.** $150° + 360n°$, $210° + 360n°$ **2a.** 0 **b.** $\approx 21.9°$, $\approx 158.1°$ **3a.** $60°, 300°$ **b.** $90°, 210°, 270°, 330°$ **4.** late March and late September

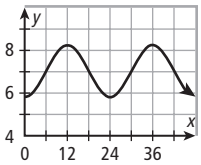
Exercises **1.** $60° + 360n°$, $300° + 360n°$ **3.** $30° + 360n°$, $330° + 360n°$ **5.** $\approx 74.5°$ or $285.5°$ **7.** π **9.** $60° + 360n°, 300° + 360n°$ **11.** $150° + 360n°, 210° + 360n°$ **13.** $\frac{\pi}{3}, \pi$, or $\frac{5\pi}{3}$ **15.** $90°, 120°, 240°, 270°$ **17a.** 10:00 A.M. and 6:00 P.M. **19.** $30°, 150°, 270°$ **21.** $30°, 90°, 150°$ **23.** $0°, 180°, 210°, 330°$ **25.** π **27.** $\frac{7\pi}{6}, \frac{11\pi}{6}$ **29.** no solution **31.** $0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}$ **33a.** $\approx \frac{\pi}{2}$; 4 sets **b.** $\approx \frac{2\pi}{5}$; 5 sets **35.** B is incorrect **37.** $x \approx -4.165, -1.797, 1.395, 5.464, 6.831$ **39.** $\approx 84.8°$, $\approx 264.8°$ **41.** $60°, 150°, 240°, 330°$ **43.** $38.5°, 141.5°$ **47.** J **49.** J

51. $\theta \approx 38.7°$ or $321.3°$. **53.** $90°, 270°, 120°, 240°, 60°, 300°$ **55.** $210°, 330°$ **57.** $30°, 150°, 210°, 330°$ **59.** $2\sqrt{5}, 4.\overline{47}, \sqrt{21}, \frac{19}{4}, \frac{\pi}{0.65}$ **61.** -1 **63.** $\cos^2\theta$

Study Guide: Review

1. cycle **2.** frequency **3.** period **4.** phase shift **5.** amplitude: 1; period: $\frac{2\pi}{3}$ **6.** amplitude: 1; period: 4π **7.** amplitude: $\frac{1}{3}$; period: 2π **8.** amplitude: 2; period: 2 **9.** amplitude: $\frac{1}{2}$; period: π **10.** amplitude: $\frac{\pi}{2}$; period: 2 **11.** x-intercepts: $\frac{\pi}{2} + \pi n$; phase shift: π left **12.** x-intercepts: $\frac{3\pi}{4} + \pi n$; phase shift: $\frac{\pi}{4}$ left **13.** x-intercepts: $\frac{\pi}{2} + \pi n$; phase shift: $\frac{3\pi}{2}$ right **14.** x-intercepts: πn; phase shift: $\frac{3\pi}{2}$ left

15.



16. 24 h **17.** 8.2; noon **18.** period: π; x-intercepts: πn; asymptotes: $\frac{\pi}{2} + \pi n$ **19.** period: 1; x-intercepts: n; asymptotes: $\frac{1}{2} + n$ **20.** period: 2; x-intercepts: $2n$; asymptotes: $1 + 2n$ **21.** period: π; x-intercepts: $\frac{\pi}{2} + \pi n$; asymptotes: πn

22. period: π; x-intercepts: $\frac{\pi}{2} + \pi n$; asymptotes: πn **23.** period: 1; x-intercepts: $\frac{1}{2} + n$; asymptotes: n

24. period: 2π; asymptotes: $\frac{\pi}{2} + \pi n$

25. period: π; asymptotes: $\frac{\pi}{2} n$

26. period: 2π; asymptotes: πn

27. period: 2π; asymptotes: $\frac{\pi}{2} + \pi n$

28. period: 2π; asymptotes: $\frac{\pi}{2} + \pi n$

29. period: 2π; asymptotes: $\pi + \pi n$

30. $\sec\theta \sin\theta \cot\theta = \left(\frac{1}{\cos\theta}\right) \sin\theta \left(\frac{\cos\theta}{\sin\theta}\right) = \left(\frac{\cos\theta}{\cos\theta}\right)\left(\frac{\sin\theta}{\sin\theta}\right) = 1$

31. $\frac{\sin^2(-\theta)}{\tan\theta} = \frac{(-\sin\theta)(-\sin\theta)}{\frac{\sin\theta}{\cos\theta}} = (\sin\theta)(\sin\theta)\left(\frac{\cos\theta}{\sin\theta}\right) = \sin\theta\cos\theta$

32. $(\sec\theta + 1)(\sec\theta - 1) = \sec^2\theta - 1 = \tan^2\theta$

33. $\cos\theta \sec\theta + \cos^2\theta \csc^2\theta = 1 + \cos^2\theta \frac{1}{\sin^2\theta} = 1 + \cot^2\theta = \csc^2\theta$

34. $(\tan\theta + \cot\theta)^2 = \tan^2\theta + 2\tan\theta\cot\theta + \cot^2\theta = \tan^2\theta + 2 + \cot^2\theta = (\tan^2\theta + 1) + (1 + \cot^2\theta) = \sec^2\theta + \csc^2\theta$

35. $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \sec\theta\csc\theta$

36. $\sin^2\theta \tan\theta = (1 - \cos^2\theta)\tan\theta = \tan\theta - \cos^2\tan\theta = \tan\theta - \cos^2\theta\left(\frac{\sin\theta}{\cos\theta}\right) = \tan\theta - \sin\theta\cos\theta$

37. $\frac{\tan\theta}{1 - \cos^2\theta} = \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{(\sin^2\theta)} = \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin^2\theta}\right) = \left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right) = \sec\theta\csc\theta$

38. $\csc\theta$ **39.** $\tan^2\theta$ **40.** $-\tan^2\theta$ **41.** $\sin\theta$ **42.** $\frac{-\sqrt{2} - \sqrt{6}}{4}$ **43.** $\frac{-\sqrt{2} - \sqrt{6}}{4}$ **44.** $\frac{\sqrt{6} + \sqrt{2}}{4}$ **45.** $2 - \sqrt{3}$ **46.** $-\frac{16}{65}$ **47.** $-\frac{63}{65}$ **48.** $\frac{56}{33}$ **49.** $\frac{16}{63}$ **50.** $-\frac{56}{65}$ **51.** $-\frac{33}{65}$ **52.** $\frac{36 - 5\sqrt{7}}{52}$ **53.** $\frac{-15 - 12\sqrt{7}}{52}$ **54.** $\frac{5\sqrt{7} + 36}{15 - 12\sqrt{7}}$ **55.** $\frac{5\sqrt{7} - 36}{15 + 12\sqrt{7}}$ **56.** $\frac{-36 - 5\sqrt{7}}{52}$ **57.** $\frac{-15 + 12\sqrt{7}}{52}$

58. $\approx \begin{bmatrix} 0 & 2.60 & 2.46 & -0.13 \\ 0 & 1.50 & 3.73 & 2.23 \end{bmatrix}$

59. $\approx \begin{bmatrix} 0 & 2.12 & 1.41 & -0.71 \\ 0 & 2.12 & 4.24 & 2.12 \end{bmatrix}$

60. $\approx \begin{bmatrix} 0 & 1.5 & 0.27 & -1.23 \\ 0 & 2.60 & 4.46 & 1.87 \end{bmatrix}$

61. $\begin{bmatrix} 0 & 0 & -2 & -2 \\ 0 & 3 & 4 & 1 \end{bmatrix}$

62. $\approx \begin{bmatrix} 0 & -4.23 & -3.46 & 0.77 \\ 0 & 3.33 & -2 & -5.33 \end{bmatrix}$

63. $\begin{bmatrix} 0 & -5 & 0 & 5 \\ 0 & -2 & -4 & -2 \end{bmatrix}$

64. $\approx \begin{bmatrix} 0 & -0.77 & 3.46 & 4.23 \\ 0 & -5.33 & -2 & 3.33 \end{bmatrix}$

65. $\begin{bmatrix} 0 & 2 & 4 & 2 \\ 0 & -5 & 0 & 5 \end{bmatrix}$ **66.** $\frac{24}{25}$ **67.** $-\frac{7}{25}$ **68.** $\frac{1}{2}$ **69.** $\frac{\sqrt{5}}{5}$ **70.** $-3\sqrt{7}$ **71.** $\frac{1}{8}$ **72.** $-\frac{\sqrt{14}}{4}$ **73.** $\frac{\sqrt{2}}{4}$ **74.** $\frac{\sqrt{2 - \sqrt{3}}}{2}$ **75.** $\frac{\sqrt{2 - \sqrt{3}}}{2}$

76. $135° + 360n°, 225° + 360n°$ **77.** $180° + 360n°$ **78.** $0° + 180n°, 135° + 180n°$ **79.** $60° + 180n°, 120° + 180n°$ **80.** $\frac{2\pi}{3}, \frac{4\pi}{3}$ **81.** 0 **82.** $\frac{\pi}{2}, \frac{3\pi}{2}$ **83.** $\frac{3\pi}{2}$ **84.** $0, \frac{2\pi}{3}, \frac{4\pi}{3}$ **85.** $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

86a. 900 min; late June **b.** 540 min; late December

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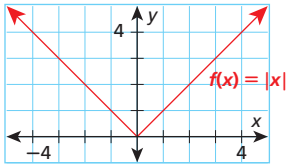
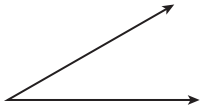
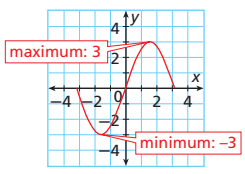
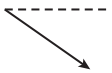
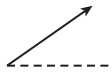
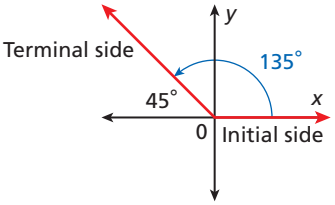
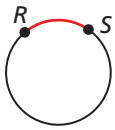
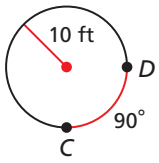
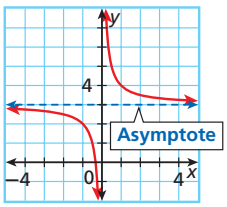
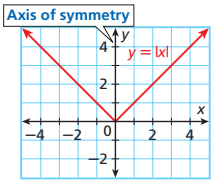
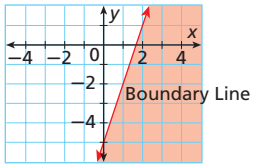
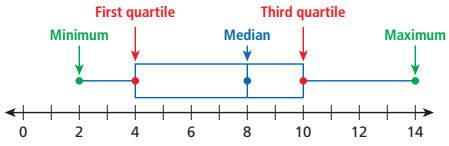
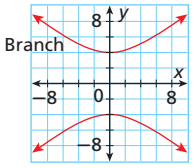
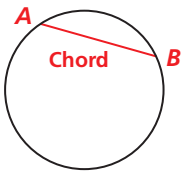
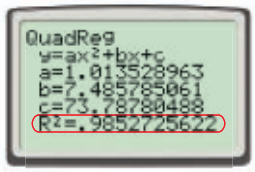
A

ENGLISH	SPANISH	EXAMPLES
absolute value of a complex number (p. 382) The absolute value of $a + bi$ is the distance from the origin to the point (a, b) in the complex plane and is denoted $\lvert a + bi \rvert = \sqrt{a^2 + b^2}$.	**valor absoluto de un número complejo** El valor absoluto de $a + bi$ es la distancia desde el origen hasta el punto (a, b) en el plano complejo y se expresa $\lvert a + bi \rvert = \sqrt{a^2 + b^2}$.	$\lvert 2 + 3i \rvert = \sqrt{2^2 + 3^2} = \sqrt{13}$
absolute value of a real number (p. 151) The absolute value of x is the distance from zero to x on a number line, denoted $\lvert x \rvert$. $\lvert x \rvert = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$	**valor absoluto de un número real** El valor absoluto de x es la distancia desde cero hasta x en una recta numérica y se expresa $\lvert x \rvert$. $\lvert x \rvert = \begin{cases} x & \text{si } x \geq 0 \\ -x & \text{si } x < 0 \end{cases}$	$\lvert 3 \rvert = 3$ $\lvert -3 \rvert = 3$
absolute-value function (p. 158) A function whose rule contains absolute-value expressions.	**función de valor absoluto** Función cuya regla contiene expresiones de valor absoluto.	
acute angle (p. 960) An angle that measures greater than 0° and less than 90°.	**ángulo agudo** Ángulo que mide más de 0° y menos de 90°.	
additive inverse of a matrix (p. 249) A matrix where each entry is the opposite of each entry in another matrix. Two matrices are additive inverses if their sum is the zero matrix.	**inverso aditivo de una matriz** Matriz en la cual cada entrada es el opuesto de cada entrada en otra matriz. Dos matrices son inversos aditivos si su suma es la matriz cero.	$\begin{bmatrix} 1 & -2 \\ 0 & 4 \end{bmatrix}$ and $\begin{bmatrix} -1 & 2 \\ 0 & -4 \end{bmatrix}$ are additive inverses.
address (p. 246) The location of an entry in a matrix, given by the row and column in which the entry appears. In matrix A, the address of the entry in row i and column j is a_{ij}.	**dirección** Ubicación de una entrada en una matriz, indicada por la fila y la columna en las que aparece la entrada. En la matriz A, la dirección de la entrada de la fila i y la columna j es a_{ij}.	In the matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$, the address of the entry 2 is a_{11}, the address of the entry 3 is a_{12}.
algebraic expression (p. 27) An expression that contains at least one variable.	**expresión algebraica** Expresión que contiene por lo menos una variable.	$2x + 3y$
amplitude (p. 991) The amplitude of a periodic function is half the difference of the maximum and minimum values (always positive).	**amplitud** La amplitud de una función periódica es la mitad de la diferencia entre los valores máximo y mínimo (siempre positivos).	 amplitude $= \frac{1}{2}\left[3 - (-3)\right] = 3$

ENGLISH	SPANISH	EXAMPLES
angle of depression (p. 931) The angle formed by a horizontal line and a line of sight to a point below.	**ángulo de depresión** Ángulo formado por una recta horizontal y una línea visual a un punto inferior.	
angle of elevation (p. 931) The angle formed by a horizontal line and a line of sight to a point above.	**ángulo de elevación** Ángulo formado por una recta horizontal y una línea visual a un punto superior.	
angle of rotation (p. 936) An angle formed by a rotating ray, called the terminal side, and a stationary reference ray, called the initial side.	**ángulo de rotación** Ángulo formado por un rayo en rotación, denominado lado terminal, y un rayo de referencia estático, denominado lado inicial.	
arc (p. 943) An unbroken part of a circle consisting of two points on the circle, called the endpoints, and all the points on the circle between them.	**arco** Parte continua de un círculo formada por dos puntos del círculo denominados extremos y todos los puntos del círculo comprendidos entre éstos.	
arc length (p. 943) The distance along an arc measured in linear units.	**longitud de arco** Distancia a lo largo de un arco medida en unidades lineales.	 $m\overset{\frown}{CD} = 5\pi$ ft
arithmetic sequence (p. 879) A sequence whose successive terms differ by the same nonzero number d, called the *common difference*.	**sucesión aritmética** Sucesión cuyos términos sucesivos difieren en el mismo número distinto de cero d, denominado *diferencia común*.	4, 7, 10, 13, 16, ... +3 +3 +3 +3 $d = 3$
arithmetic series (p. 882) The indicated sum of the terms of an arithmetic sequence.	**serie aritmética** Suma indicada de los términos de una sucesión aritmética.	4 + 7 + 10 + 13 + 16 + ...
asymptote (p. 490) A line that a graph approaches as the value of a variable becomes extremely large or small.	**asíntota** Línea recta a la cual se aproxima una gráfica a medida que el valor de una variable se hace sumamente grande o pequeño.	
augmented matrix (p. 287) A matrix that consists of the coefficients and the constant terms in a system of linear equations.	**matriz aumentada** Matriz formada por los coeficientes y los términos constantes de un sistema de ecuaciones lineales.	System of equations $3x + 2y = 5$ $2x - 3y = 1$ Augmented matrix $\left[\begin{array}{cc\|c} 3 & 2 & 5 \\ 2 & -3 & 1 \end{array}\right]$
axis of symmetry (p. 323) A line that divides a plane figure or a graph into two congruent reflected halves.	**eje de simetría** Línea que divide una figura plana o una gráfica en dos mitades reflejadas congruentes.	

ENGLISH	SPANISH	EXAMPLES
B		
base of a power (p. 34) The number in a power that is used as a factor.	**base de una potencia** Número de una potencia que se utiliza como factor.	$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$ base
base of an exponential function (p. 490) The value of b in a function of the form $f(x) = ab^x$, where a and b are real numbers with $a \neq 0$, $b > 0$, and $b \neq 1$.	**base de una función exponencial** Valor de b en una función del tipo $f(x) = ab^x$, donde a y b son números reales con $a \neq 0$, $b > 0$, y $b \neq 1$.	$f(x) = 5(2)^x$ base
binomial (pp. 336, 406) A polynomial with two terms.	**binomio** Polinomio con dos términos.	$x + y$ $2a^2 + 3$ $4m^3n^2 + 6mn^4$
binomial experiment (p. 837) A probability experiment consists of n identical and independent trials whose outcomes are either successes or failures, with a constant probability of success p and a constant probability of failure q, where $q = 1 - p$ or $p + q = 1$.	**experimento binomial** Experimento de probabilidades que comprende n pruebas idénticas e independientes cuyos resultados son éxitos o fracasos, con una probabilidad constante de éxito p y una probabilidad constante de fracaso q, donde $q = 1 - p$ o $p + q = 1$.	A multiple-choice quiz has 10 questions with 4 answer choices. The number of trials is 10. If each question is answered randomly, the probability of success for each trial is $\frac{1}{4} = 0.25$ and the probability of failure is $\frac{3}{4} = 0.75$.
binomial probability (p. 838) In a binomial experiment, the probability of r successes $(0 \leq r \leq n)$ is $P(r) = {}_nC_r \cdot p^r q^{n-r}$.	**probabilidad binomial** En un experimento binomial, la probabilidad de r éxitos $(0 \leq r \leq n)$ es $P(r) = {}_nC_r \cdot p^r q^{n-r}$.	In the binomial experiment above, the probability of randomly guessing 6 problems correctly is $P = {}_{10}C_6\,(0.25)^6\,(0.75)^4 \approx 0.016$.
Binomial Theorem (p. 837) For any positive integer n, $(x + y)^n = {}_nC_0\,x^n\,y^0 + {}_nC_1\,x^{n-1}\,y^1 + {}_nC_2\,x^{n-2}\,y^2 + \cdots + {}_nC_{n-1}\,x^1\,y^{n-1} + {}_nC_n\,x^0\,y^n$	**Teorema de los binomios** Dado un entero positivo n, $(x + y)^n = {}_nC_0\,x^n\,y^0 + {}_nC_1\,x^{n-1}\,y^1 + {}_nC_2\,x^{n-2}\,y^2 + \cdots + {}_nC_{n-1}\,x^1\,y^{n-1} + {}_nC_n\,x^0\,y^n$	$(x + 2)^4 = {}_4C_0\,x^4 2^0 + {}_4C_1\,x^3 2^1 + {}_4C_2\,x^2 2^2 + {}_4C_1\,x^1 2^3 + {}_4C_4\,x^0 2^4$ $= x^4 + 8x^3 + 24x^2 + 32x + 16$
boundary line (p. 124) A line that divides a coordinate plane into two half-planes.	**línea de límite** Línea que divide un plano cartesiano en dos semiplanos.	
box-and-whisker plot (p. 829) A method of showing how data is distributed by using the median, quartiles, and minimum and maximum values; also called a *box plot*.	**gráfica de mediana y rango** Método para demostrar la distribución de datos utilizando la mediana, los cuartiles y los valores mínimos y máximos; también llamado *gráfica de caja*.	
branch of a hyperbola (p. 744) One of the two symmetrical parts of the hyperbola.	**rama de una hipérbola** Una de las dos partes simétricas de la hipérbola.	

C

ENGLISH	SPANISH	EXAMPLES
circle (p. 729) The set of points in a plane that are a fixed distance from a given point called the center of the circle.	**círculo** Conjunto de puntos en un plano que se encuentran a una distancia fija de un punto determinado denominado centro del círculo.	
circumference (p. 943) The distance around a circle.	**circunferencia** Distancia alrededor del círculo.	
closure (p. 15) A set of numbers is said to be closed, or to have closure, under a given operation if the result of the operation on any two numbers in the set is also in the set.	**cerradura** Se dice que un conjunto de números es cerrado, o tiene cerradura, respecto de una operación determinada, si el resultado de la operación entre dos números cualesquiera del conjunto también está en el conjunto.	The natural numbers are closed under addition because the sum of two natural numbers is always a natural number.
coefficient (p. 47) A number multiplied by a variable.	**coeficiente** Número multiplicado por una variable.	In the expression $2x + 3y$, 2 is the coefficient of x and 3 is the coefficient of y.
coefficient matrix (p. 271) The matrix of the coefficients of the variables in a linear system of equations.	**matriz de coeficientes** Matriz de los coeficientes de las variables en un sistema lineal de ecuaciones.	System of equations: $2x + 3y = 11$, $5x - 4y = 16$ — Coefficient matrix: $\begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix}$
coefficient of determination (p. 376) The number R^2, with $0 \le R^2 \le 1$, that shows the fraction of the data that are close to the curve of best fit and, thus, how well the curve fits the data.	**coeficiente de determinación** El número R^2, con $0 \le R^2 \le 1$, que muestra la fracción de los datos cercanos a la línea de mejor ajuste y, por lo tanto, cuánto se ajusta la línea de mejor ajuste a los datos.	
combination (p. 796) A selection of a group of objects in which order is *not* important. The number of combinations of r objects chosen from a group of n objects is denoted ${}_nC_r$.	**combinación** Selección de un grupo de objetos en la cual el orden *no* es importante. El número de combinaciones de r objetos elegidos de un grupo de n objetos se expresa así: ${}_nC_r$.	For 4 objects *A, B, C,* and *D*, there are ${}_4C_2 = 6$ different combinations of 2 objects: *AB, AC, AD, BC, BD, CD.*
combined variation (p. 572) A relationship containing both direct and inverse variation.	**variación combinada** Relación que contiene variaciones directas e inversas.	$y = \frac{kx}{z}$, where k is the constant of variation
common difference (p. 879) In an arithmetic sequence, the nonzero constant difference of any term and the previous term.	**diferencia común** En una sucesión aritmética, diferencia constante distinta de cero entre cualquier término y el término anterior.	In the arithmetic sequence 3, 5, 7, 9, 11, ..., the common difference is 2.
common logarithm (p. 506) A logarithm whose base is 10, denoted $\log_{10}$ or just log.	**logaritmo común** Logaritmo de base 10, que se expresa $\log_{10}$ o simplemente log.	$\log 100 = \log_{10} 100 = 2$, since $10^2 = 100$.

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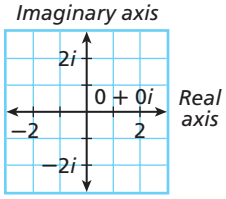
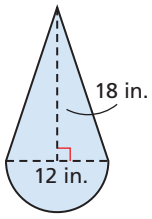
common ratio (p. 879) In a geometric sequence, the constant ratio of any term and the previous term.	**razón común** En una sucesión geométrica, la razón constante r entre cualquier término y el término anterior.	In the geometric sequence 32, 16, 18, 4, 2 ..., the common ratio is $\frac{1}{2}$.
complement of an event (p. 803) All outcomes in the sample space that are not in an event E, denoted $\overline{E}$.	**complemento de un suceso** Todos los resultados en el espacio muestral que no están en el suceso E y se expresan $\overline{E}$.	In the experiment of rolling a number cube, the complement of rolling a 3 is rolling a 1, 2, 4, 5, or 6.
completing the square (p. 342) A process used to form a perfect-square trinomial. To complete the square of $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.	**completar el cuadrado** Proceso utilizado para formar un trinomio cuadrado perfecto. Para completar el cuadrado de $x^2 + bx$, hay que sumar $\left(\frac{b}{2}\right)^2$.	$x^2 + 6x + \blacksquare$ Add $\left(\frac{6}{2}\right)^2 = 9$. $x^2 + 6x + 9$ $(x + 3)^2$ *is a perfect square.*
complex conjugate (p. 352) The complex conjugate of any complex number $a + bi$, denoted $\overline{a + bi}$, is $a - bi$.	**conjugado complejo** El conjugado complejo de cualquier número complejo $a + bi$, expresado como $\overline{a + bi}$, es $a - bi$.	$\overline{4 + 3i} = 4 - 3i$ $\overline{4 - 3i} = 4 + 3i$
complex fraction (p. 586) A fraction that contains one or more fractions in the numerator, the denominator, or both.	**fracción compleja** Fracción que contiene una o más fracciones en el numerador, en el denominador, o en ambos.	$\frac{\frac{1}{2}}{1 + \frac{2}{3}}$
complex number (p. 351) Any number that can be written as $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$.	**número complejo** Todo número que se puede expresar como $a + bi$, donde a y b son números reales e $i = \sqrt{-1}$.	$4 + 2i$ $5 + 0i = 5$ $0 - 7i = -7i$
complex plane (p. 382) A set of coordinate axes in which the horizontal axis is the real axis and the vertical axis is the imaginary axis; used to graph complex numbers.	**plano complejo** Conjunto de ejes cartesianos en el cual el eje horizontal es el eje real y el eje vertical es el eje imaginario; se utiliza para representar gráficamente números complejos.	
composite figure (p. 349) A plane figure made up of triangles, rectangles, trapezoids, circles, and other simple shapes, or a three-dimensional figure made up of prisms, cones, pyramids, cylinders, and other simple three-dimensional figures.	**figura compuesta** Figura plana compuesta por triángulos, rectángulos, trapecios, círculos y otras formas simples, o figura tridimensional compuesta por prismas, conos, pirámides, cilindros y otras figuras tridimensionales simples.	
composition of functions (p. 683) The composition of functions f and g, written as $(f \circ g)(x)$ and defined as $f(g(x))$ uses the output of $g(x)$ as the input for $f(x)$.	**composición de funciones** La composición de las funciones f y g, expresada como $(f \circ g)(x)$ y definida como $f(g(x))$ utiliza la salida de $g(x)$ como la entrada para $f(x)$.	If $f(x) = x^2$ and $g(x) = x + 1$, the composite function $(f \circ g)(x) = (x + 1)^2$.
compound event (p. 819) An event made up of two or more simple events.	**suceso compuesto** Suceso formado por dos o más sucesos simples.	In the experiment of tossing a coin and rolling a number cube, the event of the coin landing heads and the number cube landing on 3.

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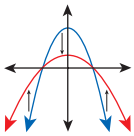
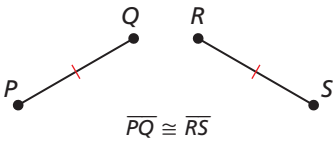
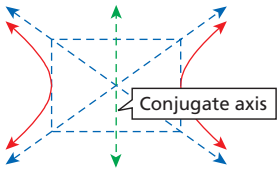
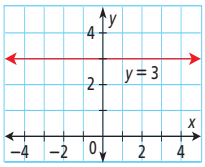
compression (p. 61) A transformation that pushes the points of a graph horizontally toward the y-axis or vertically toward the x-axis.	**compresión** Transformación que desplaza los puntos de una gráfica horizontalmente hacia el eje y o verticalmente hacia el eje x.	
conditional probability (p. 812) The probability of event B, given that event A has already occurred or is certain to occur, denoted $P(B \mid A)$; used to find probability of dependent events.	**probabilidad condicional** Probabilidad del suceso B, dado que el suceso A ya ha ocurrido o es seguro que ocurrirá, expresada como $P(B \mid A)$; se utiliza para calcular la probabilidad de sucesos dependientes.	
congruent (p. 60) Having the same size and shape, denoted by $\cong$.	**congruente** Que tiene el mismo tamaño y forma, expresado por $\cong$.	
conic section (p. 722) A plane figure formed by the intersection of a double right cone and a plane. Examples include circles, ellipses, hyperbolas, and parabolas.	**sección cónica** Figura plana formada por la intersección de un cono regular doble y un plano. Algunos ejemplos son círculos, elipses, hipérbolas y parábolas.	
conjugate axis (p. 744) The axis of symmetry of a hyperbola that separates the two branches of the hyperbola.	**eje conjugado** Eje de simetría de una hipérbola que separa las dos ramas de la hipérbola.	
conjunction (p. 150) A compound statement that uses the word *and*.	**conjunción** Enunciado compuesto que contiene la palabra *y*.	3 is less than 5 AND greater than 0.
consistent system (p. 183) A system of equations or inequalities that has at least one solution.	**sistema consistente** Sistema de ecuaciones o desigualdades que tiene por lo menos una solución.	$\begin{cases} x + y = 6 \\ x - y = 4 \end{cases}$ solution: $(5, 1)$
constant function (p. 67) A function of the form $f(x) = c$, where c is a constant.	**función constante** Función del tipo $f(x) = c$, donde c es una constante.	
constant matrix (p. 279) The matrix of the constants in a linear system of equations.	**matriz de constantes** Matriz de las constantes de un sistema lineal de ecuaciones.	System of equations: $\begin{cases} 2x + 3y = 11 \\ 5x - 4y = 16 \end{cases}$ Constant matrix: $\begin{bmatrix} 11 \\ 16 \end{bmatrix}$
constant of variation (p. 569) The constant k in direct, inverse, joint, and combined variation equations.	**constante de variación** La constante k en ecuaciones de variación directa, inversa, conjunta y combinada.	$y = 5x$ constant of variation

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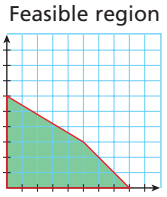
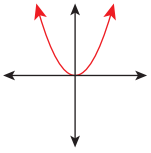
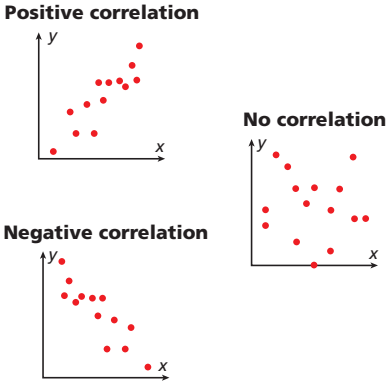
constant term (p. 28, 569) A term in a function or expression that does not contain variables.	**término constante** Término de una función o expresión que no contiene variables.	$f(x) = 3x + 5$ ↑ Constant term
constraint (p. 205) One of the inequalities that define the feasible region in a linear-programming problem.	**restricción** Una de las desigualdades que definen la región factible en un problema de programación lineal.	Constraints: $x > 0$, $y > 0$, $x + y \leq 8$, $3x + 5y \leq 30$ 
continuous data (p. 846) Data that can take on any real-value measurement within an interval.	**datos continuos** Datos obtenidos por medición que pueden asumir cualquier valor real dentro de un intervalo.	The quantity of water in a glass as the water evaporates is continuous data.
continuous function (p. 593) A function whose graph is an unbroken line or curve with no gaps or breaks.	**función continua** Función cuya gráfica es una línea recta o curva continua, sin espacios ni interrupciones.	
contradiction (p. 92) An equation that has no solutions.	**contradicción** Ecuación que no tiene soluciones.	$x + 1 = x$ $1 = 0$ ✗
converge (p. 900) An infinite series converges when the partial sums approach a fixed number.	**convergir** Una sucesión o serie infinita converge cuando las sumas parciales se aproximan a un número fijo.	$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ converges to 1.
conversion factor (p. S57) The ratio of two equal quantities, each measured in different units.	**factor de conversión** Razón entre dos cantidades iguales, cada una medida en unidades diferentes.	$\frac{12 \text{ inches}}{1 \text{ foot}}$
correlation (p. 142) A measure of the strength and direction of the relationship between two variables or data sets.	**correlación** Medida de la fuerza y dirección de la relación entre dos variables o conjuntos de datos.	

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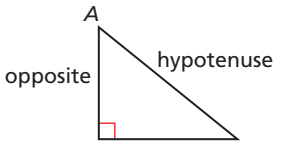
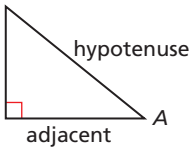
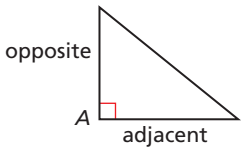
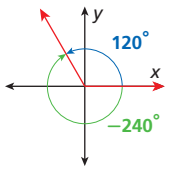
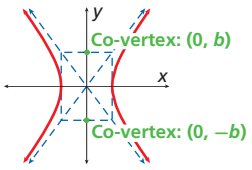
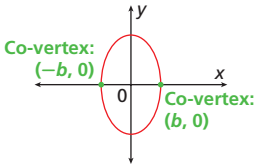
correlation coefficient (p. 143) A number r, where $-1 \le r \le 1$, that describes how closely the points in a scatter plot cluster around the least-squares line.	**coeficiente de correlación** Número r, donde $-1 \le r \le 1$, que describe a qué distancia de la recta de mínimos cuadrados se agrupan los puntos de un diagrama de dispersión.	An r-value close to 1 describes a strong positive correlation. An r-value close to 0 describes a weak correlation or no correlation. An r-value close to -1 describes a strong negative correlation.
cosecant (p. 932) In a right triangle, the cosecant of angle A is the ratio of the length of the hypotenuse to the length of the side opposite A. It is the reciprocal of the sine function.	**cosecante** En un triángulo rectángulo, la cosecante del ángulo A es la razón entre la longitud de la hipotenusa y la longitud del cateto opuesto a A. Es la inversa de la función seno.	 $\csc A = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\sin A}$
cosine (p. 929) In a right triangle, the cosine of angle A is the ratio of the length of the side adjacent to angle A to the length of the hypotenuse. It is the reciprocal of the secant function.	**coseno** En un triángulo rectángulo, el coseno del ángulo A es la razón entre la longitud del cateto adyacente al ángulo A y la longitud de la hipotenusa. Es la inversa de la función secante.	 $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sec A}$
cotangent (p. 932) In a right triangle, the cotangent of angle A is the ratio of the length of the side adjacent to A to the length of the side opposite A. It is the reciprocal of the tangent function.	**cotangente** En un triángulo rectángulo, la cotangente del ángulo A es la razón entre la longitud del cateto adyacente a A y la longitud del cateto opuesto a A. Es la inversa de la función tangente.	 $\cot A = \frac{\text{adjacent}}{\text{opposite}} = \frac{1}{\tan A}$
coterminal angles (p. 937) Two angles in standard position with the same terminal side.	**ángulos coterminales** Dos ángulos en posición estándar con el mismo lado terminal.	
counterexample (p. 903) An example that proves that a conjecture or statement is false.	**contraejemplo** Ejemplo que demuestra que una conjetura o enunciado es falso.	
co-vertices of a hyperbola (p. 744) The endpoints of the conjugate axis.	**co-vértices de una hipérbola** Extremos de un eje conjugado.	
co-vertices of an ellipse (p. 736) The endpoints of the minor axis.	**co-vértices de una elipse** Extremos del eje menor.	

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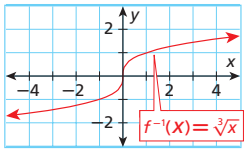
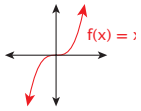
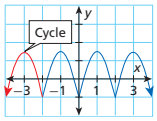
Cramer's rule (p. 271) A method of solving systems of linear equations by using determinants.	**regla de Cramer** Método para resolver sistemas de ecuaciones lineales utilizando determinantes.	For the system $\begin{cases} x - y = 3 \\ 2x - y = -1 \end{cases}$, $D = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = 1(-1) - 2(-1) = 1$; $x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{D} = \frac{\begin{vmatrix} 3 & -1 \\ -1 & -1 \end{vmatrix}}{1} = \frac{-3-1}{1} = -4$; $y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{D} = \frac{\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}}{1} = \frac{-1-6}{1} = -7$
critical values (p. 367) Values that separate the number line into intervals that either contain solutions or do not contain solutions.	**valores críticos** Valores que separan la recta numérica en intervalos que contienen o no contienen soluciones.	
cross products (p. 97) In the statement $\frac{a}{b} = \frac{c}{d}$, bc and ad are the cross products.	**productos cruzados** En el enunciado $\frac{a}{b} = \frac{c}{d}$, bc y ad son los productos cruzados.	$\frac{1}{2} = \frac{3}{6}$ Cross products: $2 \cdot 3 = 6$ and $1 \cdot 6 = 6$
cube-root function (p. 619) The function $f(x) = \sqrt[3]{x}$.	**función de raíz cúbica** La función $f(x) = \sqrt[3]{x}$.	
cubic function (p. 67) A polynomial function of degree 3.	**función cúbica** Función polinomial de grado 3.	
cycle of a periodic function (p. 990) The shortest repeating part of a periodic graph or function.	**ciclo de una función periódica** La parte repetida más corta de una gráfica o función periódica.	

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decay factor (p. 481) The base $1 - r$ in an exponential expression.	**factor decremental** Base $1 - r$ en una expresión exponencial.	$2(0.93)^t$ ↑ decay factor (representing $1 - 0.07$)
degenerate conic (p. 728) A degenerate conic is formed when a plane passes through the vertex of a hollow double cone. A point, a line, and a pair of intersecting lines are all degenerate conics.	**cónica degenerada** Una cónica degenerada se forma cuando un plano atraviesa el vértice de un cono doble hueco. Un punto, una línea y un par de líneas secantes son cónicas degeneradas.	A point is a circle with no radius.
degree of a monomial (p. 406) The sum of the exponents of the variables in the monomial.	**grado de un monomio** Suma de los exponentes de las variables del monomio.	$4x^2y^5z^3$ Degree: $2 + 5 + 3 = 10$ 5 Degree: 0 $(5 = 5x^0)$
degree of a polynomial (p. 406) The degree of the term of the polynomial with the greatest degree.	**grado de un polinomio** Grado del término del polinomio con el grado máximo.	$3x^2y^2 + 4xy^5 - 12x^3y^2$ Degree 6 ↑ ↑ ↑ Degree 4 Degree 6 Degree 5

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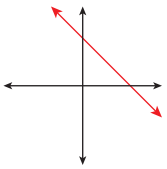
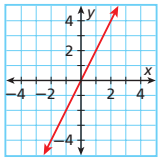
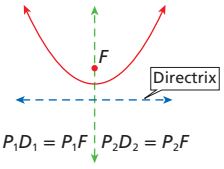
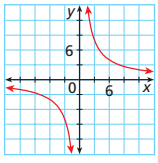
dependent events (p. 812) Events for which the occurrence or nonoccurrence of one event affects the probability of the other event.	**sucesos dependientes** Dos sucesos son dependientes si el hecho de que uno de ellos se cumpla o no afecta la probabilidad del otro.	From a bag containing 3 red marbles and 2 blue marbles, drawing a red marble, and then drawing a blue marble without replacing the first marble.
dependent system (p. 184) A system of equations that has infinitely many solutions.	**sistema dependiente** Sistema de ecuaciones que tiene infinitamente muchas soluciones.	 $\begin{cases} x + y = 3 \\ 2x + 2y = 6 \end{cases}$
dependent variable (p. 52) The output of a function; a variable whose value depends on the value of the input, or independent variable.	**variable dependiente** Salida de una función; variable cuyo valor depende del valor de la entrada, o variable independiente.	$y = 2x + 1$ ↑ dependent variable.
determinant (p. 270) A real number associated with a square matrix. The determinant of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\lvert A \rvert = ad - bc$.	**determinante** Número real asociado con una matriz cuadrada. El determinante de $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ es $\lvert A \rvert = ad - bc$.	$\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 2(4) - (-1)(3) = 11$
difference of two squares (p. 336) A polynomial of the form $a^2 - b^2$, which may be written as the product $(a + b)(a - b)$.	**diferencia de dos cuadrados** Polinomio del tipo $a^2 - b^2$, que se puede expresar como el producto $(a + b)(a - b)$.	$x^2 - 4 = (x + 2)(x - 2)$
dimensions of a matrix (p. 246) A matrix with m rows and n columns has dimensions $m \times n$, read "m by n."	**dimensiones de una matriz** Una matriz con m filas y n columnas tiene dimensiones $m \times n$, expresadas "m por n".	$\begin{bmatrix} -3 & 2 & 1 & -1 \\ 4 & 0 & -5 & 2 \end{bmatrix}$ Dimensions 2×4
direct variation (p. 569) A linear relationship between two variables, x and y, that can be written in the form $y = kx$, where k is a nonzero constant.	**variación directa** Relación lineal entre dos variables, x e y, que puede expresarse en la forma $y = kx$, donde k es una constante distinta de cero.	 $y = 2x$
directrix (p. 751) A fixed line used to define a *parabola*. Every point on the parabola is equidistant from the directrix and a fixed point called the *focus*.	**directriz** Línea fija utilizada para definir una *parábola*. Cada punto de la parábola es equidistante de la directriz y de un punto fijo denominado *foco*.	 $P_1D_1 = P_1F \quad P_2D_2 = P_2F$
discontinuous function (p. 593) A function whose graph has one or more jumps, breaks, or holes.	**función discontinua** Función cuya gráfica tiene uno o más saltos, interrupciones u hoyos.	
discrete data (p. 846) Data that cannot take on any real-value measurement within an interval.	**datos discretos** Datos que no admiten cualquier medida de valores reales dentro de un intervalo.	the number of pennies in a jar over time

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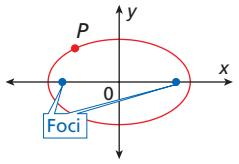
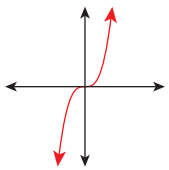
discriminant (p. 357) The discriminant of the quadratic equation $ax^2 + bx + c = 0$ is $b^2 - 4ac$.	**discriminante** El discriminante de la ecuación cuadrática $ax^2 + bx + c = 0$ es $b^2 - 4ac$.	The discriminant of $2x^2 - 5x - 3$ is $(-5)^2 - 4(2)(-3) = 25 + 24 = 49$.
disjunction (p. 150) A compound statement that uses the word *or*.	**disyunción** Enunciado compuesto que contiene la palabra *o*.	John will walk to work OR he will stay home.
Distance Formula (p. 724) In a coordinate plane, the distance from (x_1, y_1) to (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.	**Fórmula de distancia** En un plano cartesiano, la distancia desde (x_1, y_1) hasta (x_2, y_2) es $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.	The distance from $(2, 1)$ to $(6, 4)$ is $d = \sqrt{(6-2)^2 + (4-1)^2}$ $= \sqrt{4^2 + 3^2} = \sqrt{9 + 16} = 5$.
diverge (p. 900) An infinite series diverges when the partial sums do not approach a fixed number.	**divergir** Una serie infinita diverge cuando las sumas parciales no se aproximan a un número fijo.	$1 + 2 + 4 + 8 + 16 + \ldots$ diverges.
domain (p. 44) The set of all possible input values of a relation or function.	**dominio** Conjunto de todos los posibles valores de entrada de una función o relación.	The domain of the function $f(x) = \sqrt{x}$ is $\{x \mid x \geq 0\}$.

E

element of a set (p. 6) An item in a set.	**elemento de un conjunto** Componente de un conjunto.	4 is an element of the set of even numbers. $4 \in \{\text{even numbers}\}$
elimination (p. 191) A method used to solve systems of equations in which one variable is eliminated by adding or subtracting two equations of the system.	**eliminación** Método utilizado para resolver sistemas de ecuaciones por el cual se elimina una variable sumando o restando dos ecuaciones del sistema.	
ellipse (p. 736) The set of all points P in a plane such that the sum of the distances from P to two fixed points F_1 and F_2, called the foci, is constant.	**elipse** Conjunto de todos los puntos P de un plano tal que la suma de las distancias desde P hasta los dos puntos fijos F_1 y F_2, denominados focos, es constante.	
empty set (p. 6, 153) A set with no elements.	**conjunto vacío** Conjunto sin elementos.	The solution set of $\lvert x \rvert < 0$ is the empty set, $\{\ \}$, or $\varnothing$.
end behavior (p. 453) The trends in the y-values of a function as the x-values approach positive and negative infinity.	**comportamiento extremo** Tendencia de los valores de y de una función a medida que los valores de x se aproximan al infinito positivo y negativo.	 End behavior: $f(x) \to \infty$ as $x \to \infty$ $f(x) \to -\infty$ as $x \to -\infty$
entry (p. 246) Each value in a matrix; also called an element.	**entrada** Cada valor de una matriz; también denominado elemento.	3 is the entry in the first row and second column of $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$, denoted a_{12}.

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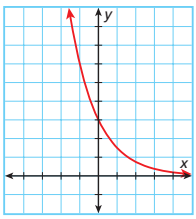
equally likely outcomes (p. 802) Outcomes are equally likely if they have the same probability of occurring. If an experiment has n equally likely outcomes, then the probability of each outcome is $\frac{1}{n}$.	**resultados igualmente probables** Los resultados son igualmente probables si tienen la misma probabilidad de ocurrir. Si un experimento tiene n resultados igualmente probables, entonces la probabilidad de cada resultado es $\frac{1}{n}$.	If a coin is tossed, and heads and tails are equally likely, then $P(\text{heads}) = P(\text{tails}) = \frac{1}{2}$.
equation (p. 90) A mathematical statement that two expressions are equivalent.	**ecuación** Enunciado matemático que indica que dos expresiones son equivalentes.	$x + 4 = 7$ $2 + 3 = 6 - 1$ $(x - 1)^2 + (y + 2)^2 = 4$
evaluate (p. 28) To find the value of an algebraic expression by substituting a number for each variable and simplifying by using the order of operations.	**evaluar** Calcular el valor de una expresión algebraica sustituyendo cada variable por un número y simplificando mediante el orden de las operaciones.	Evaluate $2x + 7$ for $x = 3$. $2x + 7$ $2(3) + 7$ $6 + 7$ 13
event (p. 802) An outcome or set of outcomes in a probability experiment.	**suceso** Resultado o conjunto de resultados en un experimento de probabilidad.	In the experiment of rolling a number cube, the event "an odd number" consists of the outcomes 1, 3, and 5.
expected value (p. 828) The weighted average of the numerical outcomes of a probability experiment.	**valor esperado** Promedio ponderado de los resultados numéricos de un experimento de probabilidad.	The table shows the probability of getting a given score by guessing on a three-question quiz. **Score** | 0 | 1 | 2 | 3 **Probability** | 0.42 | 0.42 | 0.14 | 0.02 The expected value is a score of $0(0.42) + 1(0.42) + 2(0.14) + 3(0.02) = 0.76$.
experiment (p. 805) An operation, process, or activity in which outcomes can be used to estimate probability.	**experimento** Una operación, proceso o actividad cuyo resultado se puede usar para estimar la probabilidad.	Tossing a coin 10 times and noting the number of heads.
experimental probability (p. 805) The ratio of the number of times an event occurs to the number of trials, or times, that an activity is performed.	**probabilidad experimental** Razón entre la cantidad de veces que ocurre un suceso y la cantidad de pruebas, o veces, que se realiza una actividad.	Kendra made 6 of 10 free throws. The experimental probability that she will make her next free throw is $P(\text{free throw}) = \frac{\text{number made}}{\text{number attempted}} = \frac{6}{10}$.
explicit formula (p. 863) A formula that defines the nth term a_n, or general term, of a sequence as a function of n.	**fórmula explícita** Fórmula que define el enésimo término a_n, o término general, de una sucesión como una función de n.	Sequence: 4, 7, 10, 13, 16, 19, ... Explicit formula: $a_n = 1 + 3n$
exponent (p. 34) The number that indicates how many times the base in a power is used as a factor.	**exponente** Número que indica la cantidad de veces que la base de una potencia se utiliza como factor.	$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$ exponent
exponential decay (p. 490) An exponential function of the form $f(x) = ab^x$ in which $0 < b < 1$. If r is the rate of decay, then the function can be written $y = a(1 - r)^t$, where a is the initial amount and t is the time.	**decremento exponencial** Función exponencial del tipo $f(x) = ab^x$ en la cual $0 < b < 1$. Si r es la tasa decremental, entonces la función se puede expresar como $y = a(1 - r)^t$, donde a es la cantidad inicial y t es el tiempo.	 $y = 3\left(\frac{1}{2}\right)^x$

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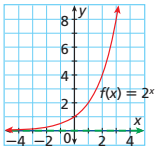
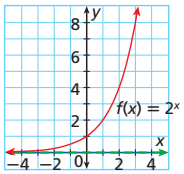
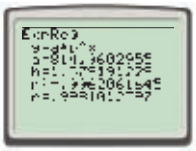
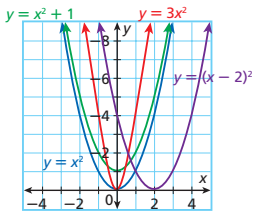
exponential equation (p. 522) An equation that contains one or more exponential expressions.	**ecuación exponencial** Ecuación que contiene una o más expresiones exponenciales.	$2^{x+1} = 8$
exponential function (p. 490) A function of the form $f(x) = ab^x$, where a and b are real numbers with $a \neq 0$, $b > 0$, and $b \neq 1$.	**función exponencial** Función del tipo $f(x) = ab^x$, donde a y b son números reales con $a \neq 0$, $b > 0$ y $b \neq 1$.	
exponential growth (p. 490) An exponential function of the form $f(x) = ab^x$ in which $b > 1$. If r is the rate of growth, then the function can be written $y = a(1 + r)^t$, where a is the initial amount and t is the time.	**crecimiento exponencial** Función exponencial del tipo $f(x) = ab^x$ en la que $b > 1$. Si r es la tasa de crecimiento, entonces la función se puede expresar como $y = a(1 + r)^t$, donde a es la cantidad inicial y t es el tiempo.	
exponential regression (p. 546) A statistical method used to fit an exponential model to a given data set.	**regresión exponencial** Método estadístico utilizado para ajustar un modelo exponencial a un conjunto de datos determinado.	
extraneous solution (p. 524, 600) A solution of a derived equation that is not a solution of the original equation.	**solución extraña** Solución de una ecuación derivada que no es una solución de la ecuación original.	To solve $\sqrt{x} = -2$, square both sides; $x = 4$. ***Check*** $\sqrt{4} = -2$ is false; so 4 is an extraneous solution.

F

Factor Theorem (p. 430) For any polynomial $P(x)$, $(x - a)$ is a factor of $P(x)$ if and only if $P(a) = 0$.	**Teorema del factor** Dado el polinomio $P(x)$, $(x - a)$ es un factor de $P(x)$ si y sólo si $P(a) = 0$.	$(x - 1)$ is a factor of $P(x) = x^2 - 1$ because $P(1) = 1^2 - 1 = 0$.
factorial (p. 795) If n is a positive integer, then n factorial, written $n!$, is $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$. The factorial of 0 is defined to be 1.	**factorial** Si n es un entero positivo, entonces el factorial de n, expresado como $n!$, es $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$. Por definición, el factorial de 0 será 1.	$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$ $0! = 1$
factoring (p. 334) The process of writing a number or algebraic expression as a product.	**factorización** Proceso por el que se expresa un número o expresión algebraica como un producto.	$x^2 - 4x - 21 = (x - 7)(x + 3)$
family of functions (p. 67) A set of functions whose graphs have basic characteristics in common. Functions in the same family are transformations of their parent function.	**familia de funciones** Conjunto de funciones cuyas gráficas tienen características básicas en común. Las funciones de la misma familia son transformaciones de su función madre.	Some members of the family of quadratic functions with the parent function $f(x) = x^2$ are: $f(x) = 3x^2$ $f(x) = x^2 + 1$ $f(x) = (x - 2)^2$ 

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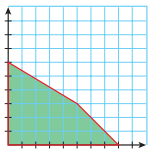
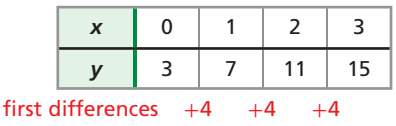
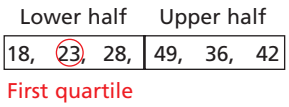
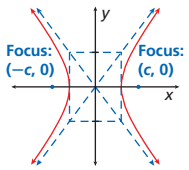
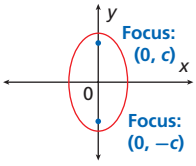
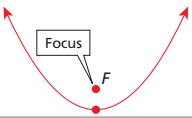
favorable outcome (p. 802) The occurrence of one of several possible outcomes of a specified event or probability experiment.	**resultado favorable** Cuando se produce uno de varios resultados posibles de un suceso específico o experimento de probabilidad.	In the experiment of rolling an odd number on a number cube, the favorable outcomes are 1, 3, and 5.
feasible region (p. 205) The set of points that satisfy the constraints in a linear-programming problem.	**región factible** Conjunto de puntos que cumplen con las restricciones de un problema de programación lineal.	Constraints: $x > 0$, $y > 0$, $x + y \leq 8$, $3x + 5y \leq 30$. Feasible region 
Fibonacci sequence (p. 862) The infinite sequence of numbers beginning with 1, 1 such that each term is the sum of the two previous terms.	**sucesión de Fibonacci** Sucesión infinita de números que comienza con 1, 1 de forma tal que cada término es la suma de los dos términos anteriores.	1, 1, 2, 3, 5, 8, 13, 21, ...
finite sequence (p. 862) A sequence with a finite number of terms.	**sucesión finita** Sucesión con un número finito de términos.	1, 2, 3, 4, 5
finite set (p. 7) A set with a definite, or finite, number of elements.	**conjunto finito** Conjunto con un número de elementos definido o finito.	$\{2, 4, 6, 8, 10\}$
first differences (p. 105) The differences between y-values of a function for evenly spaced x-values.	**primeras diferencias** Diferencias entre los valores de y de una función para valores de x espaciados uniformemente.	
first quartile (p. 829) The median of the lower half of a data set, denoted Q_1. Also called *lower quartile.*	**primer cuartil** Mediana de la mitad inferior de un conjunto de datos, expresada como Q_1. También se llama *cuartil inferior.*	
focus (pl. foci) of a hyperbola (p. 744) One of two fixed points F_1 and F_2 that are used to define a hyperbola. For every point P on the hyperbola, $PF_1 - PF_2$ is constant.	**foco de una hipérbola** Uno de los dos puntos fijos F_1 y F_2 utilizados para definir una hipérbola, Para cada punto P de la hipérbola, $PF_1 - PF_2$ es constante.	
focus (pl. foci) of an ellipse (p. 736) One of two fixed points F_1 and F_2 that are used to define an ellipse. For every point P on the ellipse, $PF_1 + PF_2$ is constant.	**foco de una elipse** Uno de los dos puntos fijos F_1 y F_2 utilizados para definir una elipse. Para cada punto P de la elipse, $PF_1 + PF_2$ es constante.	
focus (pl. foci) of a parabola (p. 751) A fixed point F used with a *directrix* to define a *parabola.*	**foco de una parábola** Punto fijo F utilizado con una *directriz* para definir una *parábola.*	
frequency of a data value (p. 828) The number of times the value appears in the data set.	**frecuencia de un valor de datos** Cantidad de veces que aparece el valor en un conjunto de datos.	In the data set 5, 6, 6, 6, 8, 9, the data value 6 has a frequency of 3.

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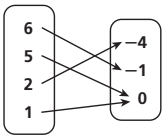
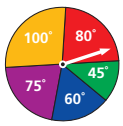
frequency of a periodic function (p. 992) The number of cycles per unit of time. Also the reciprocal of the period.	**frecuencia de una función periódica** Cantidad de ciclos por unidad de tiempo. También es la inversa del periodo.	The function $y = \sin(2x)$ has a period of π and a frequency of $\frac{1}{\pi}$.
function (p. 45) A relation in which every input is paired with exactly one output.	**función** Una relación en la que cada entrada corresponde exactamente a una salida.	
function notation (p. 51) If x is the independent variable and y is the dependent variable, then the function notation for y is $f(x)$, read "f of x," where f names the function.	**notación de función** Si x es la variable independiente e y es la variable dependiente, entonces la notación de función para y es $f(x)$, que se lee "f de x", donde f nombra la función.	equation: $y = 2x$ function notation: $f(x) = 2x$
function rule (p. 53) An algebraic expression that defines a function.	**regla de función** Expresión algebraica que define una función.	$f(x) = 2x^2 + 3x - 7$ function rule
Fundamental Counting Principle (p. 794) For n items, if there are m_1 ways to choose a first item, m_2 ways to choose a second item after the first item has been chosen, and so on, then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to choose n items.	**Principio fundamental de conteo** Dados n elementos, si existen m_1 formas de elegir un primer elemento, m_2 formas de elegir un segundo elemento después de haber elegido el primero, y así sucesivamente, entonces existen $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ formas de elegir n elementos.	If there are 4 colors of shirts, 3 colors of pants, and 2 colors of shoes, then there are $4 \cdot 3 \cdot 2 = 24$ possible outfits.

G

general form of a conic section (p. 761) $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where A and B are not both 0.	**forma general de una sección cónica** $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, donde A y B no son los dos 0.	A circle with a vertex at $(1, 2)$ and radius 3 has the general form $x^2 + y^2 - 2x - 4y - 4 = 0$.
geometric mean (p. 892) In a geometric sequence, a term that comes between two given nonconsecutive terms of the sequence. For positive numbers a and b, the geometric mean is $\sqrt{ab}$.	**media geométrica** En una sucesión geométrica, un término que se encuentra entre dos términos no consecutivos dados de la sucesión. Dados los números positivos a y b, la media geométrica es $\sqrt{ab}$.	The geometric mean of 4 and 9 is $\sqrt{4(9)} = \sqrt{36} = 6$.
geometric probability (p. 804) A form of theoretical probability determined by a ratio of geometric measures such as lengths, areas, or volumes.	**probabilidad geométrica** Una forma de la probabilidad teórica determinada por una razón de medidas geométricas, como longitud, área o volumen.	 The probability of the pointer landing on red is $\frac{2}{9}$.

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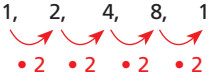
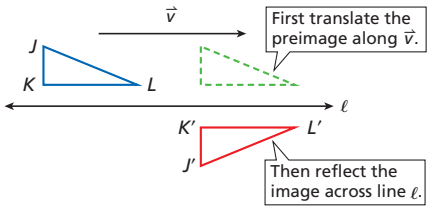
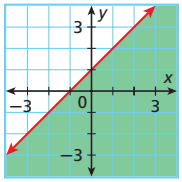
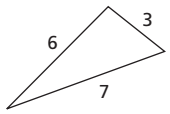
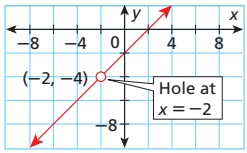
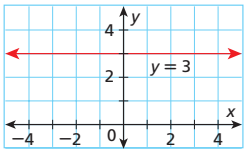
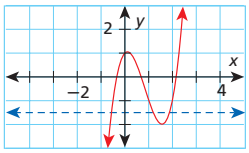
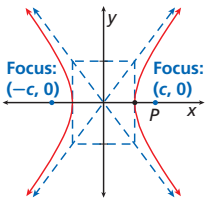
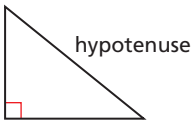
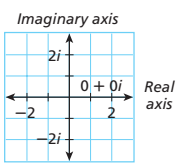
geometric sequence (p. 890) A sequence in which the ratio of successive terms is a constant r, called the common ratio, where $r \neq 0$ and $r \neq 1$.	**sucesión geométrica** Sucesión en la que la razón de los términos sucesivos es una constante r, denominada razón común, donde $r \neq 0$ y $r \neq 1$.	1, 2, 4, 8, 16, ...  $r = 2$
geometric series (p. 893) The indicated sum of the terms of a geometric sequence.	**serie geométrica** Suma indicada de los términos de una sucesión geométrica.	$1 + 2 + 4 + 8 + 16 + \ldots$
glide reflection (p. 261) A composition of a translation and a reflection across a line parallel to the translation vector.	**deslizamiento con inversión** Composición de una traslación y una reflexión sobre una línea paralela al vector de traslación.	
grade (p. 102) A measure of the steepness of surfaces, expressed as a percent.	**grado** Medida de la inclinación de las superficies, expresada como un porcentaje.	A ramp that rises 1 foot for every 5 feet of the horizontal distance has a grade of 20%.
greatest common factor (GCF) (p. 331) The product of the greatest integer and the greatest power of each variable that divides evenly into each term.	**máximo común divisor (MCD)** Producto del entero mayor y la potencia mayor de cada variable que divide exactamente cada término.	The GCF of $4x^3y$ and $6x^2y$ is $2x^2y$. The GCF of 27 and 45 is 9.
greatest-integer function (p. 669) A function denoted by $f(x) = [x]$ or $f(x) = \lfloor x \rfloor$ in which the number x is rounded down to the greatest integer that is less than or equal to x.	**función de entero mayor** Función expresada como $f(x) = [x]$ o $f(x) = \lfloor x \rfloor$ en la cual el número x se redondea hacia abajo hasta el entero mayor que sea menor que o igual a x.	$\lfloor 4.98 \rfloor = 4$ $\lfloor -2.1 \rfloor = -3$
growth factor (p. 491) The base $1 + r$ in an exponential expression.	**factor de crecimiento** La base $1 + r$ en una expresión exponencial.	$12{,}000(1 + 0.14)^t$ ↑ growth factor

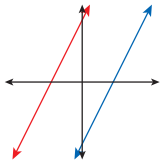
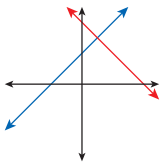
H

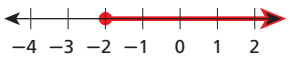
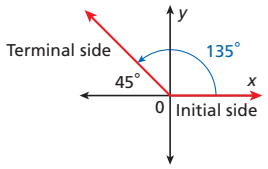
half-life (p. 532) The half-life of a substance is the time it takes for one-half of the substance to decay into another substance.	**vida media** La vida media de una sustancia es el tiempo que tarda la mitad de la sustancia en desintegrarse y transformarse en otra sustancia.	Carbon-14 has a half-life of 5730 years, so 5 g of an initial amount of 10 g will remain after 5730 years.
half-plane (p. 124) The part of the coordinate plane on one side of a line, which may include the line.	**semiplano** Parte del plano cartesiano de un lado de una línea, que puede incluir la línea.	

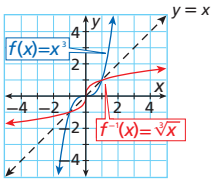
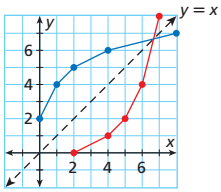
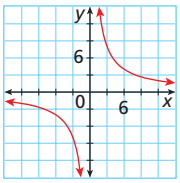
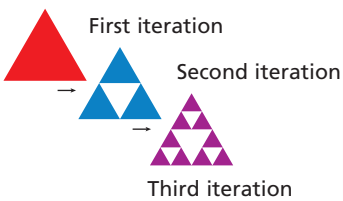
ENGLISH	SPANISH	EXAMPLES
Heron's Formula (p. 969) A triangle with side lengths a, b, and c has area $A = \sqrt{s(s-a)(s-b)(s-c)}$, where s is one-half the perimeter, or $s = \frac{1}{2}(a+b+c)$.	**fórmula de Herón** Un triángulo con longitudes de lado a, b y c tiene un área $A = \sqrt{s(s-a)(s-b)(s-c)}$, donde s es la mitad del perímetro ó $s = \frac{1}{2}(a+b+c)$.	 $s = \frac{1}{2}(3+6+7) = 8$ $A = \sqrt{8(8-3)(8-6)(8-7)}$ $= \sqrt{80} = 4\sqrt{5}$ square units
hole (in a graph) (p. 596) An omitted point on a graph. If a rational function has the same factor $x - b$ in both the numerator and the denominator, and the line $x = b$ is not a vertical asymptote, then there is a hole in the graph at the point where $x = b$.	**hoyo (en una gráfica)** Punto omitido en una gráfica. Si una función racional tiene el mismo factor $x - b$ tanto en el numerador como en el denominador, y la línea $x = b$ no es una asíntota vertical, entonces hay un hoyo en la gráfica en el punto donde $x = b$.	$f(x) = \frac{(x-2)(x+2)}{(x+2)}$ has a hole at $x = -2$. 
horizontal line (p. 108) A line described by the equation $y = b$, where b is the y-intercept.	**línea horizontal** Línea descrita por la ecuación $y = b$, donde b es la intersección con el eje y.	
horizontal line test (p. 690) If a horizontal line crosses the graph of a function f at more than one point, then the inverse is not a function.	**prueba de la línea horizontal** Si una línea horizontal cruza la gráfica de una función f en más de un punto, entonces la inversa no es una función.	 The inverse is not a function.
hyperbola (p. 744) The set of all points P in a plane such that the difference of the distances from P to two fixed points F_1 and F_2, called the foci, is a constant $d = \lvert PF_1 - PF_2 \rvert$.	**hipérbola** Conjunto de todos los puntos P en un plano tal que la diferencia de las distancias de P a dos puntos fijos F_1 y F_2, llamados focos, es una constante $d = \lvert PF_1 - PF_2 \rvert$.	
hypotenuse (p. 20) The side opposite the right angle in a right triangle.	**hipotenusa** Lado opuesto al ángulo recto de un triángulo rectángulo.	

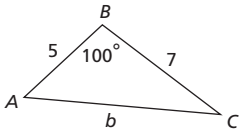
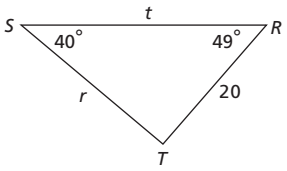
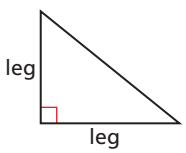
I

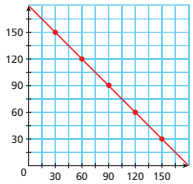
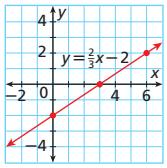
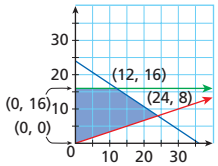
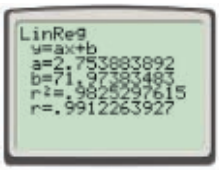
identity (p. 92) An equation that is true for all values of the variables.	**identidad** Ecuación verdadera para todos los valores de las variables.	$3 = 3$ $2(x-1) = 2x - 2$
imaginary axis (p. 382) The vertical axis in the complex plane, it graphically represents the purely imaginary part of complex numbers.	**eje imaginario** Eje vertical de un plano complejo. Representa gráficamente la parte puramente imaginaria de los números complejos.	

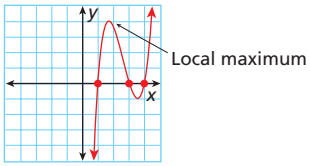
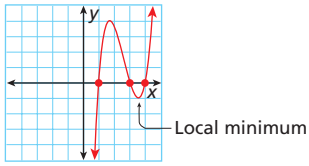
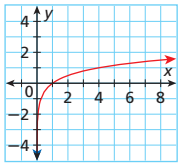
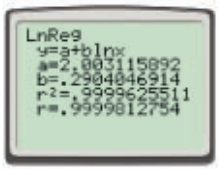
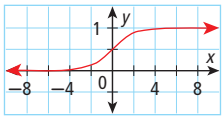
ENGLISH	SPANISH	EXAMPLES
imaginary number (p. 350) The square root of a negative number, written in the form bi, where b is a real number and i is the imaginary unit, $\sqrt{-1}$. Also called a *pure imaginary number*.	**número imaginario** Raíz cuadrada de un número negativo, expresado como bi, donde b es un número real e i es la unidad imaginaria, $\sqrt{-1}$. También se denomina *número imaginario puro*.	$\sqrt{-16} = \sqrt{16} \cdot \sqrt{-1} = 4i$
imaginary part of a complex number (p. 351) For a complex number of the form $a + bi$, the real number b is called the imaginary part, represented graphically as b units on the imaginary axis of a complex plane.	**parte imaginaria de un número complejo** Dado un número complejo del tipo $a + bi$, el número real b se denomina parte imaginaria y se representa gráficamente como b unidades en el eje imaginario de un plano complejo.	$5 + 6i$ real part — imaginary part
imaginary unit (p. 350) The unit in the imaginary number system, $\sqrt{-1}$.	**unidad imaginaria** Unidad del sistema de números imaginarios, $\sqrt{-1}$.	$\sqrt{-1} = i$
inclusive events (p. 820) Events that have one or more outcomes in common.	**sucesos inclusivos** Sucesos que tienen uno o más resultados en común.	In the experiment of rolling a number cube, rolling an even number and rolling a number less than 3 are inclusive events because the outcome 2 is both even and less than 3.
inconsistent system (p. 183) A system of equations or inequalities that has no solution.	**sistema inconsistente** Sistema de ecuaciones o desigualdades que no tiene solución.	 $\begin{cases} y = 2.5x + 5 \\ y = 2.5x - 5 \end{cases}$ is inconsistent.
independent events (p. 811) Events for which the occurrence or non-occurrence of one event does not affect the probability of the other event.	**sucesos independientes** Dos sucesos son independientes si el hecho de que se produzca o no uno de ellos no afecta la probabilidad del otro suceso.	From a bag containing 3 red marbles and 2 blue marbles, drawing a red marble, replacing it, and then drawing a blue marble.
independent system (p. 184) A system of equations that has exactly one solution.	**sistema independiente** Sistema de ecuaciones que tiene exactamente una solución.	 $\begin{cases} y = -x + 4 \\ y = x + 2 \end{cases}$ Solution: $(1, 3)$
independent variable (p. 52) The input of a function; a variable whose value determines the value of the output, or dependent variable.	**variable independiente** Entrada de una función; variable cuyo valor determina el valor de la salida, o variable dependiente.	$y = 2x + 1$ independent variable
index (p. 610) In the radical $\sqrt[n]{x}$, which represents the nth root of x, n is the index. In the radical $\sqrt{x}$, the index is understood to be 2.	**índice** En el radical $\sqrt[n]{x}$, que representa la enésima raíz de x, n es el índice. En el radical $\sqrt{x}$, se da por sentado que el índice es 2.	The radical $\sqrt[3]{8}$ has an index of 3.

ENGLISH	SPANISH	EXAMPLES
indirect measurement (p. 99) A method of measurement that uses formulas, similar figures, and/or proportions.	**medición indirecta** Método para medir objetos mediante fórmulas, figuras semejantes y/o proporciones.	
inequality (p. 92) A statement that compares two expressions by using one of the following signs: $<$, $>$, $\leq$, $\geq$, or $\neq$.	**desigualdad** Enunciado que compara dos expresiones utilizando uno de los siguientes signos: $<$, $>$, $\leq$, $\geq$, ó $\neq$.	 $x \geq -2$
infinite geometric series (p. 900) A geometric series with infinitely many terms.	**serie geométrica infinita** Serie geométrica con una cantidad infinita de términos.	$\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10{,}000} + \ldots$
infinite sequence (p. 862) A sequence with infinitely many terms.	**sucesión infinita** Sucesión con infinitos términos.	1, 3, 5, 7, 9, 11, ...
infinite set (p. 7) A set with an unlimited, or infinite, number of elements.	**conjunto infinito** Conjunto con un número de elementos ilimitado o infinito.	The set of all integers is an infinite set.
initial side (p. 936) The ray that lies on the positive x-axis when an angle is drawn in standard position.	**lado inicial** El rayo que se encuentra en el eje positivo x cuando se traza un ángulo en la posición estándar.	
integer (p. 6) A member of the set of whole numbers and their opposites.	**entero** Miembro del conjunto de números cabales y sus opuestos.	... −3, −2, −1, 0, 1, 2, 3 ...
interquartile range (IQR) (p. 829) The difference of the third (upper) and first (lower) quartiles in a data set, representing the middle half of the data.	**rango entre cuartiles** Diferencia entre el tercer cuartil (superior) y el primer cuartil (inferior) de un conjunto de datos, que representa la mitad central de los datos.	Lower half / Upper half 18, 23, 28, 29, 36, 42 First quartile / Third quartile Interquartile range: 36 − 23 = 13.
interval notation (p. 7) A way of writing the set of all real numbers between two endpoints. The symbols [and] are used to include an endpoint in an interval, and the symbols (and) are used to exclude an endpoint from an interval.	**notación de intervalo** Forma de expresar el conjunto de todos los números reales entre dos extremos. Los símbolos [y] se utilizan para incluir un extremo en un intervalo y los símbolos (y) se utilizan para excluir un extremo de un intervalo.	Interval notation / Set-builder notation (a, b) $\{x \mid a < x < b\}$ $(a, b]$ $\{x \mid a < x \leq b\}$ $[a, b)$ $\{x \mid a \leq x < b\}$ $[a, b]$ $\{x \mid a \leq x \leq b\}$
inverse cosine function (p. 951) If the domain of the cosine function is restricted to $[0, \pi]$, then the function $\text{Cos}\,\theta = a$ has an inverse function $\text{Cos}^{-1} a = \theta$, also called *arccosine*.	**función coseno inverso** Si el dominio de la función coseno se restringe a $[0, \pi]$, entonces la función $\text{Cos}\,\theta = a$ tiene una función inversa $\text{Cos}^{-1} a = \theta$, también llamada *arco coseno*.	$\text{Cos}^{-1}\frac{1}{2} = \frac{\pi}{3}$

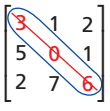
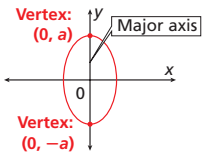
ENGLISH	SPANISH	EXAMPLES
inverse function (p. 499) The function that results from exchanging the input and output values of a one-to-one function. The inverse of $f(x)$ is denoted $f^{-1}(x)$.	**función inversa** Función que resulta de intercambiar los valores de entrada y salida de una función uno a uno. La función inversa de $f(x)$ se expresa $f^{-1}(x)$.	
inverse relation (p. 498) The inverse of the relation consisting of all ordered pairs (x, y) is the set of all ordered pairs (y, x). The graph of an inverse relation is the reflection of the graph of the relation across the line $y = x$.	**relación inversa** La inversa de la relación que consta de todos los pares ordenados (x, y) es el conjunto de todos los pares ordenados (y, x). La gráfica de una relación inversa es el reflejo de la gráfica de la relación sobre la línea $y = x$.	
inverse sine function (p. 951) If the domain of the sine function is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then the function $\text{Sin}\,\theta = a$ has an inverse function, $\text{Sin}^{-1}\,a = \theta$, also called *arcsine*.	**función seno inverso** Si el dominio de la función seno se restringe a $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, entonces la función $\text{Sen}\,\theta = a$ tiene una función inversa, $\text{Sen}^{-1}\,a = \theta$, también llamada *arco seno*.	$\text{Sin}^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$
inverse tangent function (p. 951) If the domain of the tangent function is restricted to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then the function $\text{Tan}\,\theta = a$ has an inverse function, $\text{Tan}^{-1}\,a = \theta$, also called *arctangent*.	**función tangente inversa** Si el dominio de la función tangente se restringe a $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, entonces la función $\text{Tan}\,\theta = a$ tiene una función inversa, $\text{Tan}^{-1}\,a = \theta$, también llamada *arco tangente*.	$\text{Tan}^{-1}\sqrt{3} = \frac{\pi}{3}$
inverse variation (p. 570) A relationship between two variables, x and y, that can be written in the form $y = \frac{k}{x}$, where k is a nonzero constant and $x \neq 0$.	**variación inversa** Relación entre dos variables, x e y, que puede expresarse en la forma $y = \frac{k}{x}$, donde k es una constante distinta de cero y $x \neq 0$.	 $y = \frac{24}{x}$
irrational number (p. 6) A real number that cannot be expressed as the ratio of two integers.	**número irracional** Número real que no se puede expresar como una razón de enteros.	$\sqrt{2}$, π, e
iteration (p. 864) The repetitive application of the same rule.	**iteración** Aplicación repetitiva de la misma regla.	

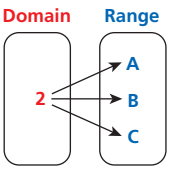
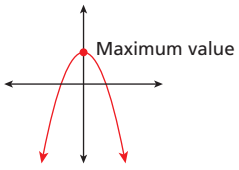
ENGLISH	SPANISH	EXAMPLES
J		
joint variation (p. 570) A relationship among three variables that can be written in the form $y = kxz$, where k is a nonzero constant.	**variación conjunta** Relación entre tres variables que se puede expresar de la forma $y = kxz$, donde k es una constante distinta de cero.	$y = 3xz$
L		
Law of Cosines (p. 966) For $\triangle ABC$ with side lengths a, b, and c, $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$.	**Ley de cosenos** Dado $\triangle ABC$ con longitudes de lado a, b y c, $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$.	 $b^2 = 7^2 + 5^2 - 2(7)(5) \cos 100°$ $b^2 \approx 86.2$ $b \approx 9.3$
law of large numbers (p. 809) The tendency of experimental probability to approach theoretical probability as the number of trials gets very large.	**Ley de los números grandes** Tendencia de la probabilidad experimental a acercarse a la probabilidad teórica cuando el número de pruebas es muy grande.	The more times you toss a coin, the closer the experimental probability will be to $\frac{1}{2}$.
Law of Sines (p. 959) For $\triangle ABC$ with side lengths a, b, and c, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.	**Ley de senos** Dado $\triangle ABC$ con longitudes de lado a, b y c, $\frac{\text{sen } A}{a} = \frac{\text{sen } B}{b} = \frac{\text{sen } C}{c}$.	 $\frac{\sin 49°}{r} = \frac{\sin 40°}{20}$ $r = \frac{20 \sin 49°}{\sin 40°} \approx 23.5$
leading coefficient (p. 406) The coefficient of the first term of a polynomial in standard form.	**coeficiente principal** Coeficiente del primer término de un polinomio en forma estándar.	$3x^2 + 7x - 2$ Leading coefficient
least common denominator (LCD) (p. 583) The least common multiple of two or more given denominators.	**mínimo común denominador (mcd)** Mínimo común múltiplo de dos o más denominadores dados.	The LCD of $\frac{3}{4}$ and $\frac{5}{6}$ is 12.
least common multiple (LCM) (p. 583) The product of the smallest positive number and the lowest power of each variable that divides evenly into each term.	**mínimo común múltiplo (mcm)** El producto del número positivo más pequeño y la potencia más baja de cada variable que divide exactamente cada término.	The LCM of 10 and 18 is 90. The LCM of $2x^2$ *and* $5x^3$ *is* $10x^3$.
leg of a right triangle (p. 20) One of the two sides of the right triangle that form the right angle.	**cateto de un triángulo rectángulo** Uno de los dos lados de un triángulo rectángulo que forman el ángulo recto.	

ENGLISH	SPANISH	EXAMPLES
like radical terms (p. 23) Radical terms having the same radicand and index.	**radicales semejantes** Términos radicales que tienen el mismo radicando e índice.	$3\sqrt{2x}$ and $\sqrt{2x}$ Like radicals $\sqrt{3x}$ and $\sqrt{2x}$ Unlike radicals
like terms (p. 28) Terms with the same variables raised to the same exponents.	**términos semejantes** Términos con las mismas variables elevadas a los mismos exponentes.	$3a^3b^2$ and $7a^3b^2$ Like terms $4xy^2$ and $6x^2y$ Unlike terms
limit (p. 900) A number (or infinity) that the terms of an infinite sequence or series approach as the term number increases.	**límite** Número (o infinito) al que se aproximan los términos de una sucesión o serie infinita a medida que aumenta el número de términos.	The series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ has a limit of 1.
line of best fit (p. 142) The line that comes closest to all of the points in a data set.	**línea de mejor ajuste** Línea que más se acerca a todos los puntos de un conjunto de datos.	
linear equation in one variable (p. 90) An equation that can be written in the form $ax = b$, where a and b are constants and $a \neq 0$.	**ecuación lineal en una variable** Ecuación que puede expresarse en la forma $ax = b$, donde a y b son constantes y $a \neq 0$.	$x + 1 = 7$
linear function (p. 105) A function that can be written in the form $f(x) = mx + b$, where x is the independent variable and m and b are real numbers. Its graph is a line.	**función lineal** Función que puede expresarse en la forma $f(x) = mx + b$, donde x es la variable independiente y m y b son números reales. Su gráfica es una línea.	
linear inequality in two variables (p. 124) An inequality that can be written in one of the following forms: $y < mx + b$, $y > mx + b$, $y \leq mx + b$, $y \geq mx + b$, or $y \neq mx + b$, where m and b are real numbers.	**desigualdad lineal en dos variables** Desiqualdad que puede expresarse de una de las siguientes formas: $y < mx + b$, $y > mx + b$, $y \leq mx + b$, $y \geq mx + b$, o $y \neq mx + b$, donde m y b son números reales.	$2x + 3y \leq 6$ $y > \frac{1}{2}x - 7$
linear programming (p. 205) A method of finding a maximum or minimum value of a linear function, called the *objective function*, that satisfies a given set of conditions, called *constraints*.	**programación lineal** Método para calcular un valor máximo o mínimo de una función lineal, denominada *función objetiva*, que cumple con una serie dada de condiciones, denominadas *restricciones*.	Constraints $x \geq 0$ $40x + 60y \leq 1440$ $y \geq \frac{1}{3}x$ $y \leq 16$ Feasible Region  For the given constraints, the objective function $P = 18x + 25y$ is maximized at $(24, 8)$.
linear regression (p. 143) A statistical method used to fit a linear model to a given data set.	**regresión lineal** Método estadístico utilizado para ajustar un modelo lineal a un conjunto de datos determinado.	

ENGLISH	SPANISH	EXAMPLES
linear system (p. 182) A system of equations containing only linear equations.	**sistema lineal** Sistema de ecuaciones que contiene sólo ecuaciones lineales.	$\begin{cases} y = 2x + 1 \\ x + y = 8 \end{cases}$
local maximum (p. 455) For a function f, $f(a)$ is a local maximum if there is an interval around a such that $f(x) < f(a)$ for every x-value in the interval except a.	**máximo local** Dada una función f, $f(a)$ es el máximo local si hay un intervalo en a tal que $f(x) < f(a)$ para cada valor de x en el intervalo excepto a.	
local minimum (p. 455) For a function f, $f(a)$ is a local minimum if there is an interval around a such that $f(x) > f(a)$ for every x-value in the interval except a.	**mínimo local** Dada una función f, $f(a)$ es el mínimo local si hay un intervalo en a tal que $f(x) > f(a)$ para cada valor de x en el intervalo excepto a.	
logarithm (p. 505) The exponent that a specified base must be raised to in order to get a certain value.	**logaritmo** Exponente al cual debe elevarse una base determinada a fin de obtener cierto valor.	$\log_2 8 = 3$, because 3 is the power that 2 is raised to in order to get 8; or $2^3 = 8$.
logarithmic equation (p. 523) An equation that contains a logarithm of a variable.	**ecuación logarítmica** Ecuación que contiene un logaritmo de una variable.	$\log x + 3 = 7$
logarithmic function (p. 507) A function of the form $f(x) = \log_b x$, where $b \neq 1$ and $b > 0$, which is the inverse of the exponential function $f(x) = b^x$.	**función logarítmica** Función del tipo $f(x) = \log_b x$, donde $b \neq 1$ y $b > 0$, que es la inversa de la función exponencial $f(x) = b^x$.	 $f(x) = \log_4 x$
logarithmic regression (p. 546) A statistical method used to fit a logarithmic model to a given data set.	**regresión logarítmica** Método estadístico utilizado para ajustar un modelo logarítmico a un conjunto de datos determinado.	
logistic function (p. 535) An exponential growth function that tapers off at an asymptote.	**función logística** Función de crecimiento exponencial que disminuye en una asíntota.	

M

main diagonal (of a matrix) (p. 255) The diagonal from the upper left corner to the lower right corner of a matrix.	**diagonal principal (de una matriz)** Diagonal que se extiende desde la esquina superior izquierda hasta la esquina inferior derecha de una matriz.	
major axis (p. 736) The longer axis of an ellipse. The foci of the ellipse are located on the major axis, and its endpoints are the *vertices of the ellipse*.	**eje mayor** El eje más largo de una elipse. Los focos de la elipse se encuentran sobre el eje mayor y sus extremos son los *vértices de la elipse*.	

ENGLISH	SPANISH	EXAMPLES
mapping diagram (p. 44) A diagram that shows the relationship of elements in the domain to elements in the range of a relation or function.	**diagrama de correspondencia** Diagrama que muestra la relación entre los elementos del dominio y los elementos del rango de una función.	
mathematical induction (p. 902) A type of mathematical proof. To prove that a statement is true for all natural numbers n, first show that the statement is true for $n = 1$; then assume it is true for some number k and prove that it is true for $k + 1$. It follows that the statement is true for all values of n.	**inducción matemática** Tipo de demostración matemática. Para demostrar que un enunciado se cumple para todos los números naturales n, primero se demuestra que el enunciado se cumple para $n = 1$; luego se supone que se cumple para un número k y se demuestra que se cumple para $k + 1$. Por lo tanto, el enunciado se cumplirá para todos los valores de n.	
matrix (p. 246) A rectangular array of numbers.	**matriz** Arreglo rectangular de números.	$\begin{bmatrix} 1 & 0 & 3 \\ -2 & 2 & -5 \\ 7 & -6 & 3 \end{bmatrix}$
matrix equation (p. 279) An equation of the form $AX = B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix of a system of equations.	**ecuación matricial** Ecuación del tipo $AX = B$, donde A es la matriz de coeficientes, X es la matriz de variables y B es la matriz de constantes de un sistema de ecuaciones.	System of equations: $2x + 3y = 7$, $4x - 6y = 5$ Matrix equation: $\begin{bmatrix} 2 & 3 \\ 4 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$
matrix product (p. 253) The product of two matrices, where each entry in P_{ij} is the sum of the products of consecutive entries in row i in matrix A and column j in matrix B.	**producto matricial** Producto de dos matrices, donde cada entrada de P_{ij} es la suma de los productos de las entradas consecutivas de la fila i de la matriz A y de la columna j de la matriz B.	$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1(5) + 2(7) & 1(6) + 2(8) \\ 3(5) + 4(7) & 3(6) + 4(8) \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$
maximum value of a function (p. 326) The y-value of the highest point on the graph of the function.	**máximo de una función** Valor de y del punto más alto en la gráfica de la función.	
mean (p. 828) The sum of all the values in a data set divided by the number of data values. Also called the *average*.	**media** Suma de todos los valores de un conjunto de datos dividida entre el número de valores de datos. También llamada *promedio*.	Data set: 4, 6, 7, 8, 10 Mean: $\frac{4 + 6 + 7 + 8 + 10}{5} = \frac{35}{5} = 7$
measure of central tendency (p. 828) A measure that describes the center of a data set.	**medida de tendencia dominante** Medida que describe el centro de un conjunto de datos.	the mean, median, or mode
measure of variation (p. 830) A measure that describes the spread of a data set.	**medida de variación** Medida que describe la amplitud de un conjunto de datos.	the range, variance, standard deviation, or interquartile range

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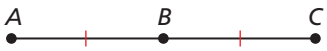
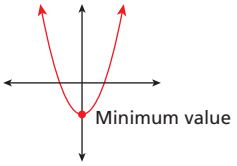
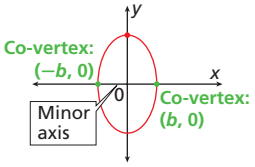
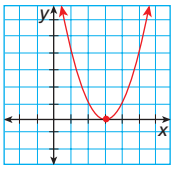
median of a data set (p. 828) For an ordered data set with an odd number of values, the median is the middle value. For an ordered data set with an even number of values, the median is the average of the two middle values.	**mediana de un conjunto de datos** Dado un conjunto de datos ordenados con un número impar de valores, la mediana es el valor del medio. Dado un conjunto de datos ordenados con un número par de valores, la mediana es el promedio de los dos valores del medio.	8, 9, 9, 12, 15 Median: 9 4, 6, 7, 10, 10, 12 Median: $\frac{7 + 10}{2} = 8.5$
midpoint (p. 724) The point that divides a segment into two congruent segments.	**punto medio** Punto que divide un segmento en dos segmentos congruentes.	 Point B is the midpoint of $\overline{AC}$.
minimum value of a function (p. 326) The y-value of the lowest point on the graph of the function.	**mínimo de una función** Valor de y del punto más bajo en la gráfica de la función.	
minor axis (p. 736) The shorter axis of an ellipse. Its endpoints are the *co-vertices of the ellipse*.	**eje menor** El eje más corto de una elipse. Sus extremos son los *co-vértices de la elipse*.	
mode (p. 828) The value or values that occur most frequently in a data set; if all values occur with the same frequency, the data set is said to have no mode.	**moda** El valor o los valores que se presentan con mayor frecuencia en un conjunto de datos. Si todos los valores se presentan con la misma frecuencia, se dice que el conjunto de datos no tiene moda.	Data set: 3, 6, 8, 8, 10 Mode: 8 Data set: 2, 5, 5, 7, 7 Modes: 5 and 7 Data set: 2, 3, 6, 9, 11 No mode
monomial (p. 406) A number or a product of numbers and variables with whole-number exponents, or a polynomial with one term.	**monomio** Número o producto de números y variables con exponentes de números cabales, o polinomio con un término.	$8x$, 9, $3x^2y^4$
multiple root (p. 439) A root r is a multiple root when the factor $(x - r)$ appears in the equation more than once.	**raíz múltiple** Una raíz r es una raíz múltiple cuando el factor $(x - r)$ aparece en la ecuación más de una vez.	 3 is a multiple root of $P(x) = (x - 3)^2$.
multiplicative identity matrix (p. 255) A square matrix with 1 in every entry of the main diagonal and 0 in every other entry.	**matriz de identidad multiplicativa** Una matriz cuadrada que contiene 1 en cada entrada de la diagonal principal y 0 en las demás entradas.	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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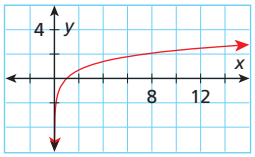
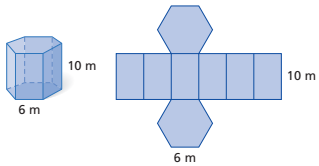
multiplicative inverse of a square matrix (p. 278) The multiplicative inverse of square matrix A, if it exists, is notated A^{-1}, where the product of A and A^{-1} is the identity matrix.	**inverso multiplicativo de una matriz cuadrada** El inverso multiplicativo de una matriz cuadrada A, si existe, se escribe A^{-1}, donde el producto de A y A^{-1} es la matriz de identidad.	The multiplicative inverse of $A = \begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} -3 & -5 \\ -1 & -2 \end{bmatrix}$, because $AA^{-1} = A^{-1}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
multiplicity (p. 439) If a polynomial $P(x)$ has a multiple root at r, the multiplicity of r is the number of times $(x - r)$ appears as a factor in $P(x)$.	**multiplicidad** Si un polinomio $P(x)$ tiene una raíz múltiple en r, la multiplicidad de r es la cantidad de veces que $(x - r)$ aparece como factor en $P(x)$.	For $P(x) = (x - 3)^2$, the root 3 has a multiplicity of 2.
mutually exclusive events (p. 819) Two events are mutually exclusive if they cannot both occur in the same trial of an experiment.	**sucesos mutuamente excluyentes** Dos sucesos son mutuamente excluyentes si ambos no pueden ocurrir en la misma prueba de un experimento.	In the experiment of rolling a number cube, rolling a 3 and rolling an even number are mutually exclusive events.
natural logarithm (p. 532) A logarithm with base e, written as ln.	**logaritmo natural** Logaritmo con base e, que se escribe ln.	$\ln 5 = \log_e 5 \approx 1.6$
natural logarithmic function (p. 532) The function $f(x) = \ln x$, which is the inverse of the natural exponential function $f(x) = e^x$. Domain is $\{x \mid x > 0\}$; range is all real numbers.	**función logarítmica natural** Función $f(x) = \ln x$, que es la inversa de la función exponencial natural $f(x) = e^x$. El dominio es $\{x \mid x > 0\}$; el rango es todos los números reales.	
natural number (p. 6) A counting number.	**número natural** Número que sirve para contar.	1, 2, 3, 4, 5, 6, …
negative exponent (p. 35) A base raised to a negative exponent is equal to the reciprocal of that base raised to the opposite exponent: $b^{-n} = \frac{1}{b^n}$.	**exponente negativo** Una base elevada a un exponente negativo es igual al recíproco de dicha base elevado al exponente opuesto: $b^{-n} = \frac{1}{b^n}$.	$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$
net (p. S65) A diagram of the faces of a three-dimensional figure arranged in such a way that the diagram can be folded to form the three-dimensional figure.	**plantilla** Diagrama de las caras de una figura tridimensional que se puede plegar para formar la figura tridimensional.	
nonlinear system of equations (p. 768) A system in which at least one of the equations is not linear.	**sistema no lineal de ecuaciones** Sistema en el cual por lo menos una de las ecuaciones no es lineal.	$\begin{cases} y = 2x^2 \\ y = -3x^2 + 5 \end{cases}$

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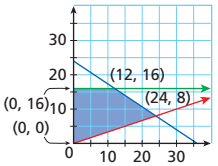
nth root (p. 610) The nth root of a number a, written as $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$, is a number that is equal to a when it is raised to the nth power.	**enésima raíz** La enésima raíz de un número a, que se escribe como $\sqrt[n]{a}$ o $a^{\frac{1}{n}}$, es un número igual a a cuando se eleva a la enésima potencia.	$\sqrt[5]{32} = 2$, because $2^5 = 32$.

O

objective function (p. 206) The function to be maximized or minimized in a linear programming problem.

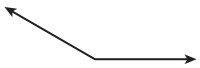
función objetiva Función que se debe maximizar o minimizar en un problema de programación lineal.



The objective function $P = 18x + 25y$ is maximized at $(24, 8)$.

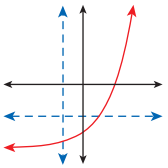
obtuse angle (p. 198) An angle that measures greater than 90° and less than 180°.

ángulo obtuso Ángulo que mide más de 90° y menos de 180°.



one-to-one function (p. 691) A function in which each y-value corresponds to only one x-value. The inverse of a one-to-one function is also a function.

función uno a uno Función en la que cada valor de y corresponde a sólo un valor de x. La inversa de una función uno a uno es también una función.



opposite (p. 14) The opposite of a number a, denoted $-a$, is the number that is the same distance from zero as a, on the opposite side of the number line. The sum of opposites is 0.

opuesto El opuesto de un número a, expresado $-a$, es el número que se encuentra a la misma distancia de cero que a, del lado opuesto de la recta numérica. La suma de los opuestos es 0.

5 and -5 are opposites.

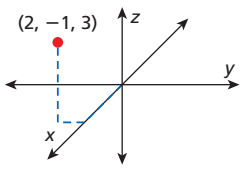
order of operations (p. 28) A process for evaluating expressions:
First, perform operations in parentheses or other grouping symbols.
Second, evaluate powers and roots.
Third, perform all multiplication and division from left to right.
Fourth, perform all addition and subtraction from left to right.

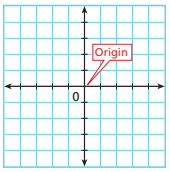
orden de las operaciones
Proceso para evaluar las expresiones:
Primero, realizar las operaciones entre paréntesis u otros símbolos de agrupación.
Segundo, evaluar las potencias y las raíces.
Tercero, realizar todas las multiplicaciones y divisiones de izquierda a derecha.
Cuarto, realizar todas las sumas y restas de izquierda a derecha.

$2 + 3^2 - (7 + 5) \div 4 \cdot 3$	
$2 + 3^2 - 12 \div 4 \cdot 3$	Add inside parentheses.
$2 + 9 - 12 \div 4 \cdot 3$	Evaluate the power.
$2 + 9 - 3 \cdot 3$	Divide.
$2 + 9 - 9$	Multiply.
$11 - 9$	Add.
2	Subtract.

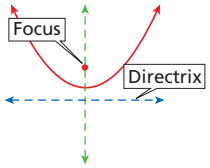
ordered triple (p. 214) A set of three numbers that can be used to locate a point (x, y, z) in a three-dimensional coordinate system.

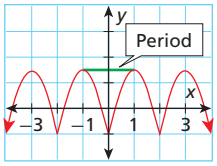
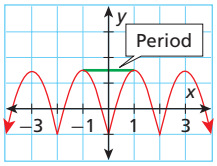
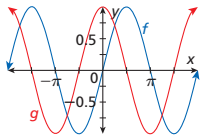
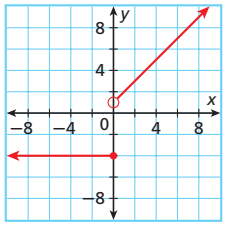
tripleta ordenada Conjunto de tres números que se pueden utilizar para ubicar un punto (x, y, z) en un sistema de coordenadas tridimensional.

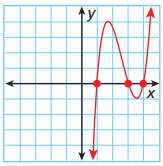


ENGLISH	SPANISH	EXAMPLES
origin (p. 3) The intersection of the x- and y-axes in a coordinate plane. The coordinates of the origin are $(0, 0)$.	**origen** Intersección de los ejes x e y en un plano cartesiano. Las coordenadas de origen son $(0, 0)$.	
outcome (p. 802) A possible result of a probability experiment.	**resultado** Resultado posible en un experimento de probabilidad.	In the experiment of rolling a number cube, the possible outcomes are 1, 2, 3, 4, 5, and 6.
outlier (p. 831) A data value that is far removed from the rest of the data. A value less than $Q_1 - 1.5(\text{IQR})$ or greater than $Q_3 + 1.5(\text{IQR})$ is considered to be an outlier.	**valor extremo** Valor de datos que está muy alejado del resto de los datos. Un valor menor que $Q_1 - 1.5(\text{IQR})$ o mayor que $Q_3 + 1.5(\text{IQR})$ se considera un valor extremo.	

P

parabola (p. 315) The shape of the graph of a quadratic function. Also, the set of points equidistant from a point F, called the *focus*, and a line d, called the *directrix*.	**parábola** Forma de la gráfica de una función cuadrática. También, conjunto de puntos equidistantes de un punto F, denominado *foco*, y una línea d, denominada *directriz*.	
parameter (p. 230) One of the constants in a function or equation that may be changed. Also the third variable in a set of parametric equations.	**parámetro** Una de las constantes en una función o ecuación que se puede cambiar. También es la tercera variable en un conjunto de ecuaciones paramétricas.	$y = (x - h)^2 + k$ parameters
parametric equations (p. 230) A pair of equations that define the x- and y-coordinates of a point in terms of a third variable called a parameter.	**ecuaciones paramétricas** Par de ecuaciones que definen las coordenadas x e y de un punto en función de una tercera variable denominada parámetro.	$x(t) = t + 1$ $y(t) = -2t$
parent function (p. 67) The simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent function.	**función madre** La función más básica con las características de la familia. Las funciones de la misma familia son transformaciones de su función madre.	$f(x) = x^2$ is the parent function for $g(x) = x^2 + 4$ and $h(x) = 5(x + 2)^2 - 3$.
partial sum (p. 870) Indicated by $S_n = \sum_{i=1}^{n} a_i$, the sum of a specified number of terms n of a sequence whose total number of terms is greater than n.	**suma parcial** Expresada por $S_n = \sum_{i=1}^{n} a_i$, la suma de un número específico n de términos de una sucesión cuyo número total de términos es mayor que n.	For the sequence $a_n = n^2$, the fourth partial sum of the infinite series $\sum_{k=1}^{\infty} k^2$ is $\sum_{k=1}^{4} k^2 = 1^2 + 2^2 + 3^2 + 4^2 = 30$.

ENGLISH	SPANISH	EXAMPLES
Pascal's triangle (p. 416) A triangular arrangement of numbers in which every row starts and ends with 1 and each other number is the sum of the two numbers above it.	**triángulo de Pascal** Arreglo triangular de números en el cual cada fila comienza y termina con 1 y cada uno de los demás números es la suma de los dos números que están encima de él.	1 1 1 1 2 1 1 3 3 1 1 4 6 4 1
perfect square (p. 21) A number whose positive square root is a whole number.	**cuadrado perfecto** Número cuya raíz cuadrada positiva es un número cabal.	36 is a perfect square because $\sqrt{36} = 6$.
perfect-square trinomial (p. 336) A trinomial whose factored form is the square of a binomial. A perfect-square trinomial has the form $a^2 - 2ab + b^2 = (a - b)^2$ or $a^2 + 2ab + b^2 = (a + b)^2$.	**trinomio cuadrado perfecto** Trinomio cuya forma factorizada es el cuadrado de un binomio. Un trinomio cuadrado perfecto tiene la forma $a^2 - 2ab + b^2 = (a - b)^2$ o $a^2 + 2ab + b^2 = (a + b)^2$.	$x^2 + 6x + 9$ is a perfect-square trinomial, because $x^2 + 6x + 9 = (x + 3)^2$.
period of a periodic function (p. 990) The length of a cycle measured in units of the independent variable (usually time in seconds). Also the reciprocal of the frequency.	**periodo de una función periódica** Longitud de un ciclo medido en unidades de la variable independiente (generalmente el tiempo en segundos). También es la inversa de la frecuencia.	
periodic function (p. 990) A function that repeats exactly in regular intervals, called *periods*.	**función periódica** Función que se repite exactamente a intervalos regulares denominados *periodos*.	
permutation (p. 795) An arrangement of a group of objects in which order is important. The number of permutations of r objects from a group of n objects is denoted ${}_nP_r$.	**permutación** Arreglo de un grupo de objetos en el cual el orden es importante. El número de permutaciones de r objetos de un grupo de n objetos se expresa ${}_nP_r$.	For 4 objects *A, B, C,* and *D*, there are ${}_4P_2 = 12$ different permutations of 2 objects: *AB, AC, AD, BC, BD, CD, BA, CA, DA, CB, DB,* and *DC*.
phase shift (p. 993) A horizontal translation of a periodic function.	**cambio de fase** Traslación horizontal de una función periódica.	 g is a phase shift of f $\frac{\pi}{2}$ units left.
piecewise function (p. 662) A function that is a combination of one or more functions.	**función a trozos** Función que es una combinación de una o más funciones.	 $f(x) = \begin{cases} -4 & \text{if } x \le 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

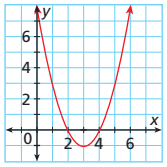
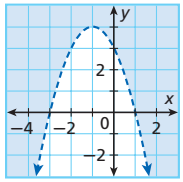
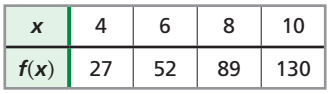
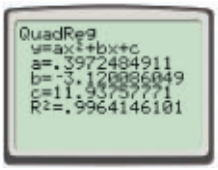
ENGLISH	SPANISH	EXAMPLES
point-slope form (p. 116) The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.	**forma de punto y pendiente** La forma de punto y pendiente de una ecuación lineal es $y - y_1 = m(x - x_1)$, donde m es la pendiente y (x_1, y_1) es un punto en la línea.	The equation of the line through $(2, 1)$ with slope 3 is $y - 1 = 3(x - 2)$.
polynomial (p. 406) A monomial or a sum or difference of monomials.	**polinomio** Monomio o suma o diferencia de monomios.	$2x^2 + 3x - 7$
polynomial function (p. 408) A function whose rule is a polynomial.	**función polinomial** Función cuya regla es un polinomio.	 $f(x) = x^3 - 8x^2 + 19x - 12$
power (p. 34) An expression written with a base and an exponent or the value of such an expression.	**potencia** Expresión escrita con una base y un exponente o el valor de dicha expresión.	$2^3 = 8$, so 8 is the third power of 2.
principal root (p. 21) The positive root of a number, indicated by the radical sign.	**raíz principal** Raíz cuadrada positiva de un número, expresada por el signo de radical.	$\sqrt{36} = 6$
probability (p. 802) A number from 0 to 1 (or 0% to 100%) that is the measure of how likely an event is to occur.	**probabilidad** Número entre 0 y 1 (o entre 0% y 100%) que describe cuán probable es que ocurra un suceso.	A bag contains 3 red marbles and 4 blue marbles. The probability of choosing a red marble is $\frac{3}{7}$.
probability distribution for an experiment (p. 828) The function that pairs each outcome with its probability.	**distribución de probabilidad para un experimento** Función que asigna a cada resultado su probabilidad.	A number cube is rolled 10 times. The results are shown in the table.

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	0	$\frac{3}{10}$	$\frac{1}{5}$

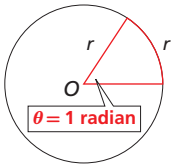
proportion (p. 97) A statement that two ratios are equal; $\frac{a}{b} = \frac{c}{d}$.	**proporción** Enunciado que establece que dos razones son iguales; $\frac{a}{b} = \frac{c}{d}$.	$\frac{2}{3} = \frac{4}{6}$
pure imaginary number (p. 351) *See* imaginary number.	**número imaginario puro** *Ver* número imaginario.	$3i$

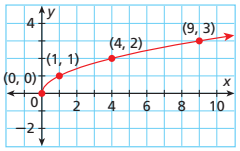
Q

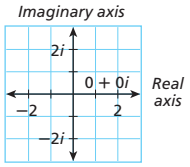
quadratic equation (p. 334) An equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.	**ecuación cuadrática** Ecuación que se puede expresar como $ax^2 + bx + c = 0$, donde a, b y c son números reales y $a \neq 0$.	$x^2 + 3x - 4 = 0$ $x^2 - 9 = 0$

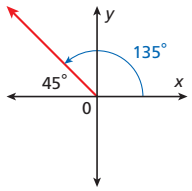
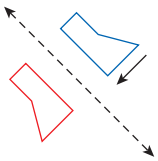
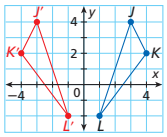
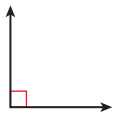
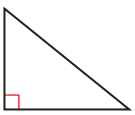
ENGLISH	SPANISH	EXAMPLES
Quadratic Formula (p. 356) The formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, which gives solutions, or roots, of equations in the form $ax^2 + bx + c = 0$, where $a \neq 0$.	**fórmula cuadrática** La fórmula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, que da soluciones, o raíces, para las ecuaciones del tipo $ax^2 + bx + c = 0$, donde $a \neq 0$.	The solutions of $2x^2 - 5x - 3 = 0$ are given by $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$ $= \frac{5 \pm \sqrt{25 + 24}}{4} = \frac{5 \pm 7}{4}$; $x = 3$ or $x = -\frac{1}{2}$.
quadratic function (p. 315) A function that can be written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$, or in the form $f(x) = a(x - h)^2 + k$, where a, h, and k are real numbers and $a \neq 0$.	**función cuadrática** Función que se puede expresar como $f(x) = ax^2 + bx + c$, donde a, b y c son números reales y $a \neq 0$, o como $f(x) = a(x - h)^2 + k$, donde a, h y k son números reales y $a \neq 0$.	 $f(x) = x^2 - 6x + 8$
quadratic inequality in two variables (p. 366) An inequality that can be written in one of the following forms: $y < ax^2 + bx + c$, $y > ax^2 + bx + c$, $y \leq ax^2 + bx + c$, $y \geq ax^2 + bx + c$, or $y \neq ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.	**desigualdad cuadrática en dos variables** Desigualdad que puede expresarse de una de las siguientes formas: $y < ax^2 + bx + c$, $y > ax^2 + bx + c$, $y \leq ax^2 + bx + c$, $y \geq ax^2 + bx + c$, o $y \neq ax^2 + bx + c$, donde a, b y c son números reales y $a \neq 0$.	 $y > -x^2 - 2x + 3$
quadratic model (p. 376) A quadratic function used to represent a set of data.	**modelo cuadrático** Función cuadrática que se utiliza para representar un conjunto de datos.	 A quadratic model for the data is $f(x) = x^2 + 3.3x - 2.6$.
quadratic regression (p. 376) A statistical method used to fit a quadratic model to a given data set.	**regresión cuadrática** Método estadístico utilizado para ajustar un modelo cuadrático a un conjunto de datos determinado.	

R

radian (p. 943) A unit of angle measure based on arc length. In a circle of radius r, if a central angle has a measure of 1 radian, then the length of the intercepted arc is r units. 2π radians $= 360°$ 1 radian $\approx 57°$	**radián** Unidad de medida de un ángulo basada en la longitud del arco. En un círculo de radio r, si un ángulo central mide 1 radián, entonces la longitud del arco abarcado es r unidades. 2π radianes $= 360°$ 1 radián $\approx 57°$	
radical (p. 21) An indicated root of a quantity.	**radical** Raíz indicada de una cantidad.	$\sqrt{36} = 6$, $\sqrt[3]{27} = 3$

ENGLISH	SPANISH	EXAMPLES
radical equation (p. 628) An equation that contains a variable within a radical.	**ecuación radical** Ecuación que contiene una variable dentro de un radical.	$\sqrt{x+3}+4=7$
radical function (p. 619) A function whose rule contains a variable within a radical.	**función radical** Función cuya regla contiene una variable dentro de un radical.	 $f(x)=\sqrt{x}$
radical inequality (p. 630) An inequality that contains a variable within a radical.	**desigualdad radical** Desigualdad que contiene una variable dentro de un radical.	$\sqrt{x+3}\le 7$
radical symbol (p. 21) The symbol $\sqrt{\ }$ used to denote a root. The symbol is used alone to indicate a square root or with an index, $\sqrt[n]{\ }$, to indicate the nth root.	**símbolo de radical** Símbolo $\sqrt{\ }$ que se utiliza para expresar una raíz. Puede utilizarse solo para indicar una raíz cuadrada, o con un índice, $\sqrt[n]{\ }$, para indicar la enésima raíz.	$\sqrt{36}=6,\ \sqrt[3]{27}=3$
radicand (p. 21) The expression under a radical sign.	**radicando** Número o expresión debajo del signo de radical.	$\sqrt{x+3}-2$ Radicand
random sample (p. S69) A sample selected from a population so that each member of the population has an equal chance of being selected.	**muestra aleatoria** Muestra seleccionada de una población tal que cada miembro de ésta tenga igual probabilidad de ser seleccionado.	Mr. Hansen chose a random sample of the class by writing each student's name on a slip of paper, mixing up the slips, and drawing five slips without looking.
range of a data set (p. 830) The difference of the greatest and least values in the data set.	**rango de un conjunto de datos** La diferencia del mayor y menor valor en un conjunto de datos.	The data set $\{3, 3, 5, 7, 8, 10, 11, 11, 12\}$ has a range of $12-3=9$.
range of a function or relation (p. 44) The set of output values of a function or relation.	**rango de una función o relación** Conjunto de los valores de salida de una función o relación.	The range of $y=x^2$ is $\{y \mid y \ge 0\}$.
rate (p. 98) A ratio that compares two quantities measured in different units.	**tasa** Razón que compara dos cantidades medidas en diferentes unidades.	$\frac{55 \text{ miles}}{1 \text{ hour}}=55$ mi/h
ratio (p. 97) A comparison of two quantities by division.	**razón** Comparación de dos cantidades mediante una división.	$\frac{1}{2}$ or 1:2
rational equation (p. 600) An equation that contains one or more rational expressions.	**ecuación racional** Ecuación que contiene una o más expresiones racionales.	$\frac{x+2}{x^2+3x-1}=6$
rational exponent (p. 611) An exponent that can be expressed as $\frac{m}{n}$ such that if m and n are integers, then $b^{\frac{m}{n}}=\sqrt[n]{b^m}=\left(\sqrt[n]{b}\right)^m$.	**exponente racional** Exponente que se puede expresar como $\frac{m}{n}$ tal que, si m y n son números enteros, entonces $b^{\frac{m}{n}}=\sqrt[n]{b^m}=\left(\sqrt[n]{b}\right)^m$.	$4^{\frac{3}{2}}=\sqrt{4^3}=\sqrt{64}=8$ $4^{\frac{3}{2}}=\left(\sqrt{4}\right)^3=2^3=8$

ENGLISH	SPANISH	EXAMPLES
rational expression (p. 577) An algebraic expression whose numerator and denominator are polynomials and whose denominator has a degree ≥ 1.	**expresión racional** Expresión algebraica cuyo numerador y denominador son polinomios y cuyo denominador tiene un grado ≥ 1.	$\frac{x+2}{x^2+3x-1}$
rational function (p. 592) A function whose rule can be written as a rational expression.	**función racional** Función cuya regla se puede expresar como una expresión racional.	$f(x) = \frac{x+2}{x^2+3x-1}$
rational inequality (p. 603) An inequality that contains one or more rational expressions.	**desigualdad racional** Desigualdad que contiene una o más expresiones racionales.	$\frac{x+2}{x^2+3x-1} \geq 6$
rational number (p. 6) A number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.	**número racional** Número que se puede expresar como $\frac{a}{b}$, donde a y b son números enteros y $b \neq 0$.	$3,\ 1.75,\ 0.\overline{3},\ -\frac{2}{3},\ 0$
rationalizing the denominator (p. 22) A method of rewriting a fraction by multiplying by another fraction that is equivalent to 1 in order to remove radical terms from the denominator.	**racionalizar el denominador** Método que consiste en escribir nuevamente una fracción multiplicándola por otra fracción equivalente a 1 a fin de eliminar los términos radicales del denominador.	$\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$
real axis (p. 382) The horizontal axis in the complex plane; it graphically represents the real part of complex numbers.	**eje real** Eje horizontal de un plano complejo. Representa gráficamente la parte real de los números complejos.	
real number (p. 6) A rational or irrational number. Every point on the number line represents a real number.	**número real** Número racional o irracional. Cada punto de la recta numérica representa un número real.	$-5,\ 0,\ \frac{2}{3},\ \sqrt{2},\ 3.1,\ \pi$
real part of a complex number (p. 351) For a complex number of the form $a + bi$, a is the real part.	**parte real de un número complejo** Dado un número complejo del tipo $a + bi$, a es la parte real.	$5 + 6i$ Real part — Imaginary part
reciprocal (p. 14) For a real number $a \neq 0$, the reciprocal of a is $\frac{1}{a}$. The product of reciprocals is 1.	**recíproco** Dado el número real $a \neq 0$, el recíproco de a es $\frac{1}{a}$. El producto de los recíprocos es 1.	$\frac{1}{2}$ is the reciprocal of 2. $\frac{5}{3}$ is the reciprocal of $\frac{3}{5}$.
recursive formula (p. 862) A formula for a sequence in which one or more previous terms are used to generate the next term.	**fórmula recurrente** Fórmula para una sucesión en la cual uno o más términos anteriores se utilizan para generar el término siguiente.	For the sequence 5, 7, 9, 11, …, a recursive formula is $a_1 = 5$ and $a_n = a_{n-1} + 2$.
reduced row-echelon form (p. 288) A form of an augmented matrix in which the coefficient columns form an identity matrix.	**forma escalonada reducida por filas** Forma de matriz aumentada en la que las columnas de coeficientes forman una matriz de identidad.	$\left[\begin{array}{cc\|c} 1 & 0 & -1 \\ 0 & 1 & 3 \end{array}\right]$

ENGLISH	SPANISH	EXAMPLES
reference angle (p. 937) For an angle in standard position, the reference angle is the positive acute angle formed by the terminal side of the angle and the x-axis.	**ángulo de referencia** Dado un ángulo en posición estándar, el ángulo de referencia es el ángulo agudo positivo formado por el lado terminal del ángulo y el eje x.	
reflection (p. 60) A transformation that reflects, or "flips," a graph or figure across a line, called the line of reflection, such that each reflected point is the same distance from the line of reflection but is on the opposite side of the line.	**reflexión** Transformación que refleja, o invierte, una gráfica o figura sobre una línea, llamada la línea de reflexión, de manera tal que cada punto reflejado esté a la misma distancia de la línea de reflexión pero que se encuentre en el lado opuesto de la línea.	
reflection matrix (p. 263) A matrix used to reflect a figure across a specified *line of symmetry*.	**matriz de reflexión** Matriz utilizada para reflejar una figura sobre un *eje de simetría* específico.	 Matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ was used to reflect the figure across the y-axis.
regression (p. 142) The statistical study of the relationship between variables.	**regresión** Estudio estadístico de la relación entre variables.	
relation (p. 44) A set of ordered pairs.	**relación** Conjunto de pares ordenados.	$\{(0, 5), (0, 4), (2, 3), (4, 0)\}$
replacement set (p. 55) A set of numbers that can be substituted for a variable.	**conjunto de reemplazo** Conjunto de números que pueden sustituir una variable.	The solution set of $y = x + 3$ for the replacement set $\{1, 2, 3\}$ is $\{4, 5, 6\}$.
right angle (p. 960) An angle that measures 90°.	**ángulo recto** Ángulo que mide 90°.	
right triangle (p. 20) A triangle with one right angle.	**triángulo rectángulo** Triángulo con un ángulo recto.	
rigid transformation (p. 261) A transformation that does not change the size or shape of a figure.	**transformación rígida** Transformación que no cambia el tamaño o la forma de una figura.	Reflections, rotations, and translations are rigid transformations.
root of an equation (p. 334) Any value of the variable that makes the equation true.	**raíz de una ecuación** Cualquier valor de la variable que transforme la ecuación en verdadera.	The roots of $(x - 2)(x + 1) = 0$ are 2 and -1.
roster notation (p. 7) A way of representing a set by listing the elements between braces, { }.	**notación de lista** Forma de representar un conjunto enumerando los elementos entre llaves, { }.	The first 5 positive odd numbers are $\{1, 3, 5, 7, 9\}$.

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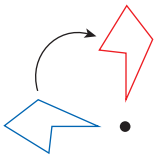
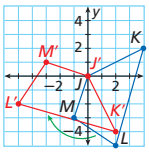
rotation (p. 261) A transformation that rotates or turns a figure about a point called the center of rotation.	**rotación** Transformación que hace rotar o girar una figura sobre un punto llamado centro de rotación.	
rotation matrix (p. 264) A matrix used to rotate a figure about the origin.	**matriz de rotación** Matriz utilizada para rotar una figura sobre el origen.	 Matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ was used to rotate the figure 90° clockwise.
row operation (p. 288) An operation performed on a row of an augmented matrix that creates an equivalent matrix.	**operación por filas** Operación realizada en una fila de una matriz aumentada que crea una matriz equivalente.	$\left[\begin{array}{cc\|c} 2 & 0 & -2 \\ 0 & 1 & 3 \end{array}\right] = \left[\begin{array}{cc\|c} \frac{1}{2}(2) & \frac{1}{2}(0) & \frac{1}{2}(-1) \\ 0 & 1 & 3 \end{array}\right]$ $= \left[\begin{array}{cc\|c} 1 & 0 & -1 \\ 0 & 1 & 3 \end{array}\right]$
row-reduction method (p. 288) The process of performing elementary row operations on an augmented matrix to transform the matrix to reduced row echelon form.	**método de reducción por filas** Proceso por el cual se realizan operaciones elementales de filas en una matriz aumentada para transformar la matriz en una forma reducida de filas escalonadas.	$\left[\begin{array}{cc\|c} 2 & 0 & -2 \\ 0 & 1 & 3 \end{array}\right] = \left[\begin{array}{cc\|c} \frac{1}{2}(2) & \frac{1}{2}(0) & \frac{1}{2}(-1) \\ 0 & 1 & 3 \end{array}\right]$ $= \left[\begin{array}{cc\|c} 1 & 0 & -1 \\ 0 & 1 & 3 \end{array}\right]$

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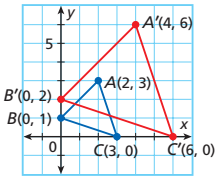
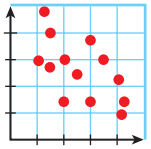
sample space (p. 802) The set of all possible outcomes of a probability experiment.	**espacio muestral** Conjunto de todos los resultados posibles en un experimento de probabilidades.	In the experiment of rolling a number cube, the sample space is $\{1, 2, 3, 4, 5, 6\}$.
scalar (p. 248) A number that is multiplied by a matrix.	**escalar** Número que se multiplica por una matriz.	$3\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & 9 \end{bmatrix}$ scalar
scale factor (p. 99) The multiplier used on each dimension to change one figure into a similar figure.	**factor de escala** El multiplicador utilizado en cada dimensión para transformar una figura en una figura semejante.	
scatter plot (p. 142) A graph with points plotted to show a possible relationship between two sets of data.	**diagrama de dispersión** Gráfica con puntos que se usa para demostrar una relación posible entre dos conjuntos de datos.	

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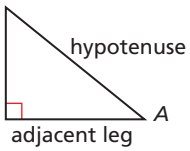
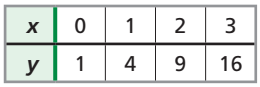
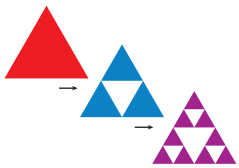
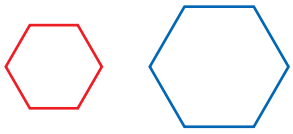
scientific notation (p. 36) A method of writing very large or very small numbers, by using powers of 10, in the form $m \times 10^n$, where $1 \leq m < 10$ and n is an integer.	**notación científica** Método que consiste en escribir números muy grandes o muy pequeños utilizando potencias de 10 del tipo $m \times 10^n$, donde $1 \leq m < 10$ y n es un número entero.	$1.256\ 10^{13} = 12{,}560{,}000{,}000$ $7.5 \times 10^{-6} = 0.0000075$
secant of an angle (p. 932) In a right triangle, the ratio of the length of the hypotenuse to the length of the side adjacent to angle A. It is the reciprocal of the cosine function.	**secante de un ángulo** En un triángulo rectángulo, la razón entre la longitud de la hipotenusa y la longitud del cateto adyacente al ángulo A. Es la inversa de la función coseno.	 $\sec A = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{1}{\cos A}$
second differences (p. 374) Differences between first differences of a function.	**segundas diferencias** Diferencias entre las primeras diferencias de una función.	 first differences +3 +5 +7 second differences +2 +2
sequence (p. 862) A list of numbers that often form a pattern.	**sucesión** Lista de números que generalmente forman un patrón.	1, 2, 4, 8, 16, …
series (p. 870) The indicated sum of the terms of a sequence.	**serie** Suma indicada de los términos de una sucesión.	$1 + 2 + 4 + 8 + 16 + \ldots$
set (p. 6) A collection of items called elements.	**conjunto** Grupo de componentes denominados elementos.	$\{1, 2, 3\}$
set-builder notation (p. 8) A notation for a set that uses a rule to describe the properties of the elements of the set.	**notación de conjuntos** Notación para un conjunto que se vale de una regla para describir las propiedades de los elementos del conjunto.	$\{x \mid x > 3\}$ is read, "The set of all x such that x is greater than 3."
Sierpinski triangle (p. 864) A fractal formed from a triangle by removing triangles with vertices at the midpoints of the sides of each remaining triangle.	**triángulo de Sierpinski** Fractal formado a partir de un triángulo al cual se le recortan triángulos cuyos vértices se encuentran en los puntos medios de los lados de cada triángulo restante.	
similar (p. 99) Two figures are similar if they have the same shape but not necessarily the same size.	**semejantes** Dos figuras son semejantes si tienen la misma forma pero no necesariamente el mismo tamaño.	
simple event (p. 819) An event consisting of only one outcome.	**suceso simple** Suceso que contiene sólo un resultado.	In the experiment of rolling a number cube, the event consisting of the outcome 3 is a simple event.
simplify (p. 28) To perform all indicated operations.	**simplificar** Realizar todas las operaciones indicadas.	$3(4) + 7$ $12 + 7$ 19

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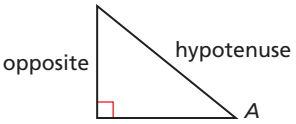
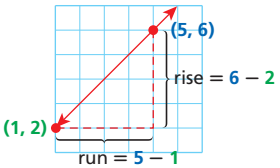
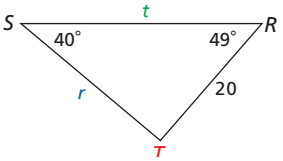
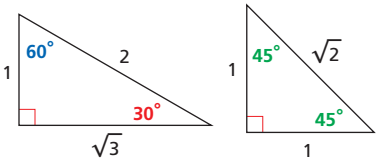
simulation (p. 810) A model of an experiment, often one that would be too difficult or time-consuming to actually perform.	**simulación** Modelo de un experimento; generalmente se recurre a la simulación cuando realizar dicho experimento sería demasiado difícil o llevaría mucho tiempo.	A random number generator is used to simulate the roll of a number cube.
sine (p. 929) In a right triangle, the ratio of the length of the side opposite $\angle A$ to the length of the hypotenuse.	**seno** En un triángulo rectángulo, razón entre la longitud del cateto opuesto a $\angle A$ y la longitud de la hipotenusa.	 $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$.
slope (p. 106) A measure of the steepness of a line. If (x_1, y_1) and (x_2, y_2) are any two points on the line, the slope of the line, known as m, is represented by the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$.	**pendiente** Medida de la inclinación de una línea. Dados dos puntos (x_1, y_1) y (x_2, y_2) en una línea, la pendiente de la línea, denominada m, se representa con la ecuación $m = \frac{y_2 - y_1}{x_2 - x_1}$.	 $m = \frac{4}{4} = 1$
slope-intercept form (p. 107) The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.	**forma de pendiente-intersección** La forma de pendiente-intersección de una ecuación lineal es $y = mx + b$, donde m es la pendiente y b es la intersección y.	$y = -2x + 4$ slope y-intercept
solution set of an equation (p. 90) The set of values that make an equation true.	**conjunto solución de una ecuación** Conjunto de valores que hacen verdadero un enunciado.	The solution set of $x^2 = 9$ is $\{-3, 3\}$.
solving a triangle (p. 959) Using given measures to find unknown angle measures or side lengths of a triangle.	**resolución de un triángulo** Utilizar medidas dadas para hallar las medidas desconocidas de los ángulos o las longitudes de los lados de un triángulo.	 $49° + 40° + m\angle T = 180°$ $m\angle T = 91°$ $\frac{\sin 49°}{r} = \frac{\sin 40°}{20}$ $\frac{\sin 91°}{t} = \frac{\sin 40°}{20}$ $r \approx 23.5$ $t \approx 31.1$
special right triangle (p. 928) A 45°-45°-90° triangle or a 30°-60°-90° triangle.	**triángulo rectángulo especial** Triángulo de 45°-45°-90° o triángulo de 30°-60°-90°.	
square matrix (p. 255) A matrix with the same number of rows as columns.	**matriz cuadrada** Matriz con el mismo número de filas y columnas.	$\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}, \begin{bmatrix} 1 & -3 & 1 \\ 2 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix}$
square root (p. 21) A number that is multiplied to itself to form a product is called a square root of that product.	**raíz cuadrada** El número que se multiplica por sí mismo para formar un producto se denomina la raíz cuadrada de ese producto.	-4 and 4 are square roots of 16 because $(-4)^2 = 16$ and $4^2 = 16$.

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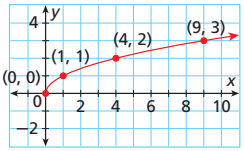
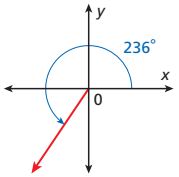
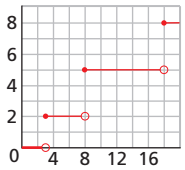
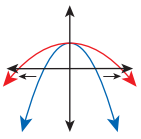
square-root function (p. 619) A function whose rule contains a variable under a square-root sign.	**función de raíz cuadrada** Función cuya regla contiene una variable bajo un signo de raíz cuadrada.	 $f(x) = \sqrt{x}$
standard deviation (p. 830) A measure of dispersion of a data set. The standard deviation σ is the square root of the variance.	**desviación estándar** Medida de dispersión de un conjunto de datos. La desviación estándar σ es la raíz cuadrada de la varianza.	Data set: $\{6, 7, 7, 9, 11\}$ Mean: $\frac{6 + 7 + 7 + 9 + 11}{5} = 8$ Variance: $\frac{1}{5}(4 + 1 + 1 + 1 + 9) = 3.2$ Standard deviation: $\sigma = \sqrt{3.2} \approx 1.8$
standard form of a linear equation (p. 111) $Ax + By = C$, where A, B, and C are real numbers.	**forma estándar de una ecuación lineal** $Ax + By = C$, donde A, B y C son números reales.	$2x + 3y = 6$
standard form of a polynomial (p. 406) A polynomial in one variable is written in standard form when the terms are in order from greatest degree to least degree.	**forma estándar de un polinomio** Un polinomio de una variable se expresa en forma estándar cuando los términos se ordenan de mayor a menor grado.	$3x^3 - 5x^2 + 6x - 7$
standard form of a quadratic equation (p. 324) $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.	**forma estándar de una ecuación cuadrática** $ax^2 + bx + c = 0$, donde a, b y c son números reales y $a \neq 0$.	$2x^2 + 3x - 1 = 0$
standard position (p. 936) An angle in standard position has its vertex at the origin and its initial side on the positive x-axis.	**posición estándar** Ángulo cuyo vértice se encuentra en el origen y cuyo lado inicial se encuentra sobre el eje x.	
step function (p. 663) A piecewise function that is constant over each interval in its domain.	**función escalón** Función a trozos que es constante en cada intervalo en su dominio.	
stretch (p. 61) A transformation that pulls the points of a graph horizontally away from the y-axis or vertically away from the x-axis.	**estiramiento** Transformación que desplaza los puntos de una gráfica en forma horizontal alejándolos del eje y o en forma vertical alejándolos del eje x.	
subset (p. 6) A set that is contained entirely within another set. Set B is a subset of set A if every element of B is contained in A, denoted $B \subset A$.	**subconjunto** Conjunto que se encuentra dentro de otro conjunto. El conjunto B es un subconjunto del conjunto A si todos los elementos de B son elementos de A; se expresa $B \subset A$.	The set of integers is a subset of the set of rational numbers, denoted $\mathbb{Z} \subset \mathbb{Q}$.

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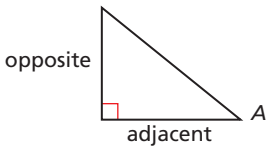
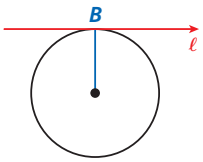
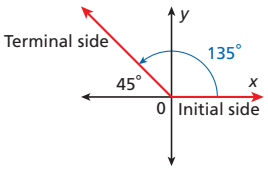
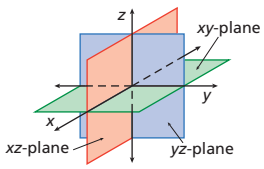
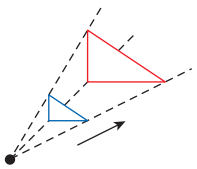
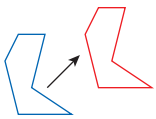
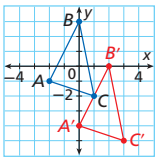
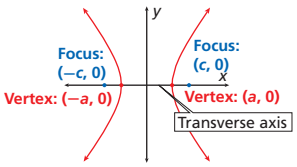
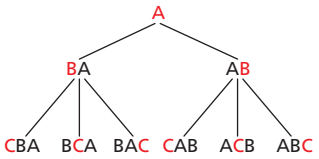
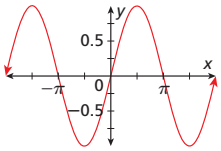
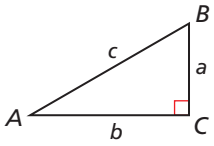
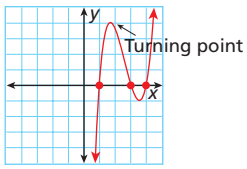
substitution (p. 190) A method used to solve systems of equations by solving an equation for one variable and substituting the resulting expression into the other equation(s).	**sustitución** Método utilizado para resolver sistemas de ecuaciones resolviendo una ecuación para una variable y sustituyendo la expresión resultante en las demás ecuaciones.	$\begin{cases} 2x + 3y = -1 \\ x - 3y = 4 \end{cases}$ Solve for x. $x = 4 + 3y$ Substitute into the first equation and solve. $2(4 + 3y) + 3y = -1$ $y = -1$ Then solve for x. $x = 4 + 3(-1) = 1$
summation notation (p. 870) A method of notating the sum of a series using the Greek letter $\sum$ (capital *sigma*).	**notación de sumatoria** Método de notación de la suma de una serie que utiliza la letra griega $\sum$ (SIGMA mayúscula).	$\sum_{k=1}^{5} 3k = 3 + 6 + 9 + 12 + 15 = 45$
synthetic division (p. 423) A shorthand method of dividing by a linear binomial of the form $(x - a)$ by writing only the coefficients of the polynomials.	**división sintética** Método abreviado de división que consiste en dividir por un binomio lineal del tipo $(x - a)$ escribiendo sólo los coeficientes de los polinomios.	$(x^3 - 7x + 6) \div (x - 2)$ $\begin{array}{r\|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & 6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$ $(x^3 - 7x + 6) \div (x - 2) = x^2 + 2x - 3$
system of equations (p. 182) A set of two or more equations that have two or more variables.	**sistema de ecuaciones** Conjunto de dos o más ecuaciones que contienen dos o más variables.	$\begin{cases} 2x + 3y = -1 \\ x^2 = 4 \end{cases}$
system of linear equations (p. 182) *See* linear system.	**sistema de ecuaciones lineales** *Ver* sistema lineal.	
system of linear inequalities (p. 199) A system of inequalities in two or more variables in which all of the inequalities are linear.	**sistema de desigualdades lineales** Sistema de desigualdades en dos o más variables en el que todas las desigualdades son lineales.	$\begin{cases} 2x + 3y \geq -1 \\ x - 3y < 4 \end{cases}$

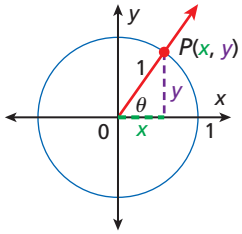
T

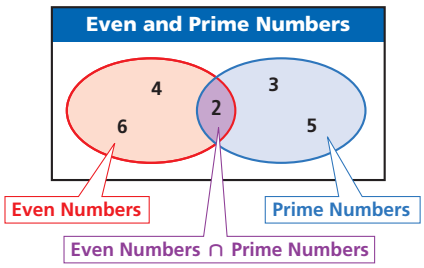
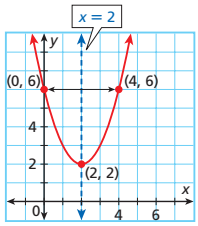
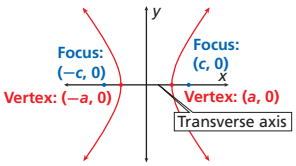
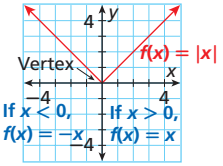
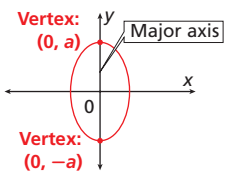
tangent of an angle (p. 929) In a right triangle, the ratio of the length of the leg opposite $\angle A$ to the length of the leg adjacent to $\angle A$.	**tangente de un ángulo** En un triángulo rectángulo, razón entre la longitud del cateto opuesto a $\angle A$ y la longitud del cateto adyacente a $\angle A$.	 $\tan A = \frac{\text{opposite}}{\text{adjacent}}$
tangent line (p. 731) A line that is in the same plane as a circle and intersects the circle at exactly one point.	**línea tangente** Línea que está en el mismo plano que un círculo y corta al círculo en exactamente un punto.	
term of an expression (p. 28) The parts of the expression that are added or subtracted.	**término de una expresión** Partes de la expresión que se suman o se restan.	$3x^2 + 6x - 8$ Term Term Term

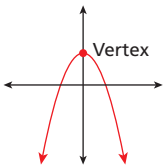
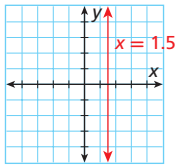
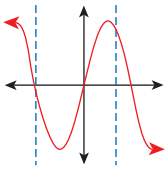
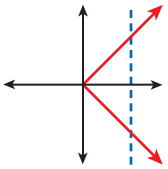
ENGLISH	SPANISH	EXAMPLES
term of a sequence (p. 862) An element or number in the sequence.	**término de una sucesión** Elemento o número de una sucesión.	5 is the third term in the sequence 1, 3, 5, 7, …
terminal side (p. 936) For an angle in standard position, the ray that is rotated relative to the positive x-axis.	**lado terminal** Dado un ángulo en una posición estándar, el rayo que rota en relación con el eje positivo x.	
theoretical probability (p. 802) The ratio of the number of equally likely outcomes in an event to the total number of possible outcomes.	**probabilidad teórica** Razón entre el número de resultados igualmente probables de un suceso y el número total de resultados posibles.	The theoretical probability of rolling an odd number on a number cube is $\frac{3}{6} = \frac{1}{2}$.
third quartile (p. 829) The median of the upper half of a data set. Also called *upper quartile*.	**tercer cuartil** La mediana de la mitad superior de un conjunto de datos. También se llama *cuartil superior.*	Lower half Upper half 18, 23, 28, 29, 36, 42 Third quartile
three-dimensional coordinate system (p. 214) A space that is divided into eight regions by an x-axis, a y-axis, and a z-axis. The locations, or coordinates, of points are given by ordered triples.	**sistema de coordenadas tridimensional** Espacio dividido en ocho regiones por un eje x, un eje y y un eje z. Las ubicaciones, o coordenadas, de los puntos son dadas por tripletas ordenadas.	
transformation (p. 59) A change in the position, size, or shape of a figure or graph.	**transformación** Cambio en la posición, tamaño o forma de una figura o gráfica.	
translation (p. 59) A transformation that shifts or slides every point of a figure or graph the same distance in the same direction.	**traslación** Transformación en la que todos los puntos de una figura se mueven la misma distancia en la misma dirección.	
translation matrix (p. 262) A matrix used to translate points on the coordinate plane.	**matriz de traslación** Matriz utilizada para trasladar puntos en el plano cartesiano.	 Matrix $\begin{bmatrix} -2 & -2 & -2 \\ 3 & 3 & 3 \end{bmatrix}$ is used to translate the figure 2 units left and 3 units up.
transpose (p. 260) A matrix that reverses the rows and columns of a matrix.	**transposición** Matriz que invierte las filas y columnas de una matriz.	$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ is the transpose of $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$.

ENGLISH	SPANISH	EXAMPLES
transverse axis (p. 744) The axis of symmetry of a hyperbola that contains the vertices and foci.	**eje transversal** Eje de simetría de una hipérbola que contiene los vértices y focos.	
tree diagram (p. 812) A branching diagram that shows all possible combinations or outcomes of an experiment.	**diagrama de árbol** Diagrama con ramificaciones que muestra todas las combinaciones o resultados posibles de un experimento.	
trial (p. 805) In probability, a single repetition or observation of an experiment.	**prueba** En probabilidad, una sola repetición u observación de un experimento.	In the experiment of rolling a number cube, each roll is one trial.
trigonometric function (p. 929) A function whose rule is given by a trigonometric ratio.	**función trigonométrica** Función cuya regla es dada por una razón trigonométrica.	 $f(x) = \sin x$
trigonometric ratio (p. 929) Ratio of the lengths of two sides of a right triangle.	**razón trigonométrica** Razón entre dos lados de un triángulo rectángulo.	 $\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$, $\tan A = \frac{a}{b}$
trigonometry (p. 929) The study of the measurement of triangles and of trigonometric functions and their applications.	**trigonometría** Estudio de la medición de los triángulos y de las funciones trigonométricas y sus aplicaciones.	
trinomial (p. 336) A polynomial with three terms.	**trinomio** Polinomio con tres términos.	$4x^2 + 3xy - 5y^2$
turning point (p. 455) A point on the graph of a function that corresponds to a local maximum (or minimum) where the graph changes from increasing to decreasing (or vice versa).	**punto de inflexión** Punto de la gráfica de una función que corresponde a un máximo (o mínimo) local donde la gráfica pasa de ser creciente a decreciente (o viceversa).	

U

unit circle (p. 944) A circle with a radius of 1, centered at the origin.	**círculo unitario** Círculo con un radio de 1, centrado en el origen.	 Unit circle

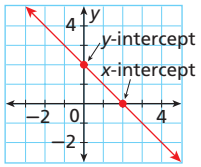
ENGLISH	SPANISH	EXAMPLES
V		
variable (p. 3) A symbol used to represent a quantity that can change.	**variable** Símbolo utilizado para representar una cantidad que puede cambiar.	$2x + 3$ ↑ variable
variable matrix (p. 279) The matrix of the variables in a linear system of equations.	**matriz de variables** Matriz de las variables de un sistema lineal de ecuaciones.	System of equations $\begin{cases} 2x + 3y = -1 \\ x - 3y = 4 \end{cases}$ Variable matrix $\begin{bmatrix} x \\ y \end{bmatrix}$
variance (p. 830) The average of squared differences from the mean. The square root of the variance is called the *standard deviation*.	**varianza** Promedio de las diferencias cuadráticas en relación con la media. La raíz cuadrada de la varianza se denomina *desviación estándar.*	Data set: is $\{6, 7, 7, 9, 11\}$ Mean: $\frac{6 + 7 + 7 + 9 + 11}{5} = 8$ Variance: $\frac{1}{5}(4 + 1 + 1 + 1 + 9) = 3.2$
Venn diagram (p. S72) A diagram used to show relationships between sets.	**diagrama de Venn** Diagrama utilizado para mostrar la relación entre conjuntos.	
vertex form of a quadratic function (p. 318) A quadratic function written in the form $f(x) = a(x - h)^2 + k$, where a, h, and k are constants and (h, k) is the vertex.	**forma en vértice de una función cuadrática** Una función cuadrática expresada en la forma $f(x) = a(x - h)^2 + k$, donde a, h y k son constantes y (h, k) es el vértice.	 $f(x) = (x - 2)^2 + 2$
vertex of a hyperbola (vertices) (p. 744) The endpoints of the transverse axis of the hyperbola.	**vértice de una hipérbola** Extremos del eje transversal de la hipérbola.	
vertex of an absolute-value graph (p. 158) The point where the axis of symmetry intersects the graph.	**vértice de una gráfica de valor absoluto** Punto donde en el eje de simetría interseca la gráfica.	
vertex of an ellipse (vertices) (p. 736) The endpoints of the major axis of the ellipse.	**vértice de una elipse** Extremos del eje mayor de la elipse.	

ENGLISH	SPANISH	EXAMPLES
vertex of a parabola (p. 318) The highest or lowest point on the parabola.	**vértice de una parábola** Punto más alto o más bajo de una parábola.	
vertical line (p. 108) A line whose equation is $x = a$, where a is the x-intercept. The slope of a vertical line is undefined.	**línea vertical** Línea cuya ecuación es $x = a$, donde a es la intersección con el eje x. La pendiente de una línea vertical es indefinida.	
vertical-line test (p. 46) A test used to determine whether a relation is a function. If any vertical line crosses the graph of a relation more than once, the relation is not a function.	**prueba de la línea vertical** Prueba utilizada para determinar si una relación es una función. Si una línea vertical corta la gráfica de una relación más de una vez, la relación no es una función.	 Function  Not a function

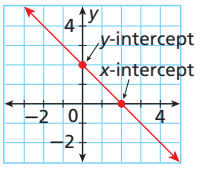
W

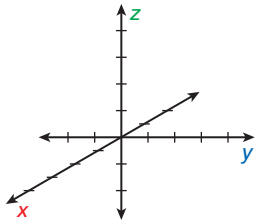
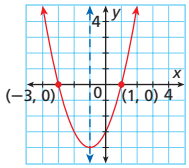
whole number (p. 6) The set of natural numbers and zero.	**número cabal** Conjunto de los números naturales y cero.	0, 1, 2, 3, 4, 5, …

X

***x*-intercept** (p. 106) The x-coordinate(s) of the point(s) where a graph intersects the x-axis.	**intersección con el eje *x*** Coordenada(s) x de uno o más puntos donde una gráfica corta el eje x.	

Y

***y*-intercept** (p. 106) The y-coordinate(s) of the point(s) where a graph intersects the y-axis.	**intersección con el eje *y*** Coordenada(s) y de uno o más puntos donde una gráfica corta el eje y.	

ENGLISH	SPANISH	EXAMPLES
Z		
z-axis (p. 214) The third axis in a three-dimensional coordinate system.	**eje *z*** Tercer eje en un sistema de coordenadas tridimensional.	
zero exponent (p. 35) For any nonzero real number x, $x^0 = 1$.	**exponente cero** Dado un número real distinto de cero x, $x^0 = 1$.	$5^0 = 1$
zero of a function (p. 333) For the function f, any number x such that $f(x) = 0$.	**cero de una función** Dada la función f, todo número x tal que $f(x) = 0$.	 The zeros of $f(x) = x^2 + 2x - 3$ are -3 and 1.

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Credits

Abbreviations used: (t) top, (c) center, (b) bottom, (l) left, (r) right, (bkgd) background

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