CHAPTER 2

Linear Functions

go.hrw.com
Chapter Project Online
KEYWORD: MB7 ChProj

Linear functions can describe the relationship between the number of stories a building has and its height.

U.S. Bank Tower
Los Angeles, CA

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. absolute value
2. function
3. transformation
4. scatter plot

A. a relation in which each first coordinate is paired with exactly one second coordinate

B. a change in the position, size, or shape of a figure

C. the distance from a number to zero on the number line

D. a symbol used to represent a quantity that can change

E. a graph on a coordinate plane with points plotted to represent relationships between data sets

Connect Words and Algebra

Write an equation for each phrase.

5. The sum of a number and 4 times another number is 25.
6. The difference of 3 times a number and 20 is greater than 10.
7. A number divided by 12 is less than 15 divided by the same number.

Solve One-Step Equations

Solve each equation for *x*.

8. $-8 + x = -20$
9. $-12 = -3x$
10. $x - 19 = -12$
11. $0.75 = \frac{x}{5}$

Percent Problems

Solve each percent problem.

12. Fifteen is 30% of what number?
13. What number is 40% of 140?
14. What percent of 140 is 105?
15. What number is 150% of 90?

Convert Units of Measure

Convert the units of measure.

16. 12 quarts to gallons
17. 15 feet to yards
18. 1.5 hours to minutes
19. 3.5 gallons to quarts
20. 17 yards to feet
21. 200 minutes to hours
22. 107 centimeters to meters
23. 2.5 kilometers to meters
24. 50 milliliters to liters

Absolute Value

Find the absolute value of each expression.

25. $|16 - 22|$
26. $|32 - 20|$
27. $|8 - 17 + 9|$
28. $|-0.75 + 0.625|$

CHAPTER 2 Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
1.0 Students solve equations and inequalities involving absolute value. (Lessons 2-8, 2-9) (Lab 2-8)	**solve** find the value of a variable that makes the left side of an equation equal to the right side of the equation **involving** needing the use of	You will write and solve equations and inequalities that contain absolute values. ***Example:*** $\|x - 7\| = 5$
Review of Algebra 1 4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x - 5) + 4(x - 2) = 12$. (Lesson 2-1)	**simplify** (simplification) make things easier **linear equation** an equation whose variable(s) have exponents not greater than 1	You simplify expressions before solving the equations that contain them. For example, you might combine like terms using the Distributive Property before solving.
Review of Algebra 1 5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step. (Lessons 2-1, 2-2)	**multistep** more than one step	You solve equations where the solution process requires two or more steps.
Review of Algebra 1 6.0 Students graph a linear equation and compute the *x*- and *y*-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequalities (e.g., they sketch the region defined by $2x + 6y < 4$) (Lessons 2-3, 2-5)	**graph** to represent data with a diagram **compute** to calculate an answer	You graph an equation of a line and solve equations for *x* and *y* to find where the graph crosses the *x*-axis and *y*-axis.
Review of Algebra 1 7.0 Students verify that a point lies on a line, given an equation of the line. **Students are able to derive linear equations by using the point-slope formula.** (Lessons 2-4, 2-7)	**derive** to reach a conclusion by reasoning	You write an equation of a line given a point on the line and its slope.

Reading and Writing Math

CHAPTER 2

Reading Strategy: Read a Lesson for Understanding

As you read a lesson, read with a purpose. Lessons are centered on one or two specific objectives given at the top of the first page. Reading with the objectives in mind will help guide you through the lesson. You can use some of the following tips to help you follow the math as you read.

Reading Tips

Objective
Identify and use properties of real numbers.

Identify the **objectives** of the lesson. Then skim through the lesson to get a sense of where the objectives are covered.

"What is an inverse?"

"What is an integer?"

As you read through the lesson, list any questions, problems, or trouble spots you may have.

EXAMPLE:
Find the additive and multiplicative inverse of –9.

Additive inverse: 9 — *The opposite of –9 is –(–9) = 9*

Check $-9 + 9 = 0$ ✓ — *The Additive Inverse Property holds.*

Multiplicative inverse: $\frac{1}{-9}$ — *The reciprocal of –9 is $\frac{1}{-9}$.*

Check $-9\left(\frac{1}{-9}\right) = 1$ ✓ — *The Multiplicative Inverse Property holds.*

Work through each example, as the examples help demonstrate the objectives.

CHECK IT OUT!

Practice your skills in the Check It Out sections to verify your understanding of the lesson.

Try This

Use Lesson 1-4 in your textbook to answer each question.

1. What is the objective of the lesson?
2. What new terms are defined in the lesson?
3. Fraction bars, square root symbols, and absolute value symbols are all forms of what type of symbol?
4. What skill is being practiced in the first Check It Out problem in the lesson?

2-1 Solving Linear Equations and Inequalities

Objectives

Solve linear equations using a variety of methods.

Solve linear inequalities.

Vocabulary

equation
solution set of an equation
linear equation in one variable
identity
contradiction
inequality

California Standards

Review of 1A5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.
Also covered:
Review of 1A4.0

Who uses this?

A hot-air balloonist can use linear equations to calculate the average speed needed to set a world record. (See Example 1.)

An **equation** is a mathematical statement that two expressions are equivalent. The **solution set of an equation** is the value or values of the variable that make the equation true. A **linear equation in one variable** can be written in the form $ax = b$, where a and b are constants and $a \neq 0$.

Linear Equations in One Variable	Nonlinear Equations
$4x = 8$	$3\sqrt{x} + 1 = 32$
$3x - \frac{2}{3}x = -9$	$\frac{2}{x^2} = 41$
$2x - 5 = 0.1x + 2$	$3 - 2^x = -5$

Notice that the variable in a linear equation is not under a radical sign and is not raised to a power other than 1. The variable is also not an exponent and is not in a denominator.

Solving a linear equation requires isolating the variable on one side of the equation by using the properties of equality.

Know it! Note

Properties of Equality

For all real numbers a, b and c,

WORDS	NUMBERS	ALGEBRA
Addition		
If you add the same quantity to both sides of an equation, the equation will still be true.	$3 = 3$ $3 + 2 = 3 + 2$	$a = b$ $a + c = b + c$
Subtraction		
If you subtract the same quantity from both sides of an equation, the equation will still be true.	$3 = 3$ $3 - 2 = 3 - 2$	$a = b$ $a - c = b - c$
Multiplication		
If you multiply both sides of an equation by the same quantity, the equation will still be true.	$3 = 3$ $3(2) = 3(2)$	$a = b$ $ac = bc$
Division		
If you divide both sides of an equation by the same nonzero quantity, the equation will still be true.	$3 = 3$ $\frac{3}{2} = \frac{3}{2}$	$a = b$ If $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$

To isolate the variable, perform the inverse, or opposite, of every operation in the equation on both sides of the equation. Do inverse operations in the reverse order of the order of operations.

EXAMPLE 1 Travel Application

Steve Fossett set a 24-hour hot-air balloon record of 3186.8 miles on July 1, 2002. Suppose a balloonist has traveled 1239 miles in 10.5 hours. What speed would the balloonist need to average during the remaining 13.5 hours to tie the record?

Let v represent the speed in miles per hour the balloonist will need to average.

Model

distance already traveled	plus	average speed	times	time remaining hours	=	total distance
1239	+	v	$\cdot$	13.5	=	3186.8

Solve

$$1239 + 13.5v = 3186.8$$

$$\underline{-1239} \qquad \underline{-1239} \qquad \textit{Subtract 1239 from both sides.}$$

$$\frac{13.5v}{13.5} = \frac{1947.8}{13.5} \qquad \textit{Divide both sides by 13.5.}$$

$$v \approx 144.3$$

The balloonist must average about 144.3 mi/h for the remaining 13.5 hours.

CHECK IT OUT!

1. Stacked cups are to be placed in a pantry. One cup is 3.25 in. high and each additional cup raises the stack 0.25 in. How many cups fit between two shelves 14 in. apart?

EXAMPLE 2 Solving Equations with the Distributive Property

Solve $5(y - 7) = 25$.

Method 1

The quantity $(y - 7)$ is multiplied by 5, so divide by 5 first.

$$\frac{5(y-7)}{5} = \frac{25}{5} \qquad \textit{Divide both sides by 5.}$$

$$y - 7 = 5$$

$$\underline{+7} \quad \underline{+7} \qquad \textit{Add 7 to both sides.}$$

$$y = 12$$

Method 2

Distribute before solving.

$$5y - 35 = 25 \qquad \textit{Distribute 5.}$$

$$\underline{+35} \quad \underline{+35} \qquad \textit{Add 35 to both sides.}$$

$$5y = 60$$

$$\frac{5y}{5} = \frac{60}{5} \qquad \textit{Divide both sides by 5.}$$

$$y = 12$$

Check

$5(y - 7)$	25
$5(12 - 7)$	25
$5(5)$	25
25	25 ✔

CHECK IT OUT!

Solve.

2a. $3(2 - 3p) = 42$

2b. $-3(5 - 4r) = -9$

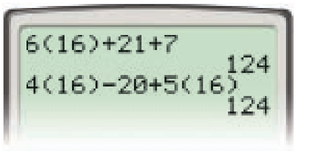
If there are variables on both sides of the equation, (1) simplify each side. (2) collect all variable terms on one side and all constant terms on the other side. (3) isolate the variable as you did in the previous problems.

EXAMPLE 3 Solving Equations with Variables on Both Sides

Solve $6y + 21 + 7 = 4y - 20 + 5y$.

$$\begin{aligned}
6y + 28 &= 9y - 20 && \textit{Simplify each side by combining like terms.}\\
\underline{-6y\quad} &\quad \underline{-6y\quad} && \textit{Collect variables on the right side.}\\
28 &= 3y - 20 && \textit{Subtract.}\\
\underline{+20} &\quad \underline{\quad +20} && \textit{Collect constants on the left side.}\\
\frac{48}{3} &= \frac{3y}{3} && \textit{Isolate the variable.}\\
16 &= y
\end{aligned}$$

Check Substitute 16 for y on both sides of the the original equation. You can use a calculator to make sure they are equal.



CHECK IT OUT!

3. Solve $3(w + 7) - 5w = w + 12$

You have solved equations that have a single solution. Equations may also have infinitely many solutions or no solution.

An equation that is true for all values of the variable, such as $x = x$, is an **identity**. An equation that has no solution, such as $3 = 5$, is a **contradiction** because there are no values that make it true.

EXAMPLE 4 Identifying Identities and Contradictions

Solve.

A $3x + 4x + 5 = 7x + 5$

$$\begin{aligned}
7x + 5 &= 7x + 5 && \textit{Simplify.}\\
\underline{-7x\quad} &\quad \underline{-7x\quad}\\
5 &= 5 \checkmark && \textit{Identity}
\end{aligned}$$

The solution set is all real numbers, or $\mathbb{R}$.

B $8(y + 7) = 6y - 8 + 2y$

$$\begin{aligned}
8y + 56 &= 8y - 8 && \textit{Simplify.}\\
\underline{-8y\quad} &\quad \underline{-8y\quad}\\
56 &= -8 \times && \textit{Contradiction}
\end{aligned}$$

The equation has no solution. The solution set is the *empty set,* which is represented by the symbol $\varnothing$.

CHECK IT OUT!

Solve.

4a. $5(x - 6) = 3x - 18 + 2x$ **4b.** $3(2 - 3x) = -7x - 2(x - 3)$

An **inequality** is a statement that compares two expressions by using the symbols $<$, $>$, $\leq$, $\geq$, or $\neq$. The graph of an inequality is the solution set, the set of all points on the number line that satisfy the inequality.

The properties of equality are true for inequalities, with one important difference. If you multiply or divide both sides by a negative number, you must *reverse* the inequality symbol.

Inequalities: Multiplying or Dividing by a Negative Number

For all real numbers *a*, *b*, and *c*,

WORDS	NUMBERS	ALGEBRA
If you multiply both sides of an inequality by the same **negative** quantity and reverse the inequality symbol, the inequality will still be true.	$4 < 6$ $4(-2) > 6(-2)$ $-8 > -12$	$a < b$ If $c < 0$, $ac > bc$
If you divide both sides of an inequality by the same **negative** quantity and reverse the inequality symbol, the inequality will still be true.	$4 < 6$ $\frac{4}{-2} > \frac{6}{-2}$ $-2 > -3$	$a < b$ If $c < 0$, $\frac{a}{c} > \frac{b}{c}$

These properties also apply to inequalities expressed with $>$, $\geq$, and $\leq$.

EXAMPLE 5 Solving Inequalities

Solve and graph $9x + 4 < 12x - 11$.

$$\begin{aligned} 9x + 4 &< 12x - 11 \\ -12x \quad & \quad -12x \end{aligned}$$ *Subtract 12x from both sides.*

$$\begin{aligned} -3x + 4 &< -11 \\ -4 \quad & \quad -4 \\ -3x &< -15 \end{aligned}$$ *Subtract 4 from both sides.*

$$\frac{-3x}{-3} > \frac{-15}{-3}$$ *Divide both sides by –3 and reverse the inequality.*

$$x > 5$$

Check Test values in the original inequality:

Test $x = 0$.	Test $x = 5$.	Test $x = 7$.
$9(0) + 4 \overset{?}{<} 12(0) - 11$	$9(5) + 4 \overset{?}{<} 12(5) - 11$	$9(7) + 4 \overset{?}{<} 12(7) - 11$
$4 < -11$ ✘	$49 < 49$ ✘	$67 < 73$ ✔
So 0 is not a solution.	So 5 is not a solution.	So 7 is a solution.

Helpful Hint

To check an inequality, test
- the value being compared with *x* (5 in Example 5),
- a value less than that, and
- a value greater than that.

CHECK IT OUT! **5.** Solve and graph $x + 8 \geq 4x + 17$.

THINK AND DISCUSS

1. Give an example of an equation containing $3x$ that has no solution and another containing $3x$ with all real numbers as solutions.

2. Explain why you must reverse the inequality symbol in an expression when you multiply by a negative number. Use the inequality $-3 < 3$ as an example.

3. GET ORGANIZED Copy and complete the graphic organizer. Note the similarities and differences in the properties and methods you use.

Similarities — **Solving Equations and Inequalities** — Differences

2-1 Exercises

California Standards
Review of 1A2.0, 1A4.0, and 1A5.0; Preparation for 24.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 2-1
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** The statement $4 = 4$ is a(n) __?__. (*identity* or *contradiction*)

SEE EXAMPLE 1 p. 91

2. **Consumer Economics** Shanti has just joined a DVD rental club. She pays a monthly membership fee of \$4.95, and each DVD rental is \$1.95. If Shanti's budget for DVD rentals in a month is \$42, how many DVDs can Shanti rent in her first month if she doesn't want to go over her budget?

SEE EXAMPLE 2 p. 91

Solve.

3. $8(x - 5) = 72$
4. $1.5(x - 4) = 9.6$
5. $-27 = 3(x - 3)$

SEE EXAMPLE 3 p. 92

6. $5 - 4c = c + 20$
7. $24 + 7x = -4x - 9$
8. $3(x - 5) = 5x + 9$
9. $x = -2(x - 3)$
10. $(t - 3)7 = 6t + 21$
11. $-0.5(r - 2) = -r - 2$

SEE EXAMPLE 4 p. 92

12. $-4t + 1 = 3t + 1 - 7t$
13. $2(3x + 1) = 3(2x + 1)$
14. $2(3n + 3) - 9 = 6n$
15. $2h + 4 - 5h = -3h + 4$
16. $4(2 - 6m) = 6(2 - 4m)$
17. $0.5(-8p + 1) = -4p + 1$

SEE EXAMPLE 5 p. 93

Solve and graph.

18. $5x - 12 > 8$
19. $62 - 18x < 20$
20. $23 + 3x \leq 15 - x$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
21	1
22–26	2
27–31	3
32–36	4
37–39	5

Extra Practice
Skills Practice p. S6
Application Practice p. S33

21. **Aerospace** A pen floating in the weightlessness of space is 30 inches above the floor of the space capsule and is rising at 1.5 inches per second. In how many seconds will it reach the 6-foot-high ceiling?

Solve.

22. $-30 = 6(x - 3)$
23. $5(x - 8) - (x + 6) = 18$
24. $2(x + 4) - 5(x - 3) = 32$
25. $\frac{1}{3}(2x - 7) = 4$
26. $3x - 8(3 - x) = 53$
27. $6n - 7 = 2n + 17$
28. $3n - 40 = \frac{1}{2}n + 35$
29. $5(x - 4) - 1 = -7x + 3$
30. $12x + 20 = 6(x + 4)$
31. $8t + 11 - 6t = 5t + 35$
32. $2x + 4(x + 1) = 6\left(x + \frac{2}{3}\right)$
33. $8 = -8x + 4(4 + 2x)$
34. $-4(2n - 5) = -8n - 20$
35. $9(3 - 2x) = -6(3x - 5)$
36. $4x - 2(3 + 2x) = -6$

Solve and graph.

37. $-3x + 8 \leq 14$
38. $3(x - 1) > 7(x + 3)$
39. $5(x - 2) \geq 4(2x + 6) + 2$

40. **Business** Pat is paid a salary of \$500 a month plus a commission of 15% of the value of the jewelry she sells. Find the value of the jewelry Pat must sell in a month to earn at least \$2000.

41. **Economics** In 1902, 44 loaves of bread cost the same amount as 1 loaf of bread in 2006. If a loaf of bread in 2006 costs \$1.72 more than in 1902, find the cost of a loaf of bread in 1902.

42. **Football** In 2004, three wide receivers for the Indianapolis Colts caught a total of 37 touchdown passes. Reggie Wayne caught 2 more than Brandon Stokely, and Marvin Harrison caught 3 more than Reggie Wayne. How many touchdown passes did each receiver catch?

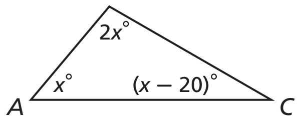
43. **Technology** A digital answering machine has a total capacity of 32 min for the personal announcement and incoming messages. Incoming messages are limited to 3 min each, and the announcement is 30 s long.

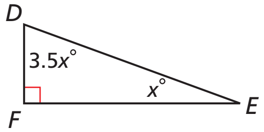
a. Find the possible number of 3 min messages the machine can record.

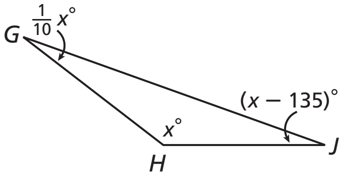
b. The average length of an incoming message is 1.5 min. How many messages of average length can the machine record?

c. **What If...?** A friend has left 2 maximum length messages on your machine. In addition you have 5 minutes worth of saved messages. How many more average length messages can your machine record?

Geometry **Find the measure of each angle in the triangles below. (*Hint:* The sum of angle measures in a triangle is 180°.)**

44. 

45. 

46. 

LINK

Literature

Shakespeare's plays have been made into more than 500 movies and television shows.

Source: IMDB.com

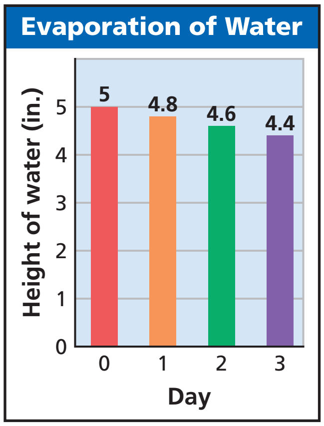
47. **Literature** William Shakespeare wrote 37 plays, including tragedies, comedies, and histories. He wrote the same number of tragedies and histories, but the number of comedies he wrote is 3 less than twice the number of tragedies. How many of each type of play did Shakespeare write?

48. **Chemistry** As an experiment, a student filled a water glass with 5 in. of water. The chart shows the height of the water after each day.

a. How much water evaporates each day?

b. When will the height of the water drop below 2.5 in.?

c. If the pattern continued, what would the height of the water be after 30 days? Is this reasonable in the context of the problem?



49. **Critical Thinking** What values of k make the equation $2(x - k) = 2x + 20$ an identity? What values of k make the equation a contradiction?

50. **Critical Thinking** Does an inequality of the form $ax > b$ always, sometimes, or never give a solution of the form $x > c$? Give examples to support your answer.

51. **Write About It** How do you recognize when an equation has no real solution or an infinite number of solutions?

Concept Connection

52. This problem will prepare you for the Concept Connection on page 132.

There are 360° of longitude at the equator.

a. The length of a nautical mile initially represented $\frac{1}{60}$ degree of longitude at the equator, and is very close to that measurement today. How many nautical miles is the circumference of the earth at the equator?

b. The circumference of the earth at the equator is 24,901.55 common or *statute* miles. Is the length of a nautical mile longer or shorter than a statute mile? Explain.

STANDARDIZED TEST PREP

53. If $5 + 3x = 17$, then which equation is true?

Ⓐ $x = \frac{5 - 17}{3}$ Ⓑ $x = \frac{17 - 3}{5}$ Ⓒ $x = \frac{17 - 5}{3}$ Ⓓ $x = \frac{3 - 17}{5}$

54. Which expression does NOT simplify to a? (for $a \neq 0$, for $b \neq 0$)

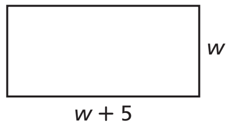
Ⓕ $(a \div b) \cdot b$ Ⓖ $(a - b) + a + b$ Ⓗ $(a \cdot b) \div b$ Ⓙ $(a + b) - b$

55. Bob has 3 times as much money as Amy has, and Sam has \$5 more than Bob has. Bob, Amy, and Sam have a total of \$75. Which equation can be used to find out how much money Amy has?

Ⓐ $x + 3x + (x - 5) = 75$ Ⓒ $x + 3x + (3x - 5) = 75$

Ⓑ $x + 3x + (x + 5) = 75$ Ⓓ $x + 3x + (3x + 5) = 75$

56. If the perimeter of the rectangle can be at most 100 feet, which inequality can be used to find the width?



Ⓕ $w + (w + 5) \leq 100$ Ⓗ $w + (w + 5) \geq 100$

Ⓖ $2w + 2(w + 5) \leq 100$ Ⓙ $2w + 2(w + 5) \geq 100$

57. **Gridded Response** If $12 = 15 - 2x$, find the value of $8x$.

CHALLENGE AND EXTEND

Solve and graph.

58. $\frac{3x - 5}{8} - \frac{4 - 5x}{5} > \frac{3 - 2x}{4}$

59. $8(x - 1) \leq 4(2 + 2x)$

60. Is the statement $4(x - 2) \neq 2(-4 + 2x)$ an identity or a contradiction? Explain.

61. **Estimation** There are 90 people in line at a theme park ride. Every 5 minutes, 40 people get on the ride and 63 join the line. Estimate how long it would take for 600 people to be in line. About how long will the 600th person have to wait?

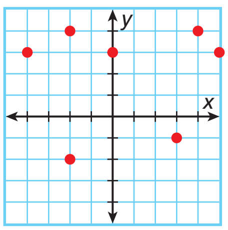
SPIRAL REVIEW

Simplify each expression. *(Lesson 1-3)*

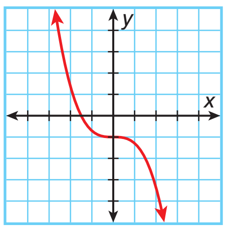
62. $\sqrt{75}$ **63.** $\sqrt{90} + \sqrt{250}$ **64.** $\frac{\sqrt{68}}{22}$ **65.** $\frac{5\sqrt{12}}{\sqrt{5}}$

Determine whether each relation is a function. *(Lesson 1-6)*

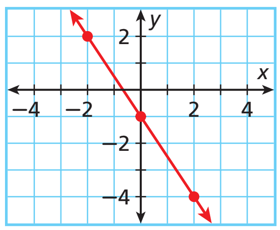
66.



67.



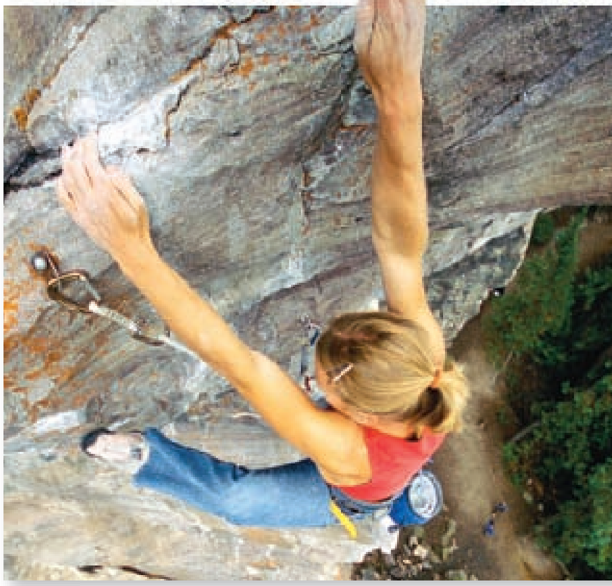
68.



69. **Economics** The table shows the federal minimum wage at four-year intervals. Is minimum wage a function of year? Explain. *(Lesson 1-6)*

Federal Minimum Wage					
Year	1988	1992	1996	2000	2004
Minimum Wage	\$3.35	\$4.25	\$4.75	\$5.15	\$5.15

2-2 Proportional Reasoning



Objective
Apply proportional relationships to rates, similarity, and scale.

Vocabulary
ratio
proportion
rate
similar
indirect measurement

Who uses this?
Rock climbers can use proportions to indirectly measure the height of cliffs. (See Example 5.)

Recall that a **ratio** is a comparison of two numbers by division and a **proportion** is an equation stating that two ratios are equal. In a proportion, the cross products are equal.

Know it! Note

California Standards

Review of 1A5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.
Also covered:
Review of 1A15.0

Cross Products Property

WORDS	NUMBERS	ALGEBRA
The cross products of a proportion are equal.	$\frac{3}{5} = \frac{9}{15}$ $3(15) = 5(9)$ $45 = 45$	For real numbers *a*, *b*, *c*, and *d*, where $b \neq 0$ and $d \neq 0$: If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

If a proportion contains a variable, you can cross multiply to solve for the variable. When you set the cross products equal, you create a linear equation that you can solve by using the skills that you learned in Lesson 2-1.

EXAMPLE 1 **Solving Proportions**

Reading Math
In $a \div b = c \div d$, *b* and *c* are the *means*, and *a* and *d* are the *extremes*. In a proportion, the product of the means is equal to the product of the extremes.

Solve each proportion.

A $\frac{22}{9} = \frac{x}{13.5}$

$\frac{22}{9} \times \frac{x}{13.5}$

$297 = 9x$ *Set cross products equal.*

$\frac{297}{9} = \frac{9x}{9}$ *Divide both sides.*

$33 = x$

B $\frac{512}{16} = \frac{64}{w}$

$\frac{512}{16} \times \frac{64}{w}$

$512w = 1024$

$\frac{512w}{512} = \frac{1024}{512}$

$w = 2$

CHECK IT OUT! Solve each proportion.

1a. $\frac{y}{12} = \frac{77}{84}$ **1b.** $\frac{15}{x} = \frac{2.5}{7}$

Because percents can be expressed as ratios, you can use the proportion $\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$ to solve percent problems.

EXAMPLE 2 Solving Percent Problems

A college brochure states that 11.5% of the students attending the college are majoring in engineering. If 2400 students are attending the college, how many are majoring in engineering?

You know the percent and the total number of students, so you are trying to find the part of the whole (the number of students who are majoring in engineering).

Remember!

Percent is a ratio that means *per hundred*. For example:

$30\% = 0.30 = \frac{30}{100}$

Method 1 Use a proportion.

$$\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$$

$$\frac{11.5}{100} = \frac{x}{2400}$$

$11.5(2400) = 100x$ *Cross multiply.*

$\frac{27600}{100} = x$ *Solve for x.*

$x = 276$

Method 2 Use a percent equation

$11.5\% = 0.115$ *Divide the percent by 100.*

Percent (as decimal) $\cdot$ whole = part

$\mathbf{0.115 \cdot 2400 = x}$

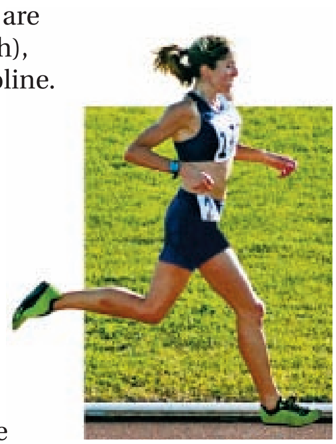
$276 = x$

So 276 students at the college are majoring in engineering.

CHECK IT OUT!

2. At Clay High School, 434 students, or 35% of the students, play a sport. How many students does Clay High School have?

A **rate** is a ratio that involves two different units. You are familiar with many rates, such as miles per hour (mi/h), words per minute (wpm), or dollars per gallon of gasoline. Rates can be helpful in solving many problems.



EXAMPLE 3 *Fitness Application*

A pedometer measures how far a jogger has run. To set her pedometer, Rita must know her stride length. Rita counts 328 strides as she runs once around a 400 m track. A meter is about 39.37 in. How long is her stride in inches?

Use a proportion to find the length of her stride in meters.

$\frac{400 \text{ m}}{328 \text{ strides}} = \frac{x \text{ m}}{1 \text{ stride}}$ *Write both ratios in the form $\frac{\text{meters}}{\text{strides}}$.*

$400 = 328x$ *Find the cross products.*

$x \approx 1.22 \text{ m}$

Convert the stride length to inches.

$\frac{1.22 \cancel{\text{m}}}{1 \text{ stride length}} \cdot \frac{39.37 \text{ in.}}{1 \cancel{\text{m}}} \approx \frac{48 \text{ in.}}{1 \text{ stride length}}$ *$\frac{39.37 \text{ in.}}{1 \text{ m}}$ is the conversion factor.*

Rita's stride length is approximately 48 inches.

CHECK IT OUT!

3. Luis ran the same 400 m track in 297 strides. Find his stride length in inches.

Similar figures have the same shape but not necessarily the same size. Two figures are **similar** if their corresponding angles are congruent and corresponding sides are proportional.

EXAMPLE 4 Scaling Geometric Figures in the Coordinate Plane

Geometry

$\triangle ABC$ has vertices $A(0, 0)$, $B(8, 4)$, and $C(8, 0)$. $\triangle ADE$ is similar to $\triangle ABC$ with a vertex at $E(2, 0)$. Graph $\triangle ABC$ and $\triangle ADE$ on the same grid.

Step 1 Graph $\triangle ABC$. Then draw $\overline{AE}$.

Step 2 To find the height of $\triangle ADE$, use a proportion.

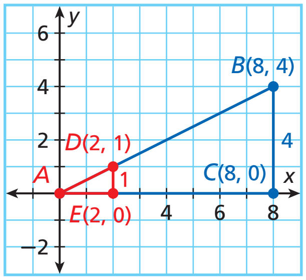
$$\frac{\text{width of } \triangle ADE}{\text{width of } \triangle ABC} = \frac{\text{height of } \triangle ADE}{\text{height of } \triangle ABC}$$

$$\frac{2}{8} = \frac{x}{4}$$

$$8x = 8, \text{ so } x = 1$$

Step 3 To graph $\triangle ADE$, first find the coordinates of D.

The height is 1 unit, and the width is 2 units, so the coordinates of D are $(2, 1)$.



> **Reading Math**
>
> The ratio of the corresponding side lengths of similar figures is often called the *scale factor*.

CHECK IT OUT!

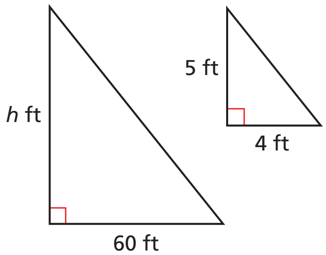
4. $\triangle DEF$ has vertices $D(0, 0)$, $E(-6, 0)$, and $F(0, -4)$. $\triangle DGH$ is similar to $\triangle DEF$ with a vertex at $G(-3, 0)$. Graph $\triangle DEF$ and $\triangle DGH$ on the same grid.

Indirect measurement uses known lengths, similar figures, and proportions to measure objects that cannot easily be measured.

EXAMPLE 5 *Recreation Application*

A rock climber wants to know the height of a cliff. The climber measures the shadow of her friend, who is 5 feet tall and standing beside the cliff, and measures the shadow of the cliff. If the friend's shadow is 4 feet long and the cliff's shadow is 60 feet long, how tall is the cliff?

Sketch the situation. The triangles formed by using the shadows are similar, so the rock climber can use a proportion to find h the height of the cliff.



$$\frac{4}{5} = \frac{60}{h} \qquad \frac{\textit{shadow of friend}}{\textit{height of friend}} = \frac{\textit{shadow of cliff}}{\textit{height of cliff}}$$

$$4h = 300$$

$$h = 75$$

The cliff is 75 feet high.

CHECK IT OUT!

5. A 6-foot-tall climber casts a 20-foot-long shadow at the same time that a tree casts a 90-foot-long shadow. How tall is the tree?

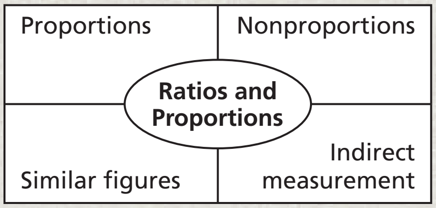
THINK AND DISCUSS

1. Use algebra to explain why equal cross products imply that two ratios are equal.

2. How is it possible to find a length or distance without physically measuring it?

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write examples of each item that relate to the concept of proportion.



2-2 Exercises

California Standards
Review of 1A5.0, 1A15.0; Preparation for 9.0, 10.0, 24.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 2-2
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** *Miles per hour* is a(n) __?__. (*rate, ratio,* or *indirect measurement*)

SEE EXAMPLE 1 p. 97

Solve each proportion.

2. $\frac{6.4}{x} = \frac{2}{3}$ **3.** $\frac{2}{13} = \frac{n}{52}$ **4.** $\frac{4}{14} = \frac{24}{x}$ **5.** $\frac{\frac{1}{3}}{3} = \frac{6}{t}$

6. $\frac{8}{x} = \frac{5}{12}$ **7.** $\frac{4}{9} = \frac{x}{45}$ **8.** $\frac{-2}{5} = \frac{18}{x}$ **9.** $\frac{x}{-15} = \frac{63}{45}$

SEE EXAMPLE 2 p. 98

10. **School** A college brochure claims that 24% of the students attending the college are majoring in business. If there are 420 students at the college who are majoring in business, how many students are attending the college?

SEE EXAMPLE 3 p. 98

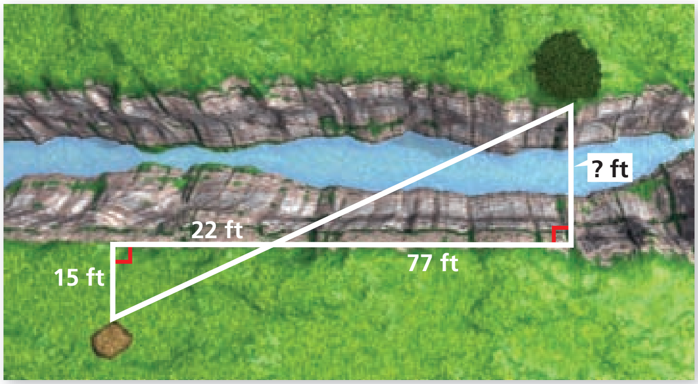
11. **Travel** Jesse drove from Los Angeles to Las Vegas, a distance of 463 km. He used 12 gal of gas on the trip. Find the gas mileage in miles per gallon of Jesse's car. (*Hint*: 1 km ≈ 0.62 mi)

SEE EXAMPLE 4 p. 98

12. **Geometry** $\triangle ABC$ has vertices $A(0, 0)$, $B(0, 8)$, and $C(-6, 8)$. $\triangle ADE$ is similar to $\triangle ABC$ with a vertex at $D(0, 4)$. Graph $\triangle ABC$ and $\triangle ADE$ on the same grid.

SEE EXAMPLE 5 p. 99

13. **Surveying** A surveyor uses similar triangles to measure the distance across a canyon. What is the distance across the canyon, according to the diagram?



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–17	1
18	2
19	3
20	4
21	5

Extra Practice

Skills Practice p. S6

Application Practice p. S33

Solve each proportion.

14. $\frac{55}{200} = \frac{143}{n}$ **15.** $\frac{1.24}{3} = \frac{y}{15}$ **16.** $\frac{22}{11} = \frac{7}{x}$ **17.** $\frac{0.1}{x} = \frac{1.1}{110}$

18. **Business** A quality control inspector has found that 3.2% of the garments produced at Standard Garments contain a defect. If Standard Garments produces 4117 garments in one day, how many of those garments are expected to have a defect?

19. **Communication** Latanya made a 17-minute phone call from her hotel in France and was charged 17 euro. At the time, \$1 was worth 0.82 euro. Find the cost per minute of the call in dollars.

20. **Geometry** $\triangle ABC$ has vertices $A(0, 0)$, $B(6, 0)$, and $C(6, -4.5)$. $\triangle ADE$ is similar to $\triangle ABC$ with a vertex at $D(8, 0)$. Graph $\triangle ABC$ and $\triangle ADE$ on the same grid.

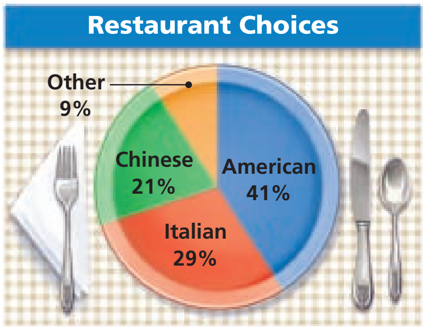
21. **Measurement** A basketball rim 10 ft high casts a shadow 15 ft long. At the same time, a nearby building casts a shadow that is 54 ft long. How tall is the building?

Solve.

22. $\frac{4}{9} = \frac{r+3}{45}$ **23.** $\frac{2.8}{1.5} = \frac{t}{0.09}$ **24.** $\frac{9+m}{5} = \frac{15}{4}$ **25.** $\frac{2}{u-5} = \frac{6}{9}$

26. $\frac{12}{27} = \frac{3r}{3}$ **27.** $\frac{-11}{0.11h} = \frac{10}{3}$ **28.** $\frac{25}{75} = \frac{80}{5x}$ **29.** $\frac{0}{17} = \frac{0.5x}{170}$

30. **Food** A sample of students was asked what type of restaurant they visit most often. Their answers are shown in the circle graph. If 126 students chose Chinese restaurants, how many students were polled?



31. **Critical Thinking** If $a \neq 0$, $b \neq 0$, $c \neq 0$, $d \neq 0$, and $\frac{a}{b} = \frac{c}{d}$, explain why $\frac{d}{c} = \frac{b}{a}$ is also true.

32. **What if...?** Suppose you double the lengths of the sides of a rectangle.

a. What is the relationship between the perimeter of the new rectangle and the perimeter of the original rectangle?

b. What is the relationship between the area of the image and the area of the preimage?

33. **Critical Thinking** In a film, the 555-feet-tall Washington Monument casts a 100-feet-long shadow, whereas the main character in the film casts a 4 feet-long shadow nearby. Why is this considered a film "goof"?

34. **Estimation** The distance from La Paz to Cabo San Lucas on Mexico's Baja Peninsula is 92 miles, or 148 kilometers.

a. The red bar representing the scale of the map represents approximately how many miles?

b. About how many kilometers is El Pescadero from Los Barilles?

CONCEPT CONNECTION

35. This problem will prepare you for the Concept Connection on page 132.

Nautical speed was once measured by throwing a rope into the water from the ship. The rope had knots every 47 ft 3 in., and its wedge-shaped end would “grab” the water. The speed was the number of rope knots that went into the water while a 28 s hourglass ran down.

a. What is the ratio of 1 hour to 28 seconds? (*Hint:* Use the same units).

b. A nautical mile is about 6076.1 ft. What is the ratio of this to the length of the rope between knots?

c. What proportion might have been set up to determine the correct length of rope between knots?

36. Environment Lake Travis, at 679.6 feet above sea level, is 99% full. It is expected to rise to between 682 and 685 feet. Would Lake Travis flood (rise above being *full*) if it reached 685 feet?

37. Chemistry There are about 1,400,000 drops in 25 gallons of a liquid. What percent of a gallon is a single drop?

Use the following for Exercises 38–40.

Grade is a measure of the steepness of surfaces, such as roads and ramps. Grade is expressed as a percent based on the ratio $\frac{\text{vertical rise}}{\text{horizontal run}}$. For example, a ramp that is 5 feet long and rises 1 foot has a grade of $\frac{1}{5}$, or 20%.

38. Construction A crew is building a stretch of road with a vertical rise of 15 m and a horizontal run of 375 m. Find the grade of the road.

39. Fitness A treadmill has a 9% grade. If the treadmill has a horizontal run of 5 feet, what is the treadmill’s vertical rise in inches?

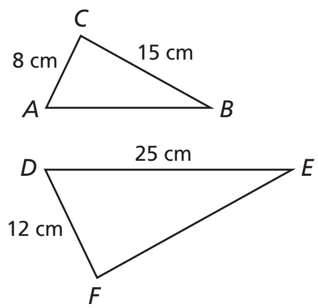
40. Accessibility The Americans with Disabilities Act set the maximum grade for wheelchair-accessible ramps at $8\frac{1}{3}$%. What is the minimum horizontal run in feet required for a ramp designed to rise 30 inches?

LINK

Hobbies

HO is the most popular model train gauge in the United States. The name may have originally meant “Half-O,” because it was thought to be about half the size of an O gauge, another model train gauge.

41. Geometry In the diagram shown, $\triangle ABC$ is similar to $\triangle DEF$. Find the lengths of sides $\overline{AB}$ and $\overline{EF}$.



Hobbies **Use the information about model trains to complete the table.**

	Railroad Gauge	Scale Length (in.)	Actual Length (ft)
42.	O	96	■
43.	■	36	261
44.	S	■	80
45.	HO	20	■

Model Trains

Railroad Gauge	HO	O	S
Model Scale	$\frac{1}{87}$	$\frac{1}{48}$	$\frac{1}{64}$

46. Show that if $b \neq 0$, $d \neq 0$, and $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

47. Write About It Explain how to justify the Cross Products Property by using the Multiplication Property of Equality.

STANDARDIZED TEST PREP

48. **Chemistry** The energy output from a chemical reaction depends on the amount of chemicals used. The table shows this relationship. What is a reasonable amount of energy from the reaction of 40 moles of the chemical?

Energy Output of a Chemical Reaction				
Amount of Chemical (moles)	5	8	12	15
Energy Output (joules)	29.89	48.01	71.96	90.12

Ⓐ 120 joules Ⓑ 160 joules Ⓒ 240 joules Ⓓ 300 joules

49. **Technology** A 38 MB file is downloading from the Internet at a constant rate. After 1 min, 18% of the file has downloaded. About how much more time should the download take?

Ⓕ 5.6 min Ⓖ 4.6 min Ⓗ 6.75 min Ⓙ 2.1 min

50. A blueprint uses a scale of $\frac{1}{4}$ inch equals 1 foot. A wall on the drawing measures $4\frac{1}{2}$ inches long. How long will the wall be in the actual building?

Ⓐ $\frac{11}{8}$ feet Ⓑ 9 feet Ⓒ 16 feet Ⓓ 18 feet

51. **Geometry** In a circle graph, how many degrees does 1% represent?

Ⓕ 1° Ⓖ 3.6° Ⓗ 6° Ⓙ 10°

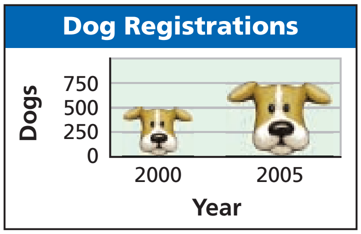
CHALLENGE AND EXTEND

Solve.

52. $\frac{-2}{x+5} = \frac{8}{x-3}$ 53. $\frac{h+4}{9} = \frac{h-3}{4}$ 54. $\frac{n-2}{4} = \frac{3n+3}{18}$ 55. $\frac{z}{12.8} = \frac{5}{z}$

56. **Construction** A concrete mix has the ratio 1 part cement, 2 parts water, and 3 parts sand. How much water can be used if 78 kg of sand and 21 kg of cement are available? How much concrete can be made?

57. **Critical Thinking** The graph intends to show the increase in the number of dogs registered. Do the icons accurately represent the data? Justify your answer.



SPIRAL REVIEW

Convert each measure using the given units. *(Previous course)*

58. $\frac{1}{5}$ h = ▢ min 59. 108 in. = ▢ yd 60. 4.5 lb = ▢ oz

61. 3.5 m = ▢ cm 62. 12 mm = ▢ cm 63. 25 mL = ▢ L

Name the three-dimensional figure that each real-world object models. *(Previous course)*

64. tennis ball 65. megaphone 66. pad of paper 67. unsharpened pencil

Identify the parent function for *h* from its function rule. Describe what transformation of the parent function it represents. *(Lesson 1-9)*

68. $h(x) = x^2 - 10$ 69. $h(x) = 3x + 4$ 70. $h(x) = 2x^3$ 71. $h(x) = -\sqrt{x+1}$

Connecting Algebra to **Geometry**

See Skills Bank page S56

Percent Increase and Decrease

California Standards

Review of 7NS1.6 Calculate the percentage of increases and decreases of a quantity.

Recall that a percent is a ratio that compares a number to 100. A proportion is a statement of two equal ratios. Review the percent change formulas below.

Percent Increase

$$\frac{\text{new measure}}{\text{original measure}} = \frac{100 + \text{percent increase}}{100}$$

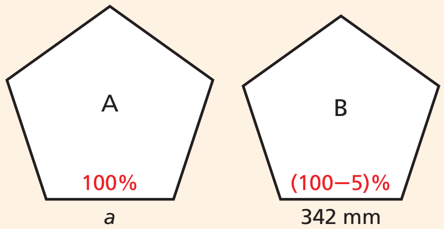
Percent Decrease

$$\frac{\text{new measure}}{\text{original measure}} = \frac{100 - \text{percent increase}}{100}$$

When solving problems involving percent change, first check to see if the change is an increase or a decrease. Then write the proportion and solve.

Example

The side lengths of Figure A are decreased by 5% to form figure B. What is the corresponding side length of figure A?



Let a represent the side length of figure A. Then 342 represents the corresponding side length of figure B. Use this information to write a proportion for percent decrease:

$$\text{new measure} \rightarrow \quad \frac{342}{a} = \frac{100 - 5}{100} \quad \leftarrow 100 - \%\text{ decrease}$$
$$\text{original measure} \rightarrow \qquad\qquad\qquad \leftarrow 100$$

$342 \cdot 100 = a \cdot (100 - 5)$	*Cross multiply.*
$34{,}200 = 95a$	*Multiply and simplify.*
$\frac{34{,}200}{95} = \frac{95a}{95}$	*Divide both sides by 95.*
$360 = a$	*Simplify.*

The corresponding length of figure A is 360 mm.

Try This

The side lengths of figure A are changed as indicated to form a similar figure B. Find the missing measure.

1. A side length of A is decreased by 30%. A side length of B is 91 in. What is the corresponding side length of A?
2. A side length of A is increased by 10%. The perimeter of B is 4200 mm. What is the perimeter of A?
3. A side length of A is increased by 75%. The area of A is 28 cm^2. What is the area of B? Is the area of B 75% greater than the area of A? (*Hint*: Consider a rectangle and look at more than one side.)

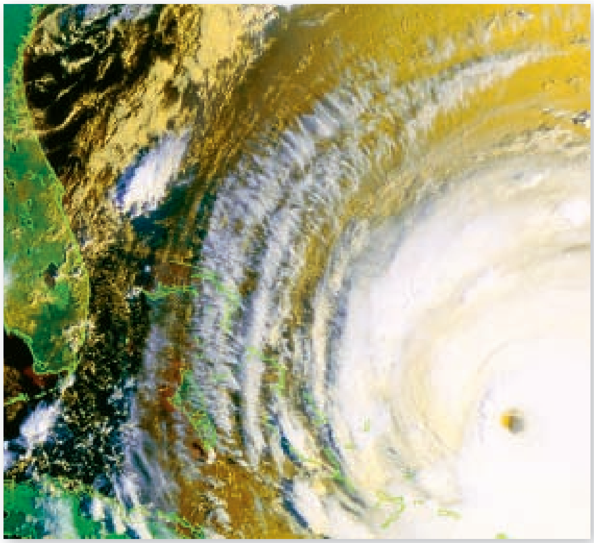
2-3 Graphing Linear Functions

Objectives
Determine whether a function is linear.

Graph a linear function given two points, a table, an equation, or a point and a slope.

Vocabulary
linear function
slope
y-intercept
x-intercept
slope-intercept form

Who uses this?
Meteorologists can use linear functions to predict when a hurricane will reach land.



Meteorologists begin tracking a hurricane's distance from land when it is 350 miles off the coast of Florida and moving steadily inland.

The meteorologists are interested in the rate at which the hurricane is approaching land.

Reading Math
The differences in the *y*-values for equally-spaced *x*-values are called *first differences.*

+1 +1 +1 +1

Time (h)	0	1	2	3	4
Distance from Land (mi)	350	325	300	275	250

−25 −25 −25 −25

This rate can be expressed as $\frac{\text{change in distance}}{\text{change in time}} = \frac{-25 \text{ miles}}{1 \text{ hour}}$. Notice that the rate of change is constant. The hurricane moves 25 miles closer each hour.

Functions with a constant rate of change are called *linear functions*. A **linear function** can be written in the form $f(x) = mx + b$, where x is the independent variable and m and b are constants. The graph of a linear function is a straight line made up of the set of all points that satisfy $y = f(x)$.

EXAMPLE 1

California Standards
Review of 1A6.0 Students graph a linear equation and compute the *x*- and *y*-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequalities (e.g., they sketch the region defined by $2x + 6y < 4$).

Recognizing Linear Functions

Determine whether each data set could represent a linear function.

A

+2 +2 +2

x	0	2	4	6
f(*x*)	−1	2	5	8

+3 +3 +3

The rate of change, $\frac{\text{change in } f(x)}{\text{change in } x}$, is constant $\frac{3}{2}$. So the data set is linear.

B

+3 +3 +3

x	−1	2	5	8
f(*x*)	0	1	3	6

+1 +2 +3

The rate of change, $\frac{\text{change in } f(x)}{\text{change in } x}$, is not constant. $\frac{1}{3} \neq \frac{2}{3} \neq \frac{3}{3}$. The data set is not linear.

CHECK IT OUT! **Determine whether each data set could represent a linear function.**

1a.

x	4	11	18	25
f(*x*)	−6	−15	−24	−33

1b.

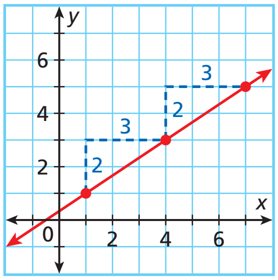
x	10	8	6	4
f(*x*)	7	5	1	−7

The constant rate of change for a linear function is its *slope*. The **slope** of a linear function is the ratio $\frac{\text{change in } f(x)}{\text{change in } x}$, or $\frac{\text{rise}}{\text{run}}$. The slope of a line is the same between any two points on the line. You can graph lines by using the slope and a point.

EXAMPLE 2 Graphing Lines Using Slope and a Point

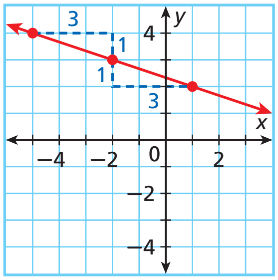
Graph each line.

A **the line with slope $\frac{2}{3}$ that passes through $(1, 1)$**



Plot the point $(1, 1)$. The slope indicates a rise of 2 and a run of 3. Move up 2 and right 3 to find another point. Repeat. Then draw a line through the points.

B **the line with slope $-\frac{1}{3}$ that passes through $(-2, 3)$**

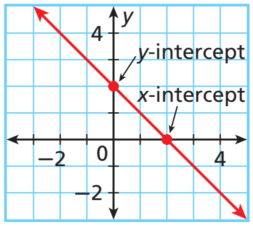


Plot the point $(-2, 3)$. The negative slope can be viewed as $\frac{-1}{3}$ or $\frac{1}{-3}$.

You can move down 1 unit and right 3 units, or move up 1 unit and left 3 units. Notice that all three points are on the same line.

CHECK IT OUT! **2.** Graph the line with slope $\frac{4}{3}$ that passes through $(3, 1)$.

Recall from geometry that two points determine a line. Often the easiest points to find are the points where a line crosses the axes. The **y-intercept** is the y-coordinate of a point where the line crosses the y-axis. The **x-intercept** is the x-coordinate of a point where the line crosses the x-axis.



EXAMPLE 3 Graphing Lines Using the Intercepts

Find the intercepts of $2x - 3y = 12$, and graph the line.

Find the x-intercept: $2x - 3y = 12$

$2x - 3(0) = 12$ *Substitute 0 for y.*

$2x = 12$

$x = 6$ *The x-intercept is 6.*

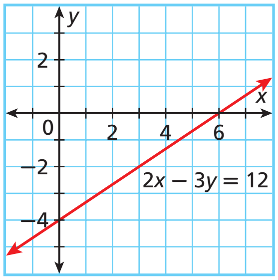
Find the y-intercept: $2x - 3y = 12$

$2(0) - 3y = 12$ *Substitute 0 for x.*

$-3y = 12$

$y = -4$ *The y-intercept is –4.*

Draw the line through $(6, 0)$ and $(0, -4)$.



Caution!

The intercept is a single value, not an ordered pair or a point.

CHECK IT OUT! **3.** Find the intercepts of $6x - 2y = -24$, and graph the line.

Linear functions can also be expressed as linear equations of the form $y = mx + b$. When a linear function is written in the form $y = mx + b$, the function is said to be in **slope-intercept form** because m is the slope of the graph and b is the y-intercept. Notice that slope-intercept form is the equation solved for y.

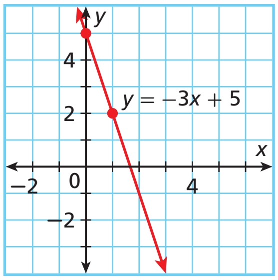
EXAMPLE 4 Graphing Functions in Slope-Intercept Form

Write each function in slope-intercept form. Then graph the function.

A $3x + y = 5$

Solve for y first.

$$\begin{aligned} 3x + y &= 5 \\ -3x \quad\; & \;\; -3x \qquad \textit{Add } -3x \textit{ to both sides.} \\ y &= -3x + 5 \end{aligned}$$

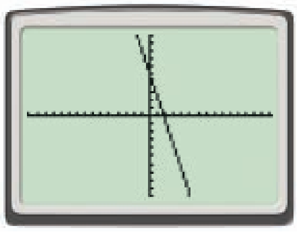


The line has y-intercept 5 and slope -3, which is $\frac{-3}{1}$. Plot the point $(0, 5)$. Then move down 3 and right 1 to find other points.

Helpful Hint

Most graphing calculators require equations to be solved for y, so slope-intercept form is the easiest to enter.

You can also use a graphing calculator to graph. Choose the standard square window to make your graph look like it would on a regular grid. Press ZOOM, choose **6:ZStandard**, press ZOOM again, and then choose **5:ZSquare**.



B $\frac{3}{2}y = x - 3$

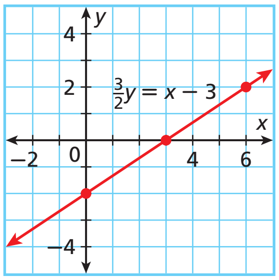
Solve for y first.

$$\frac{2}{3}\left(\frac{3}{2}y\right) = \frac{2}{3}(x - 3) \qquad \textit{Multiply both sides by } \frac{2}{3}.$$

$$y = \frac{2}{3}(x) - \frac{2}{3}(3) \qquad \textit{Distribute.}$$

$$y = \frac{2}{3}x - 2$$

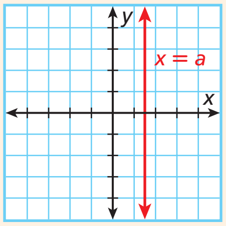
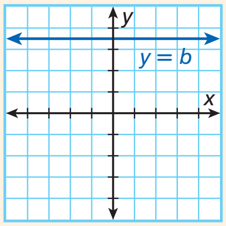
The graph of the line has y-intercept -2 and slope $\frac{2}{3}$. Plot the point $(0, -2)$. Then move up 2 and right 3 to find other points.



CHECK IT OUT! **Write each equation in slope-intercept form. Then graph the function.**

4a. $2x - y = 9$ **4b.** $5x = 15y + 30$

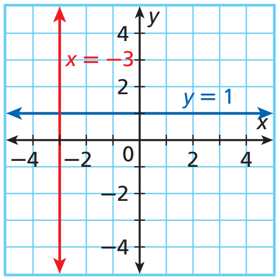
An equation with only one variable can be represented by either a vertical or a horizontal line.

Vertical and Horizontal Lines	
Vertical Lines	**Horizontal Lines**
The line $x = a$ is a vertical line at a. 	The line $y = b$ is a horizontal line at b. 

The slope of a vertical line is undefined. The slope of a horizontal line is 0.

EXAMPLE 5 Graphing Vertical and Horizontal Lines

Determine if each line is vertical or horizontal. Then graph.



A $x = -3$

This is a vertical line located at the x-value -3. (Note that it is not a function.)

B $y = 1$

This is a horizontal line located at the y-value 1.

CHECK IT OUT! **Determine if each line is vertical or horizontal. Then graph.**

5a. $y = -5$ **5b.** $x = 0.5$

EXAMPLE 6 *Travel Application*

Suppose a road rises from 2500 ft above sea level to 7000 ft in 10 mi. Find the average slope of the road. Graph the elevation against distance.

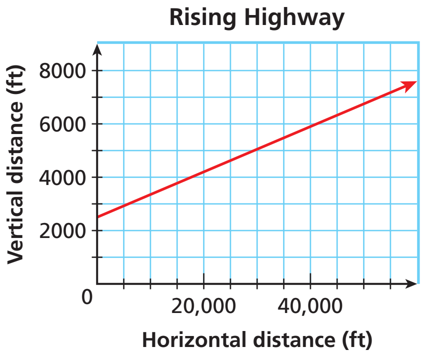
Step 1 Find the slope.

The rise is $7000 - 2500$, or 4500 ft.
The run is 10 mi.

Convert miles to feet:

$10 \text{ mi} = 10(5280) = 52{,}800 \text{ ft}$.

The slope is $\dfrac{4500}{52{,}800} \approx 0.085$.



Step 2 Graph the line.

The y-intercept is the original altitude, 2500 ft. Use $(0, 2500)$ and $(52{,}800, 7000)$ as two points on the line. Select a scale for each axis that will fit the data, and graph the function.

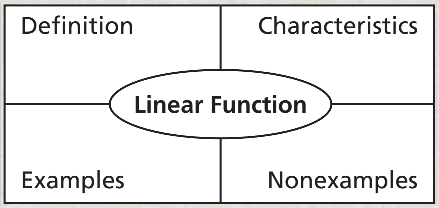
Helpful Hint

Because the graph has different scales for the x- and y-axis, the slope of the graph appears steeper than the slope of the actual road.

CHECK IT OUT! **6.** A truck driver is at mile marker 624 on Interstate 10. After 3 hours, the driver reaches mile marker 432. Find his average speed. Graph his location on I-10 in terms of mile markers.

THINK AND DISCUSS

1. Explain two different ways to graph the equation $4x = 2y - 12$.
2. Can a line have more than one slope? Explain.
3. What are the slope and y-intercept for the line that models the hurricane data at the beginning of this lesson? Explain.
4. **GET ORGANIZED** Copy and complete the graphic organizer for linear functions.



2-3 Exercises

California Standards
Review of 1A4.0, 1A5.0, 1A6.0; Preparation for 24.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 2-3
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

Apply the vocabulary from this lesson to answer each question.

1. **Vocabulary** How does the *y-intercept* differ from the *x-intercept*?
2. **Vocabulary** The rate of change of a linear function is its ___?___. (*intercept* or *slope*)

SEE EXAMPLE 1 p. 105

Determine whether each data set could represent a linear function.

3.

x	2	5	8	11
$f(x)$	9	17	25	33

4.

x	3	9	15	21
$f(x)$	1	4	10	19

SEE EXAMPLE 2 p. 106

Graph each line.

5. slope $\frac{5}{2}$; passes through $(0, 2)$
6. slope 2; passes through $(4, -5)$
7. slope $-\frac{4}{3}$; passes through $(-2, -1)$
8. slope $-\frac{2}{5}$; passes through $(3, 0)$

SEE EXAMPLE 3 p. 106

Find the intercepts of each line, and graph the line.

9. $5x + 6y = 30$
10. $2x - 3y = 24$
11. $5x - 2y = -30$
12. $-4x + 5y = 10$

SEE EXAMPLE 4 p. 107

Write each function in slope-intercept form. Then graph the function.

13. $5x + y = 4$
14. $-y = -8x$
15. $3y = 15 - 6x$
16. $2x - 5y = -6$

SEE EXAMPLE 5 p. 108

Determine if each line is vertical or horizontal. Then graph the line.

17. $x = 7$
18. $y = \frac{5}{4}$
19. $x = 0$
20. $y = -4$

SEE EXAMPLE 6 p. 108

21. **Business** Art's cash register contained \$150 when he opened the store. After 8 hours, the register contained \$738. Find the average sales per hour, and graph the hourly amount of cash in the register.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
22–23	1
24–27	2
28–31	3
32–35	4
36–39	5
40	6

Extra Practice

Skills Practice p. S6
Application Practice p. S33

Determine whether each data set could represent a linear function.

22.

x	2	3	4	5
$f(x)$	0	1	2	1

23.

x	−3	−1	1	3
$f(x)$	1	1.5	2	2.5

Graph each line.

24. slope 2; passes through $(-3, 0)$

25. slope -2.5; passes through $(1, 6)$

26. slope $-\frac{1}{4}$; passes through $(-1, -2)$

27. slope $\frac{1}{2}$; passes through $(0, -8)$

Find the intercepts of each line, and graph the line.

28. $x + y = -3$ **29.** $2x - y = 8$ **30.** $5x - 2y = 10$ **31.** $-3x + 2y = 6$

Write each function in slope-intercept form. Then graph the function.

32. $2x + y = 6$ **33.** $-y = -3x + 2$ **34.** $3y = -6 + x$ **35.** $8x - 6y = -12$

Determine if each line is vertical or horizontal. Then graph the line.

36. $x = -1$ **37.** $y = 0$ **38.** $x = 3.7$ **39.** $y = -\frac{4}{5}$

40. **Architecture** The fastest elevator in the world is in the Taipei 101 tower in Taiwan. Descending from the observation deck, the elevator travels between the two heights shown in about 7 seconds.

a. Find the average speed of the elevator, and graph the height against the time.

b. Use your graph to estimate when the elevator will reach ground level.

Graph each function.

41. $y = -\frac{1}{3}x + 2$ **42.** $x + y = 8$ **43.** $y = \frac{4}{7}x - 6$

44. $2y = 3x - 1$ **45.** $y = 4 - \frac{1}{8}x$ **46.** $0.2x + 0.6y = 1.8$

47. The beverage prices for a diner are shown.

a. Are the beverage prices a linear function of the number of ounces?

b. How much should a 32 oz drink cost?

c. What is the y-intercept? What does it represent?

d. **What if...?** Suppose the prices are all decreased by $0.20. How do your answers to parts **a**, **b**, and **c** change? What answers remain the same?

Tell whether each statement is sometimes, always, or never true.

48. If the slope of a linear function is 0, then the line is parallel to the y-axis.

49. If the y-intercept and the x-intercept of a linear function are equal, then the slope is 1.

50. If the y-intercept of a linear function is positive and the slope is negative, then the x-intercept is positive.

CONCEPT CONNECTION

51. This problem will prepare you for the Concept Connection on page 132.

The deepest point in the world's oceans, in the Marianas Trench, is 35,840 ft deep. A nautical mile is about 6,076.1 ft.

a. A league is 3 nautical miles. How many leagues deep is the Marianas Trench?

b. Graph the relationship between leagues and feet, using feet as the independent variable. Show a point representing the Marianas Trench point on your graph.

c. What does the slope of the line represent?

d. The novel *20,000 Leagues Under the Sea* was written by Jules Verne in 1870. How many feet are in 20,000 leagues? Using this answer, find out how many times the depth of the Marianas Trench 20,000 leagues is.

52. Critical Thinking If the y-intercept of a linear function is 0, what is the x-intercept? How do you know?

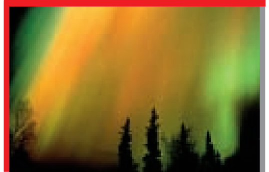
53. Critical Thinking The *standard form of a linear equation* is $Ax + By = C$.

a. Find the slope and the y-intercept of a line with this equation.

b. Use your answer to part **a** to quickly find the slope and the y-intercept for the line $12x - 4y = 18$.

LINK

Meteorology



Swedish astronomer Anders Celsius (1701–1744), for whom the Celsius temperature scale is named, published numerous observations of the aurora borealis.

54. Meteorology The table shows temperatures in both degrees Fahrenheit and degrees Celsius.

Temperature Equivalents

Temperature (°C)	–5	0	5	10	15	20
Temperature (°F)	23	32	41	50	59	68

a. Explain why this data set is linear.

b. Use Celsius temperature as the independent variable. Find the slope and the y-intercept of the line that passes through the points.

c. Graph these data. Use your graph to estimate the Celsius equivalent of 55°F.

55. School The revenue in dollars from a school play is given by the expression $5x + 2y$, where x is the number of adult tickets sold and y is the number of student tickets sold.

a. How much does each type of ticket cost if the revenue is \$220?

b. Find the x- and y-intercepts. What do the intercepts represent?

c. What if...? Suppose that after the equation is modified and graphed, the y-intercept decreases and the x-intercept remains the same. What could this indicate in the context of the problem?

56. Determine whether the data in the table are linear. Explain.

Time (s)	5	18	20	26	40
Distance (ft)	19.5	32.5	37.5	72	107

57. Write About It Explain how to find the slope of a line from a table of data.

58. Building A roof is 12 feet high at its edge and rises to a height of 20 feet at a point 10 feet horizontally from the edge. What is the slope of the roof?

STANDARDIZED TEST PREP

59. At what point does the x-intercept of the line $5x - 4y = 40$ occur?

Ⓐ $(5, 0)$ Ⓑ $(0, -10)$ Ⓒ $(0, -4)$ Ⓓ $(8, 0)$

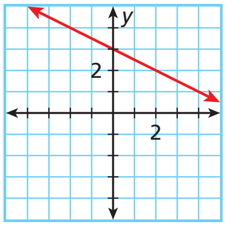
60. The graph shown could be which of these functions?

Ⓕ $y = \frac{1}{2}x + 3$

Ⓖ $y = 2x + 3$

Ⓗ $y = -\frac{1}{2}x + 3$

Ⓙ $y = -2x + 3$



61. If the slope of the line $y = 7 - 3x$ were changed to 5, what would the new equation be?

Ⓐ $y = 7 - 5x$ Ⓑ $y = 7 + 5x$ Ⓒ $y = 5 - 3x$ Ⓓ $y = -5 - 3x$

62. What is the slope of the line $5(y - 4) = -8x$?

Ⓕ $-\frac{5}{8}$ Ⓖ $\frac{5}{4}$ Ⓗ $-\frac{8}{5}$ Ⓙ $\frac{4}{5}$

63. **Gridded Response** If the rise and run are reversed for the linear equation $y = 8x + 4$, what is the slope of the new line?

CHALLENGE AND EXTEND

64. Find the slope and the y-intercept of the line $y = -4$. Does the line have an x-intercept? Explain.

65. The *double-intercept form* of a linear equation is $\frac{x}{a} + \frac{y}{b} = 1$.

a. Find the slope, the y-intercept, and the x-intercept of the line $\frac{x}{4} - \frac{y}{9} = 1$.

b. Using your answer to part **a,** explain the meaning of the values a and b in the double-intercept form of a line.

c. Write the equation of the line $5x + 2y = 30$ in double-intercept form.

66. What happens when you try to find the slope and y-intercept for the equation $3(y + 2) + 6x = 3(4 + 2x + y)$?

SPIRAL REVIEW

Multiply and simplify. *(Previous course)*

67. $\frac{3}{4}\left(\frac{2}{3}\right)$ **68.** $\frac{4}{6}\left(\frac{8}{3}\right)$ **69.** $-\frac{9}{12}\left(\frac{9}{8}\right)$

Use the set of test scores for Exercises 70–72. Find each measure.
$\{68, 72, 98, 80, 92, 76, 85, 90, 72, 86\}$ *(Previous course)*

70. mode **71.** mean **72.** median

For each function, evaluate $f(0)$ and $f(-3)$. *(Lesson 1-7)*

73. $f(x) = \frac{1}{3}x + 7$ **74.** $f(x) = -4x^2 - 1$ **75.** $f(x) = \frac{x^3}{3}$

Solve. *(Lesson 2-1)*

76. $7(x + 9) = 8(x - 3)$ **77.** $\frac{3}{4}(x + 12) + \frac{1}{2}(x + 6) = -18$

78. $7n + 4(n - 1) = 3(n + 4)$ **79.** $9t - 3(t - 5) = 51$

2-3 **Technology LAB**

Explore Graphs and Windows

go.hrw.com
Lab Resources Online
KEYWORD: MB7 Lab2

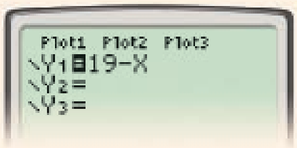
Use with Lesson 2-3

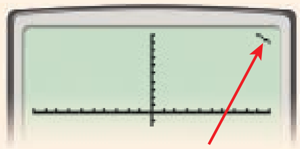
When using a graphing calculator to explore graphs, it is important to understand how the WINDOW settings affect the *visual* behavior of the graph. The standard window is usually not the best window and does not usually show the more accurate graph.

Activity 1

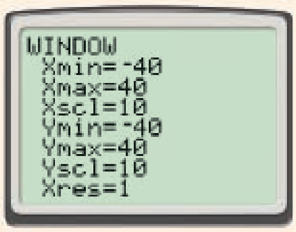
Graph $y = 19 - x$ in a window that shows both x- and y-intercepts.

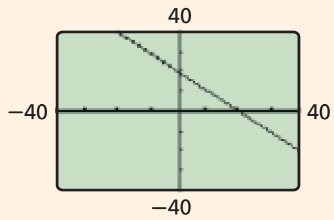
1 Enter $19 - x$ in **Y1**, and press ZOOM **6:ZStandard** to obtain the standard window, $[-10, 10]$ by $[-10, 10]$. You see only a small piece of the graph.





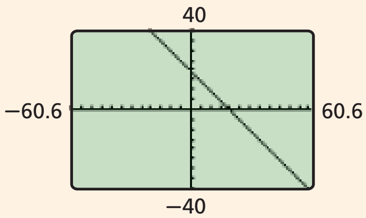
2 Press WINDOW and change the window settings as shown and graph again. By increasing the window dimensions, you can now see both intercepts, but the line looks flatter than the same line graphed on grid paper.





The graphing calculator screen is about 1.5 times as wide (95 pixels) as it is high (63 pixels), so it distorts graphs when the horizontal and vertical dimensions are the same. To correct for this, use a square viewing window.

3 Press ZOOM **5:ZSquare**. **Xmin** and **Xmax** will change to show an accurate graph that displays both intercepts. Notice the change in the window settings.



Try This

Graph each function in a window that shows both the x- and y-intercepts.

1. $y = 2x - 25$
2. $y = -3x - 50$
3. $y = 20 + 0.8x$

4. When you enter and graph a function and only a piece of the graph is visible in the lower left, what adjustments can you make to see the key features of the graph?

5. How can you see the graph of $y = 0.01x$?

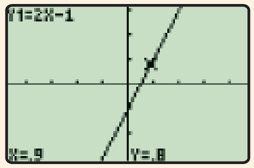
6. **What if...?** Suppose that you wanted to make $y = 0.5x$ look very steep or $y = 10x$ look flat in the calculator window. How would you change the window settings?

When you use the TRACE function, x-values are often long decimals. A *friendly* window allows you to trace along simpler x-values. There are several built-in ZOOM windows that give friendly trace values, such as **ZInteger**, and **ZDecimal**.

Activity 2

Find decimal values of the coordinates of points on $y = 2x - 1$.

1. Enter $2x - 1$ in **Y1**. Press ZOOM **4:ZDecimal**.



2. TRACE right or left to find the decimal values.

Note the following in the decimal window:

Horizontal dimensions: **Xmax** − **Xmin** = 4.7 − (−4.7) = **9.4** and

Vertical dimensions: **Ymax** − **Ymin** = 3.1 − (−3.1) = **6.2**

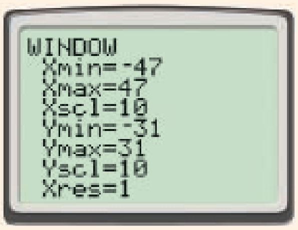
If you use multiples of 9.4 for the horizontal dimensions and multiples of 6.2 for the vertical dimensions, you will always have simple x-values and an undistorted graph.

Activity 3

Find integer values of the coordinates of points on $y = -4x + 15$.

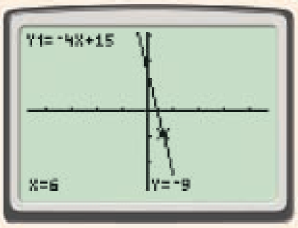
1. Enter $-4x + 15$ in **Y1**.

2. Press ZOOM **8:ZInteger** and press ENTER twice.



The window changes so that the x-values are integers and tracing to the right increases x-values by 1. The window is also square since **Xmax** − **Xmin** = 94 and **Ymax** − **Ymin** = 62.

3. TRACE to find the integer values.



After graphing the function, you can move to any location on the screen to act as the center of the next graph and use **ZInteger** again.

Try This

7. If you ZOOM out on the point (0.9, 0.8) shown in the graph in Activity 3, the window changes to [−8.5, 10.3] by [−5.4, 7]. Find **Xmax** − **Xmin** and **Ymax** − **Ymin**. Use TRACE and the arrow keys to view the coordinates of points on the line. Why is the window friendly?

8. Explain how to graph $y = 3x - 50$ so that the line "looks like" a line with a slope of 3 and allows you to trace to friendly x-values.

9. Graph $y = x + 0.5$ in the standard window.

a. How can you make the slope of the graph appear to be 1?

b. Which ZOOM window would create a space between the y-intercept and the origin while keeping an accurate representation of the slope?

2-4 Writing Linear Functions

Objectives

Use slope-intercept form and point-slope form to write linear functions.

Write linear functions to solve problems.

Vocabulary

Point-slope form

Why learn this?

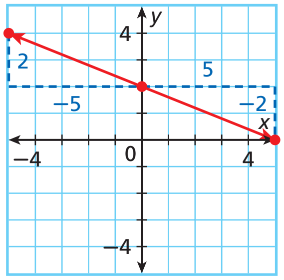
When you play Monopoly, it's easy to calculate the rent of most properties by looking at the selling price. (See Example 4.)

Recall from Lesson 2-3 that the slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and b is its y-intercept.

In Lesson 2-3, you graphed lines when you were given the slope and y-intercept. In this lesson you will write linear functions when you are given graphs of lines or problems that can be modeled with a linear function.

EXAMPLE 1 Writing the Slope-Intercept Form of the Equation of a Line

Write the equation of the graphed line in slope-intercept form.



Step 1 Identify the y-intercept.
The y-intercept b is 2.

Step 2 Find the slope.

Choose any two convenient points on the line, such as $(0, 2)$ and $(5, 0)$. Count from $(0, 2)$ to $(5, 0)$ to find the rise and the run. The rise is –2 units and the run is 5 units.

Slope is $\frac{\text{rise}}{\text{run}} = \frac{-2}{5} = -\frac{2}{5}$.

Step 3 Write the equation in slope-intercept form.

$y = mx + b$

$y = -\frac{2}{5}x + 2$ $\quad m = -\frac{2}{5}$ and $b = 2$

The equation of the line is $y = -\frac{2}{5}x + 2$.

Remember!

To express a line as a linear function, replace y with $f(x)$.

$y = -\frac{2}{5}x + 2$

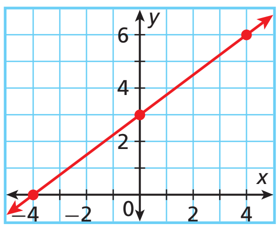
$f(x) = -\frac{2}{5}x + 2$

California Standards

Review of 1A7.0 Students verify that a point lies on a line, given an equation of the line. **Students are able to derive linear equations by using the point-slope formula.**
Also covered: **Review of 1A8.0**

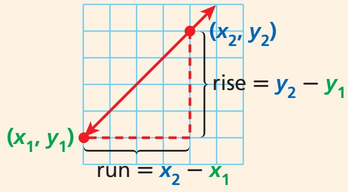
CHECK IT OUT!

1. Write the equation of the graphed line in slope-intercept form.



Notice that for two points on a line, the rise is the difference in the y-coordinates, and the run is the difference in the x-coordinates. Using this information, we can define the slope of a line by using a formula.

Slope Formula

WORDS	ALGEBRA	GRAPH
Given two points on a line, the slope is the ratio of the difference in the *y*-values to the difference in the corresponding *x*-values, or rise over run.	The slope of the line containing (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.	

EXAMPLE 2 Finding the Slope of a Line Given Two or More Points

Find the slope of each line.

A **the line through $(3, -2)$ and $(-1, 2)$**

Let be (x_1, y_1) be $(3, -2)$ and (x_2, y_2) be $(-1, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{-1 - 3} = \frac{4}{-4} = -1$$ *Use the slope formula.*

The slope of the line is -1.

Helpful Hint

If you reverse the order of the points in Example 2B, the slope is still the same.

$$m = \frac{6 - 16}{5 - 11} = \frac{-10}{-6} = \frac{5}{3}$$

B

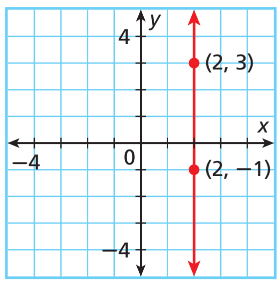
x	2	5	8	11
y	1	6	11	16

Let (x_1, y_1) be $(5, 6)$, and (x_2, y_2) be $(11, 16)$. *Choose any two points.*

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16 - 6}{11 - 5} = \frac{10}{6} = \frac{5}{3}$$ *Use the slope formula.*

The slope of the line is $\frac{5}{3}$.

C **The line shown.**



Either point may be chosen as (x_1, y_1).

Let (x_1, y_1) be $(2, -1)$ and (x_2, y_2) be $(2, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-1)}{2 - 2} = \frac{4}{0}$$

Because division by zero is undefined, the slope of the line is undefined.

CHECK IT OUT! **Find the slope of each line.**

2a.

x	−6	−4	−2
y	−3	−1	1

2b. the line through $(2, -5)$ and $(-3, -5)$

Because the slope of a line is constant, it is possible to use any point on a line and the slope of the line to write an equation of the line in **point-slope form**.

Know it! Note — Point-Slope Form

The equation of a line with a slope of m and the point (x_1, y_1) is

$$y - y_1 = m(x - x_1).$$

EXAMPLE 3 Writing Equations of Lines

In slope-intercept form, write the equation of the line that contains the points in the table.

x	-3	-1	1	3
y	1.5	1	0.5	0

First, find the slope. Let (x_1, y_1) be $(-1, 1)$ and (x_2, y_2) be $(3, 0)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{3 - (-1)} = \frac{-1}{3 + 1} = -\frac{1}{4}$$

Next, choose a point and use either form of the equation of a line.

Method A Point-Slope Form

Using $(3, 0)$:

$y - y_1 = m(x - x_1)$

$y - (0) = -\frac{1}{4}(x - 3)$ *Substitute.*

$y = -\frac{1}{4}(x - 3)$ *Simplify.*

Rewrite in slope-intercept form.

$y = -\frac{1}{4}(x - 3)$

$y = -\frac{1}{4}x + \frac{3}{4}$ *Distribute.*

Method B Slope-Intercept Form

Using $(3, 0)$, solve for b.

$y = mx + b$

$0 = \left(-\frac{1}{4}\right)3 + b$ *Substitute.*

$0 = -\frac{3}{4} + b$ *Simplify.*

$b = \frac{3}{4}$ *Solve for b.*

Rewrite the equation using m and b.

$y = -\frac{1}{4}x + \frac{3}{4}$ *$y = mx + b$*

The equation of the line is $y = -\frac{1}{4}x + \frac{3}{4}$.

CHECK IT OUT! **Write the equation of each line in slope-intercept form.**

3a. with slope –5 through $(1, 3)$

3b. through $(-2, -3)$ and $(2, 5)$

Student to Student

Slope and Point-Slope Form

Jennifer Chang
Jefferson High School

I learned the point-slope form by relating it to the formula for slope. The formula for slope and point-slope form are basically the same equation in different forms.

Begin with the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Substitute (x, y) for (x_2, y_2): $m = \frac{y - y_1}{x - x_1}$

Multiply both sides by $(x - x_1)$: $m(x - x_1) = y - y_1$

Reverse the equation: $y - y_1 = m(x - x_1)$

EXAMPLE 4 *Entertainment Application*

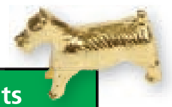
In the game of Monopoly, a player who lands on a property that is owned by another player must pay rent to the owner of the property. For most color properties, the rent can be modeled by a linear function of the selling price.

A **Express the rent as a function of the selling price.**

Let x = selling price and y = rent.

Find the slope by choosing two points. Let (x_1, y_1) be $(60, 2)$ and (x_2, y_2) be $(100, 6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{100 - 60} = \frac{4}{40} = \frac{1}{10}$$



Monopoly Prices and Rents

Property Name	Selling Price ($)	Rent ($)
Mediterranean Ave.	60	2
Vermont Ave.	100	6
Tennessee Ave.	180	14
Marvin Gardens	280	24
Pennsylvania Ave.	320	28

To find the equation for the rent function, use point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{10}(x - 60) \qquad \textit{Use the data for Mediterranean Ave.}$$

$$y = \frac{1}{10}x - 4 \qquad \textit{Simplify.}$$

B **Graph the relationship between the selling price and the rent. How much is the rent for Illinois Ave., which has a selling price of $240?**

Graph the function using a scale that fits the data.

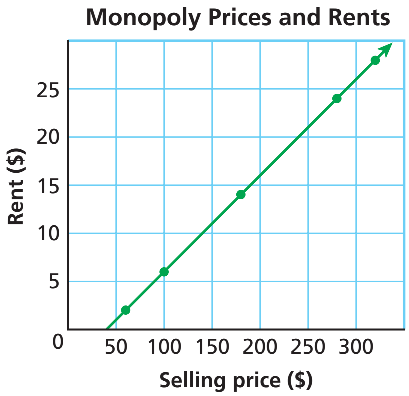
To find the rent for Illinois Avenue, use the graph or substitute its selling price of $240 into the function.

$$y = \frac{1}{10}(240) - 4 \qquad \textit{Substitute.}$$

$$y = 24 - 4$$

$$y = 20$$

The rent for Illinois Avenue is $20.



CHECK IT OUT!

4a. Express the cost as a linear function of the number of items.

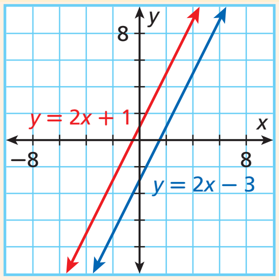
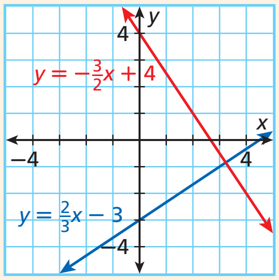
4b. Graph the relationship between the number of items and the cost. Find the cost of 18 items.

Items	Cost ($)
4	14.00
7	21.50
18	■

By comparing slopes, you can determine if lines are parallel or perpendicular. You can also write equations of lines that meet certain criteria.

Know it! Note

Parallel and Perpendicular Lines

WORDS	GRAPH	ALGEBRA
Parallel Lines		
If both slopes are defined, the slopes of parallel lines are equal. The slopes of parallel vertical lines are undefined.		$y_1 = 2x + 1$, so $m_1 = 2$ $y_2 = 2x - 3$ so $m_2 = 2$ $m_1 = m_2$ $2 = 2$
Perpendicular Lines		
If both slopes are defined, the slopes of perpendicular lines are opposite reciprocals. Their product is −1. A vertical line and a horizontal line are perpendicular.		$y_1 = -\frac{3}{2}x + 4$, so $m_1 = -\frac{3}{2}$ $y_2 = \frac{2}{3}x - 3$, so $m_2 = \frac{2}{3}$ $(m_1)(m_2) = -1$ $\left(-\frac{3}{2}\right)\left(\frac{2}{3}\right) = -1$

Remember!

A vertical line has an undefined slope.

EXAMPLE 5 Writing Equations of Parallel and Perpendicular Lines

Write the equation of each line in slope-intercept form.

A **parallel to $y = 1.5x + 6$ and through $(4, 5)$**

$m = 1.5$ — *Parallel lines have equal slopes.*

$y - 5 = 1.5(x - 4)$ — *Use $y - y_1 = m(x - x_1)$ with $(x_1, y_1) = (4, 5)$.*

$y - 5 = 1.5x - 6$ — *Distributive property.*

$y = 1.5x - 1$ — *Simplify.*

B **perpendicular to $y = -\frac{3}{4}x + 2$ and through $(6, -4)$**

The slope of the given line is $-\frac{3}{4}$, so the slope of the perpendicular line is the opposite reciprocal, $\frac{4}{3}$.

$y + 4 = \frac{4}{3}(x - 6)$ — *Use $y - y_1 = m(x - x_1)$. $y + 4$ is equivalent to $y - (-4)$.*

$y + 4 = \frac{4}{3}x - 8$ — *Distributive property.*

$y = \frac{4}{3}x - 12$ — *Simplify.*

CHECK IT OUT! **Write the equation of each line in slope-intercept form.**

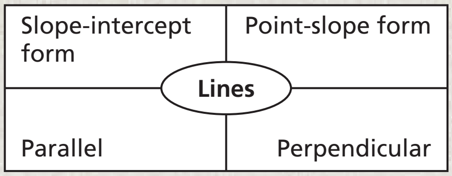
5a. parallel to $y = 5x - 3$ and through $(1, 4)$

5b. perpendicular to $y = \frac{5}{6}x - 7$ and through $(0, -2)$

THINK AND DISCUSS

1. Explain why the slope of a vertical line such as $x = 2$ is undefined.
2. Describe the information that you need in order to write the equation of a line.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write any appropriate formulas and examples of equations.

Know it! Note



2-4 Exercises

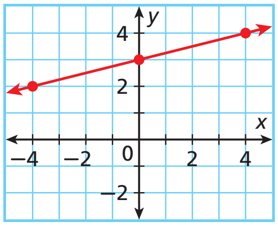
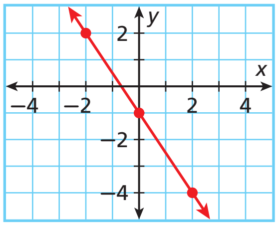
California Standards
Review of 1A5.0, 1A7.0, 1A8.0, 1A15.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 2-4
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 114

Write the equation of each line in slope-intercept form.

1. a line with slope 2 and intercept 1
2. a line with slope $-\frac{1}{7}$ and y-intercept -2
3. 
4. 

SEE EXAMPLE 2 p. 115

Find the slope of each line.

5.

x	2	7	12	17
y	3	10	17	24

6. a line through $(12, 3)$ and $(3, -4)$

SEE EXAMPLE 3 p. 116

Write the equation of each line in slope-intercept form.

7. a line with slope $-\frac{4}{3}$ passing through $(4, -8)$
8.

x	−2	2	6	10
y	−10	−7	−4	−1

SEE EXAMPLE 4 p. 117

9. **Physics** The boiling point of water can be modeled as a linear function of altitude. The boiling point of water at sea level is 212°F, and the boiling point of water at 1100 ft above sea level is 210°F.
   a. Express the boiling point as a function of altitude.
   b. Graph the relationship between boiling point and altitude.
   c. Find the boiling point of water at an altitude of 11,000 ft.

SEE EXAMPLE 5 p. 118

Write the equation of each line in slope-intercept form.

10. parallel to $y = 3x + 4$ passing through $(0, 9)$
11. perpendicular to $y = \frac{5}{9}x + 4$ passing through $(0, -4)$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–14	1
15–16	2
17–18	3
19	4
20–21	5

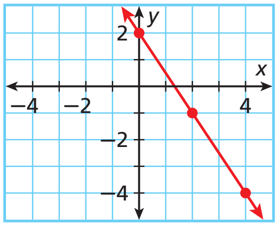
Extra Practice

Skills Practice: S6

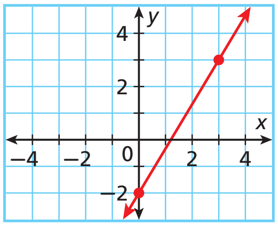
Application Practice: S33

Write the equation of each line in slope-intercept form.

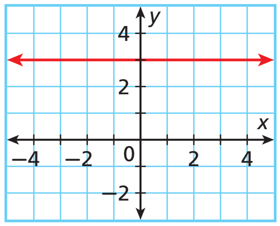
12.



13.



14.



Find the slope of each line.

15.

x	0	1	2	3
y	$-\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{5}{3}$

16. line $\overleftrightarrow{AB}$ through $A(-1, 3)$ and $B(1, -4)$

Write the equation of each line in slope-intercept form.

17. passing through $(3, 11)$ with slope $\frac{7}{3}$

18.

x	10	15	20	25
y	−2	−7	−12	−17

LINK

Math History

It is unknown why the letter *m* is used to represent slope. Some have claimed that French mathematician René Descartes used it to represent the French word *monter* (to climb). However, this theory has proven to be false.

19. Biology The table shows the number of times a firefly flashes per minute at various temperatures.

Firefly Flashing Rate

Temperature (°F)	Flashes per Minute
84	16
93	20
66	8

a. Express the flashing rate $f(T)$ as a function of temperature T.

b. Graph the relationship between temperature and the number of flashes per minute.

c. At what temperature would a firefly flash 25 times per minute?

d. How many times per minute would a firefly flash at 35°F. Is this reasonable?

Write the equation of each line in slope-intercept form.

20. parallel to $y = -\frac{1}{5}x - 7$ and through $(2, 3)$

21. perpendicular to $y = 3x$ and through $(0, 3)$

22. Clothing Men's shoe sizes are a linear function of foot length.

Men's Shoe Sizes

Foot Length (in.)	Shoe Size
10	$7\frac{1}{2}$
11	$11\frac{1}{2}$

a. Write an equation for a man's shoe size as a function of foot length. What men's size shoe is needed for a foot that measures 9.5 in.?

b. Women's shoe sizes are marked $1\frac{1}{2}$ sizes larger than men's sizes for the same foot length. What size shoe is needed for a women's foot that measures 8.5 in.?

Determine if each pair of lines is parallel, perpendicular, or neither.

23. $y = \frac{1}{4}x + 9$
$y = 4x - 9$

24. $y = 5 - \frac{1}{8}x$
$y = 8x + 2$

25. $-3x + 4y = 15$
$9x - 12y = 24$

Write each linear function.

26. $f(x)$, where $f(3) = 3$ and $f(-1) = 4$

27. $f(x)$, where $f(-2) = -5$ and $f(1) = 1$

CONCEPT CONNECTION

28. This problem will prepare you for the Concept Connection on page 132.

Steve Fossett, the balloonist in Lesson 2-1, holds the world sailing record for the fastest transatlantic crossing: 4 days, 17 hours, 28 minutes, 6 seconds, at an average speed of 25.78 knots (nautical mi/h).

a. What was his crossing time, in hours, as a decimal value to the nearest tenth?

b. How many nautical miles did he travel, to the nearest tenth?

c. Recall that a nautical mile is about 1.15 statute miles. What was Fossett's average speed in statute mi/h, to the nearest tenth?

For Exercises 29–37, write the equation of the line with the given properties.

29. a slope of 4 passing through $(1, 7)$

30. a slope of $-\frac{1}{2}$ passing through $(7, -3)$

31. passing through $(-5, 7)$ and $(3, -4)$

32. passing through $(-3, 3)$ and $(1, -1)$

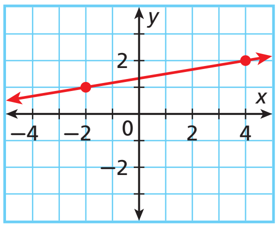
33.

x	4	7.5	8
y	44	117.5	128

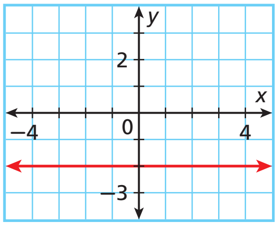
34.

x	0	30	100
y	32	86	212

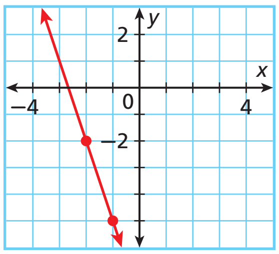
35.



36.



37.



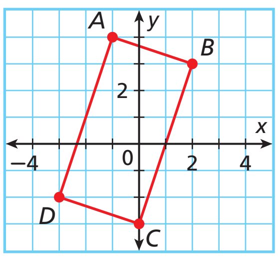
38. **Critical Thinking** Which of the Monopoly properties in the table does not conform to the rent function, $y = \frac{1}{10}x - 4$? Explain.

Monopoly Prices and Rents

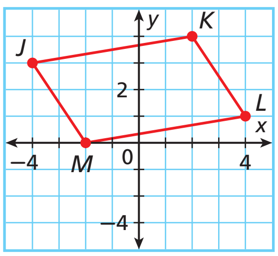
Property Name	Selling Price ($)	Rent ($)
Connecticut Ave.	120	8
Kentucky Ave.	220	18
Park Place	350	35

Geometry **Find the slope of each segment, and then classify each quadrilateral.**

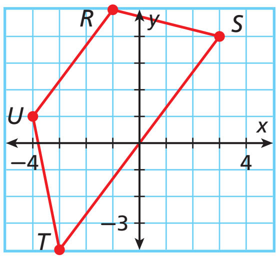
39.



40.



41.



42. **ERROR ANALYSIS** Two attempts to find the slope of the line containing $(5, 8)$ and $(1\ 2, 7)$ are shown. Identify which calculation is incorrect. Explain the error.

A

$$m = \frac{8 - 7}{5 - 12} = -\frac{1}{7}$$

B

$$m = \frac{8 - 7}{12 - 5} = \frac{1}{7}$$

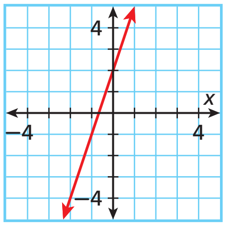
43. **Write About It** Explain how to write the equation of a line from its graph.

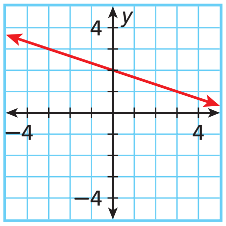
STANDARDIZED TEST PREP

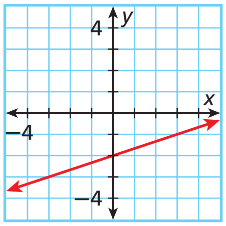
44. A carpenter determines the cost of a job by using the formula $C = 25 + 25h$, where h is the number of hours he works. He has decided to increase the amount he charges per hour to \$30. Which formula will he use now?

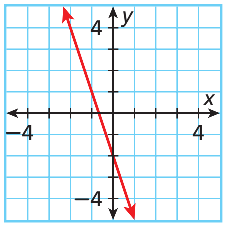
Ⓐ $C = 30 + 25h$ Ⓑ $C = 30 + 30h$ Ⓒ $C = 25 + 30h$ Ⓓ $C = 25h + 30$

45. Which graph best shows a line perpendicular to $y = 3x - 2$?

Ⓕ


Ⓖ


Ⓗ


Ⓙ


46. An equation can be used to relate the cost c of carpeting a room to the area a of the room in square feet. Which equation accurately reflects the data in the table?

Ⓐ $c = 2a - 125$ Ⓒ $c = a + 275$

Ⓑ $c = 1.5a + 75$ Ⓓ $c = 2a - 1500$

Carpeting Costs

Area (ft^2)	Cost (\$)
400	675
550	900
900	1425

CHALLENGE AND EXTEND

47. Show that $y = \frac{2}{5}x + \frac{1}{4}$ and $y - \frac{25}{4} = \frac{2}{5}(x - 15)$ represent the same line.

48. Find the value of k so that the line containing $(4, -3k)$ and $(2k, 5)$ has a slope of $m = \frac{5}{2}$.

49. Are the points $(2, 6)$, $(5, 10)$, and $(9, 15)$ on the same line? Explain.

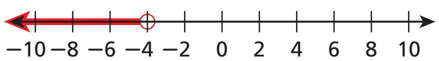
50. The slope-intercept form of a linear equation can be derived from the point-slope form. Illustrate this statement by substituting the point $(0, b)$ for (x_1, y_1) into the point-slope equation and solving for y.

51. **Aeronautics** A rule that airline pilots use to estimate outside temperature in degrees Fahrenheit at an altitude of h thousand feet is to double h, subtract 15, and multiply the result by –1. State a rule for the altitude in feet based upon the outside temperature. At what altitude is outside temperature about –51°F?

SPIRAL REVIEW

Use interval notation to represent each set of numbers. *(Lesson 1-1)*

52. $-4 \le x \le 8$ or $x > 12$

53.


Determine whether the ordered pair is a solution of both $2x + y = 5$ and $\frac{3}{4}x < -5y$. *(Lesson 2-1)*

54. $(0, 0)$ **55.** $(-1, 6)$ **56.** $(2, 1)$ **57.** $(3, -1)$

58. **Entertainment** A scaled replica of the Eiffel Tower at Kings Island Amusement Park is 331 ft 6 in. tall. The Eiffel Tower in Paris is 994 ft 6 in. tall. What percent of the height of the Eiffel Tower is the replica's height? *(Lesson 2-2)*

2-5 Linear Inequalities in Two Variables

Objectives
Graph linear inequalities on the coordinate plane.

Solve problems using linear inequalities.

Vocabulary
linear inequality
boundary line

California Standards

Review of 1A6.0 Students graph a linear equation and compute the x- and y-intercepts (e.g., graph $2x + 6y = 4$). They **are also able to sketch the region defined by linear inequalities (e.g., they sketch the region defined by $2x + 6y < 4$).**

Helpful Hint

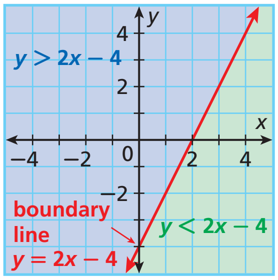
Think of the underlines in the symbols $\leq$ and $\geq$ as representing solid lines on the graph.

Who uses this?

A movie theater manager may use linear inequalities to find the numbers of different-priced tickets that must be sold to make a profit. (See Example 3.)

Linear functions form the basis of *linear inequalities*. A **linear inequality** in two variables relates two variables using an inequality symbol, such as $y > 2x - 4$. Its graph is a region of the coordinate plane bounded by a line. The line is a **boundary line**, which divides the coordinate plane into two regions.

For example, the line $y = 2x - 4$, shown at right, divides the coordinate plane into two parts: one where $y > 2x - 4$ and one where $y < 2x - 4$. In the coordinate plane higher points have larger y values, so the region where $y > 2x - 4$ is above the boundary line where $y = 2x - 4$.



To graph $y \geq 2x - 4$, make the boundary line solid, and shade the region above the line. To graph $y > 2x - 4$, make the boundary line dashed because y-values equal to $2x - 4$ are not included.

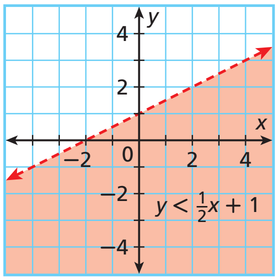
EXAMPLE 1 **Graphing Linear Inequalities**

Graph each inequality.

A $y < \dfrac{1}{2}x + 1$

The boundary line is $y = \frac{1}{2}x + 1$, which has a y-intercept of 1 and a slope of $\frac{1}{2}$.

Draw the boundary line dashed because it is not part of the solution. Then shade the region below the boundary line to show $y < \frac{1}{2}x + 1$.



Check Choose a point in the solution region, such as $(0, 0)$ and test it in the inequality.

$$y < \frac{1}{2}x + 1$$

$$0 \overset{?}{<} \frac{1}{2}(0) + 1$$

$$0 \overset{?}{<} 1 \checkmark$$

The test point satisfies the inequality, so the solution region appears to be correct.

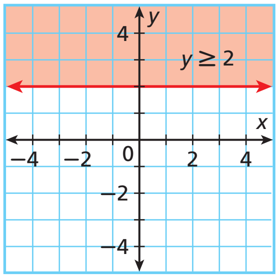
Graph each inequality.

B $y \geq 2$

Recall that $y = 2$ is a horizontal line.

Step 1 Draw a solid line for $y = 2$ because the boundary line is part of the graph.

Step 2 Shade the region above the boundary line to show where $y > 2$.



Check The point $(0, 4)$ is a solution because $4 \geq 2$. Note that any point on or above $y = 2$ is a solution, regardless of the value of x.

CHECK IT OUT! **Graph each inequality.**

1a. $y \geq 3x - 2$ **1b.** $y < -3$

If the equation of the boundary line is not in slope-intercept form, you can choose a test point that is not on the line to determine which region to shade. If the point satisfies the inequality, then shade the region containing that point. Otherwise, shade the other region.

EXAMPLE 2 Graphing Linear Inequalities Using Intercepts

Graph $2x + 3y \geq 6$ using intercepts.

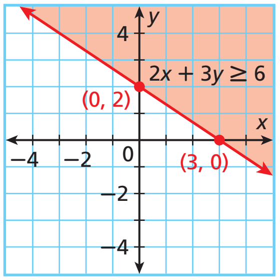
Step 1 Find the intercepts.

Substitute $x = 0$ and then $y = 0$ into $2x + 3y = 6$ to find the intercepts of the boundary line.

***y*-intercept**	***x*-intercept**
$2x + 3y = 6$	$2x + 3y = 6$
$2(0) + 3y = 6$	$2x + 3(0) = 6$
$3y = 6$	$2x = 6$
$y = 2$	$x = 3$

Step 2 Draw the boundary line.

The line goes through $(0, 2)$ and $(3, 0)$. Draw a solid line for the boundary because it is part of the graph.



Step 3 Find the correct region to shade.

Substitute $(0, 0)$ into the inequality. Because $0 + 0 \geq 6$ is false, shade the region that does *not* contain $(0, 0)$.

Helpful Hint

The point $(0, 0)$ is the easiest point to test if it is not on the boundary line.

CHECK IT OUT! **2.** Graph $3x - 4y > 12$ using intercepts.

Many applications of inequalities in two variables use only nonnegative values for the variables. Graph only the part of the plane that includes realistic solutions.

EXAMPLE 3 Problem-Solving Application

PROBLEM SOLVING

A local theater charges $7.50 for adult tickets and $5.00 for discount tickets. The theater needs to make at least $240 to cover the rent of the building. How many of each type of ticket must be sold to make a profit? If 20 discount tickets are sold, how many adult tickets must be sold?

1 Understand the Problem

The **answer** will be in two parts: (1) an inequality graph showing the number of each type of ticket that must be sold to make a profit (2) the number of adult tickets that must be sold to make at least $240 if 20 discount tickets are sold.

List the important information:

- The theater sells tickets for $7.50 and $5.00.
- The theater needs to make at least $240.

2 Make a Plan

Let x represent the number of adult tickets and y represent the number of discount tickets that must be sold. Write an inequality to represent the situation.

Adult price	times	number of adult tickets	plus	discount price	times	number of discount tickets	is at least	total.
7.50	$\cdot$	x	$+$	5.00	$\cdot$	y	$\geq$	240

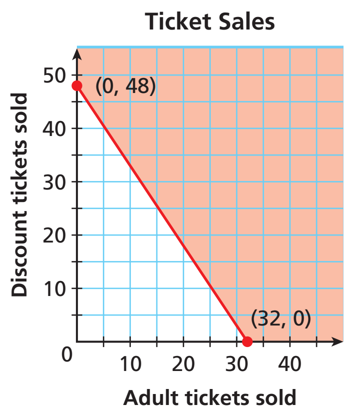
An inequality that models the problem is $7.5x + 5y \geq 240$.

3 Solve

Find the intercepts of the boundary line.

$$7.5(0) + 5y = 240 \qquad\qquad 7.5x + 5(0) = 240$$
$$y = 48 \qquad\qquad x = 32$$

Graph the boundary line through $(0, 48)$ and $(32, 0)$ as a solid line. Shade the region above the line that is in the first quadrant, as ticket sales cannot be negative.



Caution!

Don't forget which variable represents which quantity.

If 20 discount tickets are sold,

$7.5x + 5(20) \geq 240$ *Substitute 20 for y in 7.5x + 5y ≥ 240.*

$7.5x + 100 \geq 240$ *Multiply 5 by 20.*

$7.5x \geq 140$, so $x \geq 18.\overline{6}$ *A whole number of tickets must be sold.*

At least 19 adult tickets must be sold.

4 Look Back

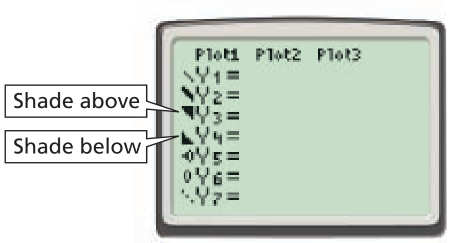
$19(\$7.50) + 20(\$5.00) = \$242.50$, so the answer is reasonable.

CHECK IT OUT! **3.** A café gives away prizes. A large prize costs the café \$125, and the small prize costs \$40. The café will not spend more than \$1500. How many of each prize can be awarded? How many small prizes can be awarded if 4 large prizes are given away?

You can graph a linear inequality that is solved for y with a graphing calculator.

Press **Y=** and use the left arrow key to move to the left side.

Each time you press **ENTER** you will see one of the graph styles shown here. You are already familiar with the line style.



EXAMPLE 4 Solving and Graphing Linear Inequalities

Solve $\frac{2}{3}(2x - y) < 2$ for y. Graph the solution.

$\frac{3}{2} \cdot \frac{2}{3}(2x - y) < \frac{3}{2} \cdot 2$ *Multiply both sides by $\frac{3}{2}$.*

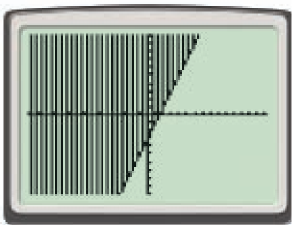
$2x - y < 3$

$-y < -2x + 3$ *Subtract 2x from both sides.*

$y > 2x - 3$ *Multiply by -1, and reverse the inequality symbol.*

Use the calculator option to shade above the line $y = 2x - 3$.

Note that the graph is shown in the standard square window (**ZOOM** **6:ZStandard** followed by **ZOOM** **5:ZSquare**).



Remember!

When multiplying or dividing an inequality by a negative number, reverse the inequality symbol.

CHECK IT OUT! **4.** Solve $2(3x - 4y) > 24$ for y. Graph the solution.

THINK AND DISCUSS

1. Compare the open and closed circles in graphs of inequalities with the dashed and solid lines in graphs of linear inequalities.

2. Describe what the graph of $x \geq 4$ would look like on a coordinate plane.

3. Explain whether you can use $(0, 0)$ to determine which side of the graph of $3x + 5y \leq 0$ to shade.

Know it! Note

4. **GET ORGANIZED** Copy and complete the graphic organizer. For each graph description, give examples of corresponding inequalities solved for y and inequalities in other forms.

Dashed Line, Shaded Above	Dashed Line, Shaded Below	Solid Line, Shaded Above	Solid Line, Shaded Below

2-5 Exercises

California Standards
Review of 1A6.0, 1A7.0, 1A8.0; Preparation for 24.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 2-5
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** Explain how the graph of $y = 3x - 4$ can be a *boundary line.*

SEE EXAMPLE 1 p. 124

Graph each inequality.

2. $y > -4$
3. $y \leq 2$
4. $y \geq x - 3$
5. $y < -\frac{1}{3}x + 2$

SEE EXAMPLE 2 p. 125

Graph each inequality using intercepts.

6. $3x + 2y > 12$
7. $5x - 2y \leq 20$
8. $-4x + 5y < -20$

SEE EXAMPLE 3 p. 126

9. **Consumer** Charisse is buying two different types of cereals from the bulk bins at the store. Granola costs \$2.29 per pound, and muesli costs \$3.75 per pound. She has \$7.00. Use x as the amount of granola and y as the amount of muesli.
   a. Write and graph an inequality for the amounts of each cereal she can buy.
   b. How many pounds of granola can she buy if she buys 1.5 pounds of muesli?

10. **School** The senior class sells hamburgers and hot dogs at a football game and makes a profit of \$1.75 on each hamburger and \$1.25 on each hot dog. The class would like a profit of at least \$280. Let x represent the number of hamburgers and y represent the number of hot dogs sold.
    a. Write and graph an inequality for the profit the senior class wants to make.
    b. If the senior class sells 100 hot dogs and 50 hamburgers, will the class make its goal?

SEE EXAMPLE 4 p. 127

Solve each inequality for y. Graph the solution.

11. $\frac{1}{2}(6x - 2y) \geq 4$
12. $-\frac{3}{5}x + y \geq 2$
13. $3(3x - y) > -12$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–16	1
17–18	2
19–21	3
22–24	4

Extra Practice
Skills Practice p. S6
Application Practice p. S33

Graph each inequality.

14. $y \geq 6$
15. $y < x + 4$
16. $y > -\frac{2}{5}x - 3$

Graph each inequality using intercepts.

17. $4x + 2y \geq 8$
18. $3x - 6y < 12$

19. **Marketing** Quarter page ads in the local papers cost \$200 per day, and one minute ads on the local radio stations cost \$500. Sheena's Lawn Care has an advertising budget of \$10,000. Let x be the number of quarter page ads in newspapers and y be the number of one minute radio ads. Write and graph an inequality for the advertising that Sheena's Lawn Care can afford.

20. **Astronomy** The rockets of a Mars probe require oxygen to lift off from the surface and return to Earth. Suppose the probe can produce 0.78 L of oxygen for every kg of water and 0.32 L of oxygen for every kg of carbon dioxide. At least 56 L of oxygen are needed. Let x represent the kg of water available and y represent the kg of carbon dioxide.
    a. Write and graph an inequality for the liters of oxygen that will be sufficient for liftoff.
    b. If the probe collects 36 kg of water and 88 kg of carbon dioxide, will it be enough for liftoff?

21. **Recreation** Amber has a \$200 gift card for boat rentals. She rents kayaks at \$8 and canoes at \$12 per hour. Let x be the number of hours of kayak rentals and y be the number of hours of canoe rentals.

   a. Write and graph an inequality for the possible number of hours of each that she can rent.

   b. If Amber rents kayaks for 10 hours, how many hours can she rent canoes for?

Solve each inequality for *y*. Graph the solution.

22. $-4y < 4(3x - 5)$

23. $-3(-10x + 2y) \geq 24$

24. $-\frac{1}{3}x + \frac{1}{5}y \leq -1$

Graph each inequality.

25. $-4y > 10x - 20$

26. $y - 5 \geq 4(x - 2)$

27. $6x + 3y < 0$

28. $y + \frac{3}{4} \leq \frac{5}{2}\left(x - \frac{1}{2}\right)$

29. $\frac{9 - 3y}{2} \geq 6x$

30. $x \leq 4$

31. $4x - 5y < 7x - 3y$

32. $2x - 5y \leq -4x + 15$

33. $x > -2$

34. **School** Tickets to the math club dance cost \$5 if bought in advance and \$6 at the door. The math club needs to make a total of at least \$600 from ticket sales for the dance.

   a. Let x be the number of tickets sold in advance and y be the number of tickets sold at the door. Write and graph an inequality for the total amount in ticket sales that the math club needs.

   b. If the math club sells 30 tickets in advance, how many tickets must be sold at the door for the math club to reach its goal?

35. **Fund-raising** The junior class is selling pizza and beverages at a basketball game. The class makes a profit of \$1.25 on each slice of pizza and \$0.50 on each beverage. Let x be the number of pizza slices and y be the number of beverages.

   a. Write and graph an inequality that shows the number of pizza slices and number of beverages the class must sell to make a profit of at least \$150.

   b. If the junior class sells 75 slices of pizza and 150 beverages, will the class make its goal?

36. **Critical Thinking** Tickets to an event cost \$5 for adults and \$2 for students. Total ticket sales were more than \$300. Jane and Erin graphed the situation as an inequality. Jane let x be the number of adult tickets sold, and Erin let x be the number of student tickets sold. How did their graphs differ? Which graph, if either, was incorrect?

Concept Connection

37. This problem will prepare you for the Concept Connection on page 132.

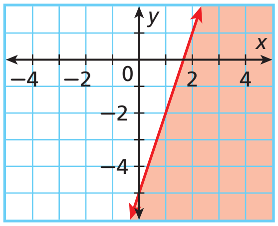
A ship starting 500 nautical miles from port can travel at a speed of 27 knots or less.

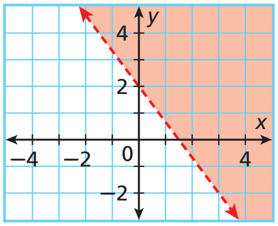
   a. How long does the trip to port take?

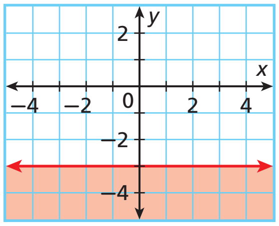
   b. Graph the ship's distance over the trip. What do the points above the boundary line represent?

   c. **What if...?** Suppose the minimum speed at any point during the trip is 10 knots. How far from port is the ship after 12 hours?

Write an inequality for each graph.

38. 

39. 

40. 

41. **Critical Thinking** Compare the graphs of $30y < 90 + x$ and $30y + x < 90$. How are they alike? How are they different?

42. **Home Economics** Omar uses almonds and raisins in a high-fiber recipe. Almonds have 3.3 g of fiber per ounce and raisins have 2.7 grams of fiber per ounce. He wants at least 5 grams of fiber from these ingredients in a recipe.

a. Let x be the number of ounces of almonds and y be the number of ounces of raisins. Write and graph an inequality for the amount of fiber from almonds and raisins that Omar wants in the recipe.

b. If Omar uses 0.5 ounce of almonds, how many ounces of raisins can he use?

c. **What if...?** Suppose Omar uses 2 ounces of almonds. What happens to the value of y in the inequality? What does this mean in the context of the problem?

43. A banquet room is to be filled with round tables and rectangular tables. The round tables have 8 chairs each, and the rectangular tables have 6 chairs each. Let x be the number of round tables and y be the number of rectangular tables.

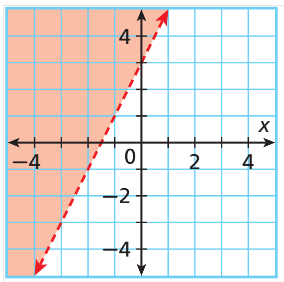
a. Write and graph an inequality for the number of each type of table needed to have at least 220 chairs.

b. Due to fire regulations, there can be no more than 300 chairs. Write and graph an inequality to reflect this.

c. Compare your graphs. How do they differ?

Standardized Test Prep

44. Which inequality best represents the set of points graphed here?



Ⓐ $y < 2x + 3$ Ⓒ $y \geq 2x + 3$

Ⓑ $4x - 2y < -6$ Ⓓ $4x + 2y > 6$

45. Which point is NOT a solution of $5x - 3y < 30$?

Ⓕ $(0, 0)$ Ⓗ $(-5, 3)$

Ⓖ $(3, -5)$ Ⓙ $(-3, 5)$

46. Which inequality is equivalent to $7x - 3y \geq 4$?

Ⓐ $y \leq \frac{7}{3}x - \frac{4}{3}$ Ⓒ $y \geq -\frac{7}{3}x - \frac{4}{3}$

Ⓑ $y \leq -\frac{7}{3}x + \frac{4}{3}$ Ⓓ $y \geq \frac{7}{3}x + \frac{4}{3}$

47. What points represent the intercepts of the boundary line of the graph of $y \le 3x - 9$?

Ⓕ $(0, 9)$ and $(3, 0)$
Ⓖ $(0, 3)$ and $(-9, 0)$
Ⓗ $(0, 9)$ and $(-3, 0)$
Ⓙ $(0, -9)$ and $(3, 0)$

48. Each dime adds 8 minutes to the time on a parking meter, and each quarter adds 20 minutes. The maximum time is 3 hours. The previous driver left 37 minutes of time. Adding which coins would NOT result in getting the maximum time?

Ⓐ 3 dimes and 6 quarters
Ⓑ 13 dimes and 2 quarters
Ⓒ 8 dimes and 4 quarters
Ⓓ 5 dimes and 5 quarters

49. **Short Response** Describe a problem situation using inequalities in which it would make sense to have negative *x*- or *y*-values.

CHALLENGE AND EXTEND

Graph each inequality.

50. $4(4x - 3y) < 5(2 + 3x) - 10y$

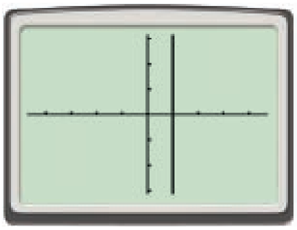
51. $\dfrac{4 + 3y - 2x}{6} \ge \dfrac{3x - 2 - 3y}{-4}$

52. **What if...?** Suppose when you graph a $y >$ inequality on a graphing calculator, you find that the entire screen is shaded. What does this indicate about the inequality? What might you do to show the graph of the inequality more accurately?

53. The graph of $y = 500(x - 1)$ is shown in the **ZDecimal** window.

a. Is the line really vertical? Explain.

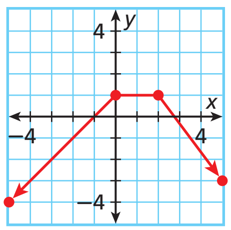
b. For the graph of $y \le 500(x - 1)$, which side of the line should be shaded? Justify your answer.



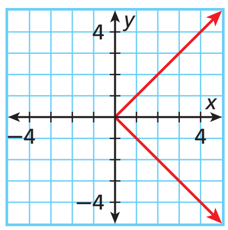
SPIRAL REVIEW

Use the vertical line test to determine whether each graph represents a function. *(Lesson 1-6)*

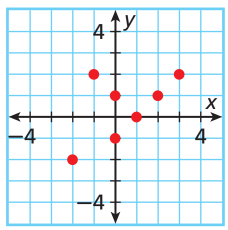
54.



55.



56.



Give the coordinates of the translated point when the original point is $(-4, 3)$. *(Lesson 1-8)*

57. horizontal translation of -1

58. reflection across the y-axis

59. vertical translation of 3

60. $(x + 7, y - 5)$

Write an equation of each line in slope-intercept form. Each line passes through the point $(1, -7)$. *(Lesson 2-4)*

61. passing through $(1, 3)$

62. parallel to $y = \frac{1}{2}x - 5$

63. with a slope of 0.25

64. perpendicular to $3x - y = -4$

CHAPTER 2

SECTION 2A

CONCEPT CONNECTION

Linear Equations and Inequalities

Sailing Away Crossing the Atlantic Ocean in a sailboat is a prestigious feat that many sailors attempt. Some of the speed records for the west-to-east trip from New York to England are shown in the table.

Transatlantic Sailing Records (New York to England)

Yacht	Year	Country	Average Speed (knots)
Atlantic	1905	USA	10.02
Royale II	1986	France	15.47
Jet Services V	1990	France	18.62
PlayStation	2001	USA	25.78

1. The length of the course that each yacht sailed, from the Ambrose Light Tower in New York to Lizard Point in England, is 3364 statute miles. How much longer did the *Atlantic* take to complete the trip than the *PlayStation*?

2. Dolphins can swim about 20 statute miles per hour. If a dolphin were racing against each of the yachts in the table, in which place would the dolphin finish?

3. Graph the distance in nautical miles that the *PlayStation* could cover over a period from 0 to 48 hours. The sailing distance from New York to Florida is 947 nautical miles. Use your graph to estimate how long it would take the *PlayStation* to make this trip.

Unit Conversions
1 knot $= \dfrac{1 \text{ nautical mi}}{1 \text{ h}}$
1 nautical mi $=$ 1.15 statute mi
1 statute mi $=$ 5280 ft

4. In 1980, the *Paul Ricard* broke the *Atlantic's* record for time crossing the Atlantic Ocean. The *Paul Ricard* finished the crossing in 10 days, 5 hours, and 14 minutes. Write a linear equation that describes the distance in nautical miles that the *Paul Ricard* covered as a function of time in hours.

5. Write and graph an inequality to show the possible distance d in statute miles that the *Atlantic* could cover in t hours. Is the point $(7.5, 85)$ a solution to the inequality? Explain the meaning of this point in the context of the problem.

READY TO GO ON?

CHAPTER 2

SECTION 2A

Quiz for Lessons 2-1 Through 2-5

2-1 Solving Linear Equations and Inequalities

Solve.

1. $15 + 8x = 3x$

2. $\frac{3}{2}(5x + 7) = 16$

3. $12 - 15x = 25 - 5x$

4. $3(x + 5) - 8(x - 3) = 20$

Solve and graph.

5. $45 \geq -25 + 10x$

6. $12 - 4x < 24$

7. $4(9 - 2x) \leq 3(4x + 2)$

8. $5x - 4(2x + 6) \geq 15$

9. Marie has \$55 in her bank account, and she would like to buy a video game system that costs \$395. Marie saves \$6 for each hour she works. How many hours must Marie work to have enough money to buy the video game system?

2-2 Proportional Reasoning

Solve each proportion.

10. $\frac{x}{12} = \frac{8}{3}$

11. $\frac{3}{5} = \frac{4x}{9}$

12. $\frac{5}{-x} = \frac{2.5}{8}$

13. $\frac{5}{9} = \frac{4}{2x - 3}$

14. A building casts a 24-foot shadow at the same time that a 6-foot-tall person casts an 8-foot shadow. How tall is the building?

2-3 Graphing Linear Functions

Find the intercepts and graph each line.

15. $2x + 3y = 18$

16. $5x - 3y = -15$

17. $\frac{1}{2}x + 2y = 6$

18. $-x - y = \frac{7}{2}$

Write each function in slope-intercept form. Then graph the function.

19. $y - 3x = 1$

20. $4x + 2y = 8$

21. $3x - 10 - 5y = 0$

22. $5 - x = \frac{y}{3}$

2-4 Writing Linear Functions

Write an equation in slope-intercept form for each line.

23. through $(3, 12)$ and $(6, 27)$

24. slope $\frac{3}{4}$ and through $(4, -6)$

25. parallel to $y = \frac{3}{2}x - 6$ and through $(-6, 2)$

26. perpendicular to $5x + 2y = 8$ and through $(5, 3)$

2-5 Linear Inequalities in Two Variables

Solve for y in each inequality. Then graph.

27. $y - 1 \leq 5$

28. $2x + 5y > 10$

29. $3x - 4y > 5x + 12$

30. $3(2x - 1) + y > 6x - 4$

31. Dorothy has \$30 to spend on holiday cards. Large cards cost \$2.50 each, and small cards cost \$1.50 each. Write and graph an inequality for the number of cards Dorothy can purchase.

2-6

Transforming Linear Functions

Objectives

Transform linear functions.

Solve problems involving linear transformations.

Why learn this?

Transformations allow you to visualize and compare many different functions at once.

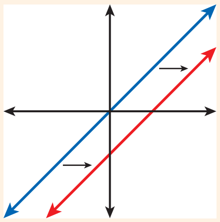
In Lesson 1-8, you learned to transform functions by transforming each point. Transformations can also be expressed by using function notation.

Know it! Note

Translations and Reflections

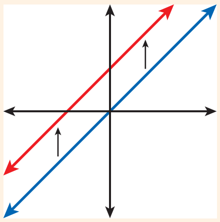
Translations

Horizontal Shift of $|h|$ Units



Input value changes.
$f(x) \to f(x - h)$
$h > 0$ moves right $h < 0$ moves left

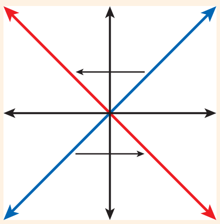
Vertical Shift of $|k|$ Units



Output value changes.
$f(x) \to f(x) + k$
$k > 0$ moves up
$k < 0$ moves down

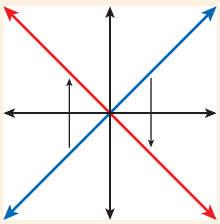
Reflections

Reflection Across y-axis



Input value changes.
$f(x) \to f(-x)$
The lines are symmetric about the y-axis.

Reflection Across x-axis



Output value changes.
$f(x) \to -f(x)$
The lines are symmetric about the x-axis.

Helpful Hint

To remember the difference between vertical and horizontal translations, think: "Add to y, go high." "Add to x, go left."

EXAMPLE 1

Translating and Reflecting Linear Functions

Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

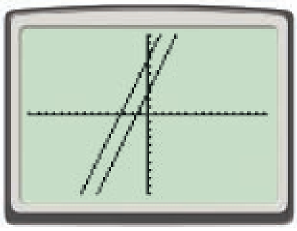
A $f(x) = 2x + 3$; **vertical translation 4 units up**

Translating $f(x)$ 4 units up adds 4 to each output value.

$g(x) = f(x) + 4$ *Add 4 to $f(x)$.*

$g(x) = (2x + 3) + 4$ *Substitute $2x + 3$ for $f(x)$.*

$g(x) = 2x + 7$ *Simplify.*



Check Graph $f(x)$ and $g(x)$ on a graphing calculator. The slopes are the same, but the y-intercept has moved 4 units up from 3 to 7. ✔

California Standards

Preparation for 9.0 Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as a, b, and c vary in the equation $y = a(x - b)^2 + c$.

Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

B **linear function defined in the table; reflection across *y*-axis**

x	$f(x)$
–1	0
0	2
1	4

Step 1 Write the rule for $f(x)$ in slope-intercept form.

The y-intercept is 2. *The table contains (0, 2).*

Find the slope:

$$m = \frac{2-0}{0-(-1)} = \frac{2}{1} = 2$$ *Use (–1, 0) and (0, 2).*

$y = mx + b$ *Slope-intercept form*

$y = 2x + 2$ *Substitute 2 for m and 2 for b.*

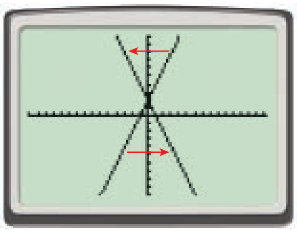
$f(x) = 2x + 2$ *Replace y with f(x).*

Step 2 Write the rule for $g(x)$. Reflecting $f(x)$ across the y-axis replaces each x with $-x$.

$g(x) = 2(-x) + 2$ *$g(x) = f(-x)$*

$g(x) = -2x + 2$

Check Graph $f(x)$ and $g(x)$ on a graphing calculator. The graphs are symmetric about the y-axis. ✔



Math Builders

For more on transformations, see the Transformation Builder on page MB2.

CHECK IT OUT! **Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.**

1a. $f(x) = 3x + 1$; translation 2 units right

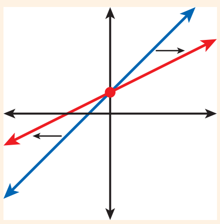
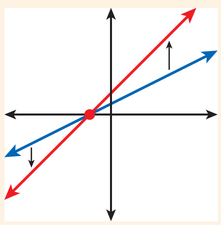
1b. linear function defined in the table; a reflection across the x-axis

x	–1	0	1
y	1	2	3

Stretches and compressions change the slope of a linear function. If the line becomes steeper, the function has been stretched vertically or compressed horizontally. If the line becomes flatter, the function has been compressed vertically or stretched horizontally.

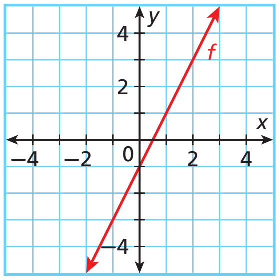
Know it! Note

Stretches and Compressions

Horizontal	Vertical
Horizontal Stretch/Compression by a Factor of *b*	**Vertical Stretch/Compression by a Factor of *a***
 Input value changes. $f(x) \rightarrow f\left(\frac{1}{b}x\right)$	 Output value changes. $f(x) \rightarrow a \cdot f(x)$
$b > 1$ stretches away from the y-axis. $0 < \lvert b \rvert < 1$ compresses toward the y-axis.	$a > 1$ stretches away from the x-axis. $0 < \lvert a \rvert < 1$ compresses toward the x-axis.

EXAMPLE 2 Stretching and Compressing Linear Functions

Let $g(x)$ be a horizontal compression of $f(x) = 2x - 1$ by a factor of $\frac{1}{3}$. Write the rule for $g(x)$, and graph the function.



Horizontally compressing $f(x)$ by a factor of $\frac{1}{3}$ replaces each x with $\frac{1}{b}x$ where $b = \frac{1}{3}$.

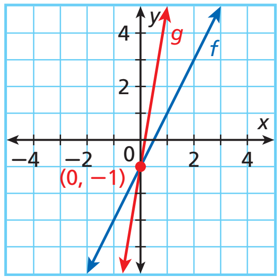
$g(x) = 2\left(\frac{1}{b}\right)x - 1$ *For horizontal compression, use $\frac{1}{b}$.*

$= 2\left(\frac{1}{\frac{1}{3}}\right)x - 1$ *Substitute $\frac{1}{3}$ for b.*

$= 2(3x) - 1$ *Replace x with 3x.*

$g(x) = 6x - 1$ *Simplify.*

Check Graph both functions on the same coordinate plane. The graph of $g(x)$ is steeper than $f(x)$, which indicates that $g(x)$ has been horizontally compressed from $f(x)$, or pushed toward the y-axis.



Helpful Hint

These don't change!
- *y*-intercepts in a horizontal stretch or compression
- *x*-intercepts in a vertical stretch or compression

CHECK IT OUT! **2.** Let $g(x)$ be a vertical compression of $f(x) = 3x + 2$ by a factor of $\frac{1}{4}$. Write the rule for $g(x)$.

Some linear functions involve more than one transformation. Combine transformations by applying individual transformations one at a time in the order in which they are given.

For multiple transformations, create a temporary function—such as $h(x)$ in Example 3 below—to represent the first transformation, and then transform it to find the combined transformation.

EXAMPLE 3 Combining Transformations of Linear Functions

Let $g(x)$ be a vertical shift of $f(x) = x$ down 2 units followed by a vertical stretch by a factor of 5. Write the rule for $g(x)$.

Step 1 First perform the translation.

Translating $f(x) = x$ down 2 units subtracts 2 from the function. You can use $h(x)$ to represent the translated function.

$h(x) = f(x) - 2$ *Subtract 2 from the function.*

$h(x) = x - 2$ *Substitute x for f(x).*

Step 2 Then perform the stretch.

Stretching $h(x)$ vertically by a factor of 5 multiplies the function by 5.

$g(x) = 5 \cdot h(x)$ *Multiply the function by 5.*

$g(x) = 5(x - 2)$ *Because h(x) = x – 2, substitute x – 2 for h(x).*

$g(x) = 5x - 10$ *Simplify.*

CHECK IT OUT! **3.** Let $g(x)$ be a vertical compression of $f(x) = x$ by a factor of $\frac{1}{2}$ followed by a horizontal shift 8 units left. Write the rule for $g(x)$.

EXAMPLE 4 *Fund-raising Application*

The Dance Club is selling beaded purses as a fund-raiser. The function $R(n) = 12.5n$ represents the club's revenue in dollars where n is the number of purses sold.

a. **The club paid \$75 for the materials needed to make the purses. Write a new function $P(n)$ for the club's profit.**

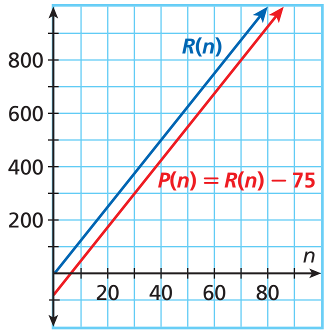
The initial costs must be subtracted from the revenue.

$R(n) = 12.5n$ — *Original function*

$P(n) = 12.5n - 75$ — *Subtract the expenses.*

b. **Graph $P(n)$ and $R(n)$ on the same coordinate plane.**

Graph both functions. The lines have the same slope but different y-intercepts.



Note that the profit can be negative but the number of purses sold cannot be less than 0.

c. **Describe the transformation(s) that have been applied.**

The graphs indicate that $P(n)$ is a translation of $R(n)$. Because 75 was subtracted, $P(n) = R(n) - 75$. This indicates a vertical shift 75 units down.

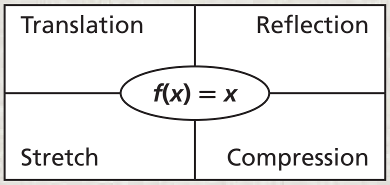
CHECK IT OUT!

4. **What if...?** The club members decided to double the price of each purse.
   a. Write a new profit function $S(n)$ for the club.
   b. Graph $S(n)$ and $P(n)$ on the same coordinate plane.
   c. Describe the transformation(s) that have been applied.

THINK AND DISCUSS

1. Identify the horizontal translation that would have the same effect on the graph of $f(x) = x$ as a vertical translation of 6 units.
2. Give an example of two different transformations of $f(x) = 2x$ that would result in $g(x) = 2x - 6$.
3. Describe the transformation that would cause all of the function values to double.
4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, give an example of the indicated transformation of the parent function $f(x) = x$. Include an equation and a graph.

Know it! Note



2-6 Exercises

California Standards
Preparation for 9.0; Review of 1A2.0, 1A16.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 2-6
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1
p. 134

Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

1. linear function defined by the table; vertical translation 1.5 units up

x	−2	−1	0
$f(x)$	3.5	2	0.5

SEE EXAMPLE 2
p. 136

2. $f(x) = -x + 5$; horizontal translation 2 units left
3. $f(x) = \frac{1}{3}x - 2$; vertical stretch by a factor of 3
4. $f(x) = -2x + 0.5$; horizontal stretch by a factor of $\frac{4}{3}$.

SEE EXAMPLE 3
p. 136

Let $g(x)$ be the indicated combined transformation of $f(x) = x$. Write the rule for $g(x)$.

5. vertical compression by a factor of $\frac{2}{3}$ followed by a vertical shift 6 units down
6. horizontal shift right 4 units followed by a horizontal stretch by a factor of $\frac{3}{2}$

SEE EXAMPLE 4
p. 137

7. **Advertising** An electronics company is changing its Internet ad from a banner ad to a pop-up ad. The cost of the banner ad in dollars is represented by $C(n) = 0.30n + 5.00$ where n is the average number of hits per hour. The cost of the pop-up ad will double the cost per hit.
   a. Write a new cost function $D(n)$ for the ads.
   b. Graph $C(n)$ and $D(n)$ on the same coordinate plane.
   c. Describe the transformation(s) that have been applied.

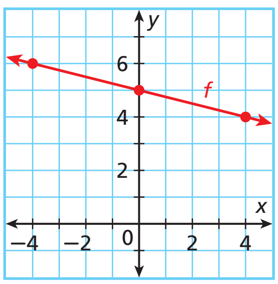
PRACTICE AND PROBLEM SOLVING

Independent Practice

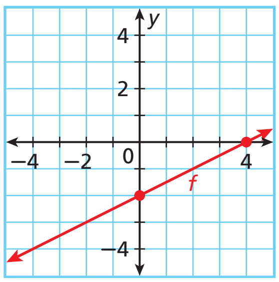
For Exercises	See Example
8–9	1
10–12	2
13–14	3
15	4

Extra Practice
Skills Practice p. S7
Application Practice p. S33

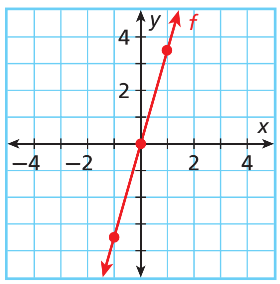
Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

8.


Reflection across the x-axis

9.


Vertical translation 2 units down

10.


Horizontal compression by a factor of 0.5

11. linear function defined by the table; vertical stretch by a factor of 1.2 units

x	1	5	9
$f(x)$	0	−2	−4

12. $f(x) = -3x + 7$; vertical compression by a factor of $\frac{3}{4}$

Let $g(x)$ be the indicated combined transformation of $f(x) = x$. Write the rule for $g(x)$.

13. horizontal stretch by a factor of 2.75 followed by a horizontal shift 1 unit left
14. vertical shift 6 units down followed by a vertical compression by a factor of $\frac{2}{3}$

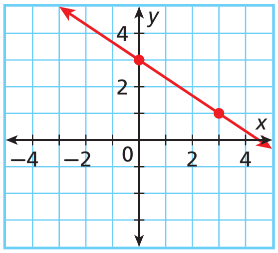
15. **Consumer Economics** In 1997, Southwestern Bell increased the price for local pay-phone calls. Before then, the price of a call could be determined by $f(x) = 0.15x + 0.25$, where x was the number of minutes after the *first* minute. The company increased the cost of the first minute by 10 cents.

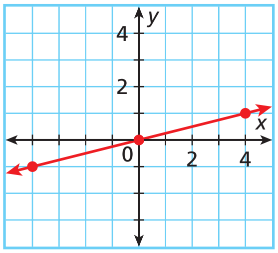
a. Write a new price function $g(x)$ for a phone call.

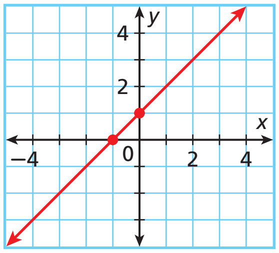
b. Graph $f(x)$ and $g(x)$ on the same coordinate plane.

c. Describe the transformation(s) that have been applied.

Write the rule for the transformed function $g(x)$ and graph.

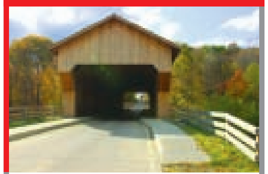
16. 
Reflection across the y-axis

17. 
Vertical stretch by a factor of 8

18. 
Horizontal stretch by a factor of 3

LINK

History



The Cumberland Road, built in the early 1800s from Cumberland, Maryland to St. Louis, Missouri, was the most ambitious road project of its time.

History **Historic tolls for traveling on the Cumberland Road in Pennsylvania are shown on the sign. Toll was paid every 15 miles.**

19. Write a function to represent the cost for 1 horse and rider to travel n miles with a score of sheep. What transformation describes the change in cost if the sheep were replaced by cattle?

20. Write a function to represent the cost for a carriage with 2 horses and 4 wheels to travel n miles. Name two different transformations that would represent a 6¢ increase in the toll rate.

21. **Critical Thinking** Consider the linear function $f(x) = x$.

a. Shift $f(x)$ 2 units up and then reflect it over the x-axis.

b. Perform the same transformations on $f(x)$ again but in reverse order.

c. Make a conjecture about the order in which transformations are performed.

22. **Write About It** Which transformations affect the slope of a linear function, and which transformations affect the y-intercept? Support your answers.

Concept Connection

23. This problem will prepare you for the Concept Connection on page 164.
Use the data set $\{1, 5, 10, 17, 23, 23, 38, 60\}$.

a. Find the mean, median, mode, and range.

b. How does adding 7 to each number affect the mean, median, mode, and range?

c. How does multiplying each number by 4 affect the mean, median, mode, and range?

d. How does multiplying each number by 2 and then adding 5 affect the mean, median, mode, and range?

STANDARDIZED TEST PREP

24. The cost function C of rent at an apartment complex increased \$50 last year and another \$60 this year. Which function accurately reflects these changes?

Ⓐ $60(C + 50)$ Ⓑ $60(50C)$ Ⓒ $(C + 50) + 60$ Ⓓ $50C + 60$

25. Given $f(x) = 28.5x + 45.6$, which function decreases the y-intercept by 20.3?

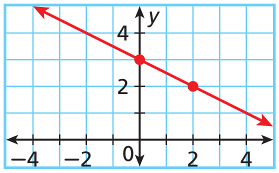
Ⓕ $g(x) = 8.2x + 45.6$ Ⓗ $g(x) = 28.5x + 25.3$

Ⓖ $g(x) = 8.2x + 66.1$ Ⓙ $g(x) = 28.5x + 66.1$

26. Which transformation describes a line that is parallel to $f(x)$?

Ⓐ $f(3x)$ Ⓑ $f\left(\frac{x}{2}\right)$ Ⓒ $f(x - 4)$ Ⓓ $f(-2x)$

27. Which transformation of $f(x) = \frac{1}{2}x - 1$ could result in the graph shown?



Ⓕ vertical shift 2 units down and reflection across x-axis

Ⓖ horizontal shift 2 units left and reflection across x-axis

Ⓗ vertical shift 2 units up and reflection across x-axis

Ⓙ horizontal shift 2 units right and reflection across x-axis

CHALLENGE AND EXTEND

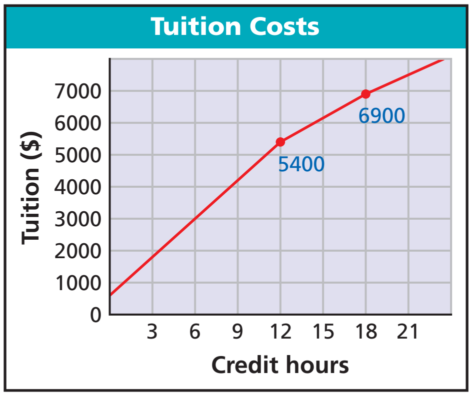
28. Give two different combinations of transformations that would transform $f(x) = 3x + 4$ into $g(x) = 15x - 10$.

29. Give an example of two transformations of $f(x) = x$ that can be performed in any order and result in the same transformed function.

30. **Education** The graph shows the tuition at a university based on the number of credit hours taken. The rate per credit hour varies according to the number of hours taken: less than 12 hours, 12 to 18 hours, and greater than 18 hours.

a. Write the linear function that represents each segment of the graph.

b. Write the linear functions that would reflect a 12% increase in all tuition costs.



SPIRAL REVIEW

Write each expression in expanded form. *(Lesson 1-5)*

31. $\left(\frac{3}{5}d^2\right)^3$ **32.** 2^{-3} **33.** $-(2n)^4$ **34.** $-a^5(6a)^{-1}$

Determine if each line is vertical, horizontal, or neither, and graph the line. *(Lesson 2-3)*

35. $y = -6$ **36.** $x = \frac{3}{7}$

37. $y = -x$ **38.** $5.1 = y$

39. **Money** Express Henry's bonus as a function of the ads that Henry sells. How many ad spots must Henry sell to earn \$520 as a bonus? *(Lesson 2-4)*

Henry's Bonus

Ads Sold	Bonus (\$)
12	65
16	195
21	357.50

Connecting Algebra to **Data Analysis**

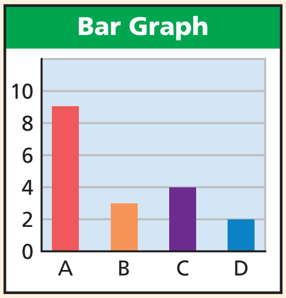
See Skills Bank page S69

California Standards

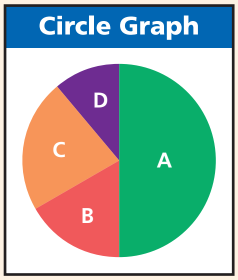
Review of 7SDAP1.1 Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

Statistical Graphs

Statistical data may be displayed in bar graphs or circle graphs. Use a bar graph to compare numerical amounts. Use a circle graph to compare parts of a whole.



A bar graph compares numerical amounts.

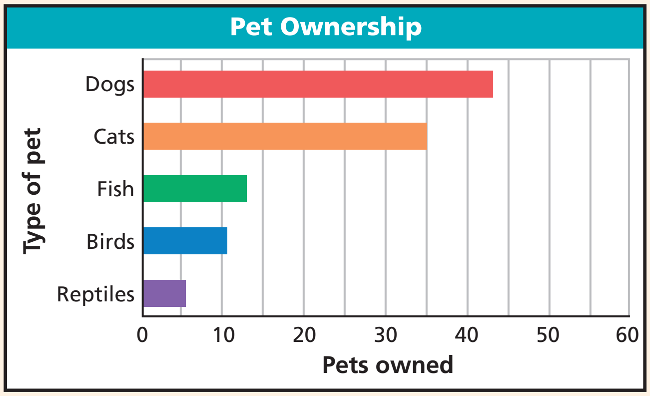


A circle graph compares parts of a whole.

The bar graph shows the numbers of pets owned by a group of students in a pet owners club.

Example

Use the bar graph. Find the central angle measure for the named category in a related circle graph, to the nearest degree.



Category: birds

1. Compute the total number of pets.
   Add the number of dogs, cats, fish, reptiles, and birds.
   $6 + 11 + 43 + 35 + 13 = 108$

2. Find the number of pets in the category.
   11 birds

3. A circle consists of 360°. **Write and solve a proportion.**
   $$\begin{matrix}\text{Part} \rightarrow \\ \text{Whole} \rightarrow\end{matrix} \frac{11}{108} = \frac{n}{360} \begin{matrix}\leftarrow \text{Circle part} \\ \leftarrow \text{Circle whole}\end{matrix}$$

4. Solve for the central angle.
   $11 \cdot 360 = 108n$
   $37° \approx n$

Try This

Find the central angle measure for each category, to the nearest degree.

1. fish **2.** reptiles **3.** dogs **4.** cats **5.** fish, birds, and reptiles combined

6. What categories combined give a central angle of approximately 207°?

2-7 Curve Fitting with Linear Models

Objectives

Fit scatter plot data using linear models with and without technology.

Use linear models to make predictions.

Vocabulary

regression
correlation
line of best fit
correlation coefficient

California Standards

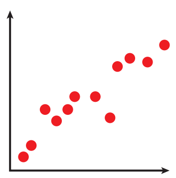
Extension of 1A7.0 Students verify that a point lies on a line, given an equation of the line. **Students are able to derive linear equations by using the point-slope formula.**

Who uses this?

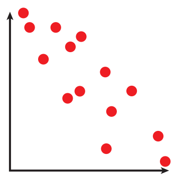
Anthropologists can use linear models to estimate the heights of ancient people from bones that the anthropologists find. (See Example 2.)

Researchers, such as anthropologists, are often interested in how two measurements are related. The statistical study of the relationship between variables is called **regression**.

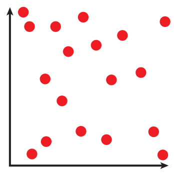
A *scatter plot* is helpful in understanding the form, direction, and strength of the relationship between two variables. **Correlation** is the strength and direction of the linear relationship between the two variables.



Positive correlation, positive slope



Negative correlation, negative slope



Relatively no correlation

If there is a strong linear relationship between two variables, a **line of best fit**, or a line that best fits the data, can be used to make predictions.

EXAMPLE 1 *Meteorology Application*

Akron, Ohio, and Wellington, New Zealand, are about the same distance from the equator. Make a scatter plot for the temperature data, identify the correlation, and then sketch a line of best fit and find its equation.

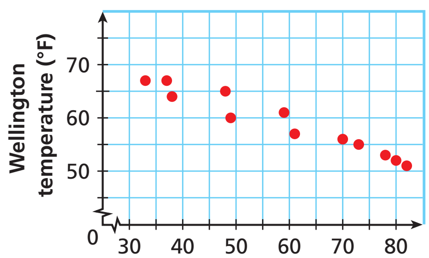
Average High Temperatures (°F)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Akron	33	37	48	59	70	78	82	80	73	61	49	38
Wellington	67	67	65	61	56	53	51	52	55	57	60	64

Step 1 Plot the data points.

Step 2 Identify the correlation.

Notice that the data set is negatively correlated—as the temperature rises in Akron, it falls in Wellington.



Helpful Hint

Try to have about the same number of points above and below the line of best fit.

Step 3 Sketch a line of best fit.

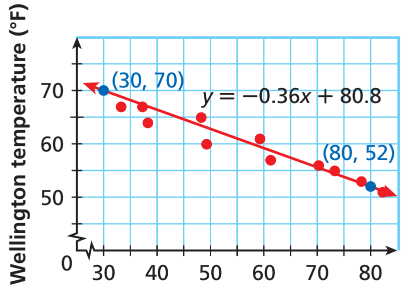
Draw a line that splits the data evenly above and below.

Step 4 Identify two points on the line.

For this data, you might select (30, 70) and (80, 52).

Step 5 Find the slope of the line that models the data.

$$m = \frac{70 - 52}{30 - 80} = \frac{18}{-50} = -0.36$$



Use the point-slope form.

$y - y_1 = m(x - x_1)$	*Point-slope form*
$y - 70 = -0.36(x - 30)$	*Substitute.*
$y = -0.36x + 80.8$	*Simplify.*

An equation that models the data is $y = -0.36x + 80.8$.

CHECK IT OUT!

1. Basketball Make a scatter plot for this set of data. Identify the correlation, sketch a line of best fit, and find its equation.

Points Scored in Ten Games

Minutes Played	28	35	8	20	39	23	19	27	15	30
Points Scored	16	13	2	12	31	10	9	15	4	19

The **correlation coefficient** r is a measure of how well the data set is fit by a model.

Know it! Note

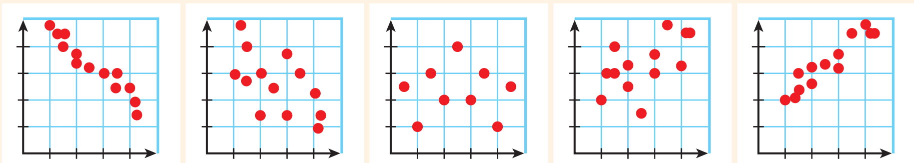
Properties of the Correlation Coefficient r

r is a value in the range $-1 \le r \le 1$.

If $r = 1$, the data set forms a straight line with a positive slope.

If $r = 0$, the data set has no correlation.

If $r = -1$, the data set forms a straight line with a negative slope.

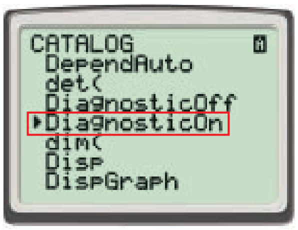


$r \approx -0.95$ | $r \approx -0.6$ | $r \approx 0$ | $r \approx 0.6$ | $r \approx 0.95$

Caution!

Don't confuse slope with the *value* of r. Whether a line has a slope of 10 or a slope of $\frac{1}{10}$, it can have an r-value of 1. The r-value and the slope have the same sign.

You can use a graphing calculator to perform a linear regression and find the correlation coefficient r. To display the correlation coefficient, you may have to turn on the diagnostic mode. To do this, press 2nd CATALOG 0, and choose the **DiagnosticOn** mode.



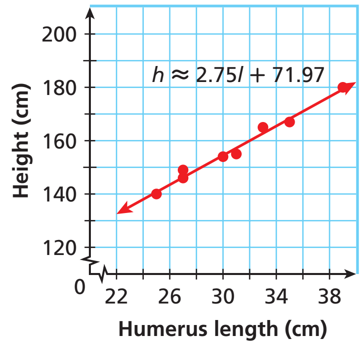
EXAMPLE 2 *Anthropology Application*

Anthropologists use known relationships between the height and length of a woman's humerus bone, the bone between the elbow and the shoulder, to estimate a woman's height. Some samples are shown in the table.

Bone Length and Height in Women								
Humerus Length (cm)	35	27	30	33	25	39	27	31
Height (cm)	167	146	154	165	140	180	149	155

a. Make a scatter plot of the data with humerus length as the independent variable.

The scatter plot is shown at right.

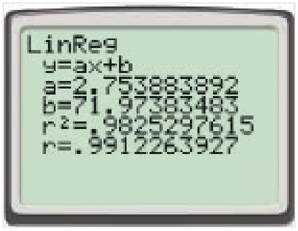


b. Find the correlation coefficient *r* and the line of best fit. Interpret the slope of the line of best fit in the context of the problem.

Enter the data into lists **L1** and **L2** on a graphing calculator. Use the linear regression feature by pressing STAT, choosing **CALC**, and selecting **4:LinReg**. The equation of the line of best fit is $h \approx 2.75\ell + 71.97$.

The slope is about 2.75, so for each 1 cm increase in humerus length, the predicted increase in a woman's height is 2.75 cm.

The correlation coefficient is $r \approx 0.991$, which indicates a strong positive correlation.



c. A humerus 32 cm long was found. Predict the woman's height.

The equation of the line of best fit is $h \approx 2.75\ell + 71.97$. Use the equation to predict the woman's height. For a 32-cm-long humerus,

$h \approx 2.75(32) + 71.97$ *Substitute 32 for ℓ.*

$h \approx 159.97$

The height of a woman with a 32-cm-long humerus would be about 160 cm.

Helpful Hint

To enter data into lists on a graphing calculator, press STAT and select **1:Edit**. Enter the *x*-values in the **L1** column and the *y*-values in the **L2** column.

CHECK IT OUT!

2. The gas mileage for randomly selected cars based upon engine horsepower is given in the table.

Gas Mileage and Horsepower of Cars										
Horsepower	175	255	140	165	115	120	190	180	110	125
Mileage (mi/gal)	22	13	25	18	32	28	15	21	35	30

a. Make a scatter plot of the data with horsepower as the independent variable.

b. Find the correlation coefficient *r* and the line of best fit. Interpret the slope of the line in the context of the problem.

c. Predict the gas mileage for a 210-horsepower engine.

EXAMPLE 3 Nutrition Application

Find the following information for this data set on the number of grams of fat and the number of calories in sandwiches served at Dave's Deli.

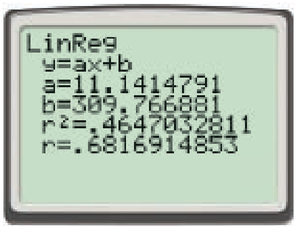
Dave's Deli Sandwiches Nutritional Information								
Fat (g)	5	9	12	15	12	10	21	14
Calories	360	455	460	420	530	375	580	390

a. Make a scatter plot of the data with fat as the independent variable.

The scatter plot is shown below.

> **Reading Math**
> A line of best fit may also be referred to as a *trend line*.

b. Find the correlation coefficient and the equation of the line of best fit. Draw the line of best fit on your scatter plot.



The correlation coefficient is $r = 0.682$.
The equation of the line of best fit is $y \approx 11.1x + 309.8$.

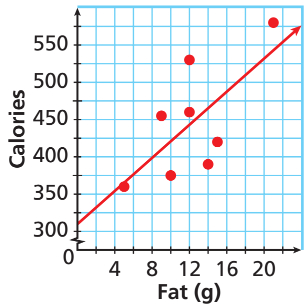
c. Predict the amount of fat in a sandwich with 500 Calories. How accurate do you think your prediction is?

$500 \approx 11.1x + 309.8$ *Calories is the dependent variable.*

$190.2 \approx 11.1x$

$17.1 \approx x$

The line predicts 17.1 grams of fat, but the scatter plot and the value of r show that fat content by itself is *not* a good predictor of the number of calories in a sandwich at Dave's.



CHECK IT OUT! 3. **What If...?** Use the equation of the line of best fit to predict the number of grams of fat in a sandwich with 420 Calories. How close is your answer to the value given in the table?

THINK AND DISCUSS

1. Explain whether the r-value is positive or negative if the line of best fit for data from two variables is $y = 3.2x - 12.5$.
2. Tell which correlation coefficient, $r = 0.65$ or $r = -0.75$, indicates a stronger linear relationship between two variables. Justify your answer.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Make a scatter plot for each type of correlation and estimate the r-value.

Correlation	Scatter Plot	Estimated r-value
Strong positive		
Weak positive		
No correlation		
Weak negative		
Strong negative		

2-7 Exercises

California Standards
Extension of 1A7.0; Review of 7AF1.1, 7AF1.2, 1A4.0, 1A5.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 2-7
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** Explain what the following *correlation coefficients* tell you about two sets of data.
   a. $r = 0.4$ **b.** $r = -0.96$ **c.** $r = -0.02$

SEE EXAMPLE 1 p. 142

2. **Driving** Make a scatter plot for this data set using gallons as the independent variable. Identify the correlation, sketch a line of best fit, and find its equation.

Distance Traveled

Gallons	11.2	9.8	10.6	10.1	12.3	8.7	10.1
Distance (mi)	338	296	332	324	368	263	305

SEE EXAMPLE 2 p. 144

3. **Home Economics** Use the data relating the average temperature in a month to the heating bill at Claire's house that month.

Claire's Heating Bills

Mean Temperature (°F)	38	42	44	36	42	49	38
Heating Bill ($)	93	79	75	83	74	67	86

   a. Make a scatter plot using mean temperature as the independent variable.
   b. Find the correlation coefficient and the equation of the line of best fit. Draw the line of best fit on your scatter plot.
   c. Predict the heating bill for a month in which the average temperature is 40° F. How accurate do you think your prediction is?

SEE EXAMPLE 3 p. 145

4. **School** Here are the number of teachers and the number of students at a randomly selected sample of high schools in a city.

Teachers and Students at Selected Schools

Teachers	92	52	114	49	110	62	76	84
Students	1050	653	753	381	1312	813	496	910

   a. Make a scatter plot of the data using teachers as the independent variable.
   b. Find the correlation coefficient and the equation of the line of best fit. Draw the line of best fit on your scatter plot.
   c. Predict the number of teachers in a high school that has 600 students. How accurate do you think your prediction is?

PRACTICE AND PROBLEM SOLVING

5. **Chemistry** Make a scatter plot for this data set using the atomic number as the independent variable. Identify the correlation, sketch a line of best fit, and find its equation.

Selected Chemical Elements

Atomic Number	89	13	95	51	18	33	85	56	97	4	83	107	5	35
Atomic Mass	227	27	243	122	40	75	210	137	247	9	209	264	11	80

Independent Practice	
For Exercises	See Example
5	1
6	2
7	3

Extra Practice

Skills Practice p. S7
Application Practice p. S33

6. Biology Hummingbird wing beat rates are much higher than those in other birds. Estimates for various species are given in the table.

Hummingbird Wing Beats							
Mass (g)	3.1	2.0	3.2	4.0	3.7	1.9	4.5
Wing Beats (per *s*)	60	85	50	45	55	90	40

a. Make a scatter plot of the data using mass as the independent variable.

b. Find the correlation coefficient and the equation of the line of best fit. Draw the line of best fit on your scatter plot.

c. Predict the wing beats rate for a Giant Hummingbird with a mass of 19 g. How accurate do you think your prediction is?

7. Ticket Pricing The manager of a band has kept track of the price of tickets and the attendance at the band's recent concerts.

Concert Attendance by Ticket Price									
Price ($)	6	5	8.5	8	10	5.50	7	7.5	8
Attendance	213	256	155	194	160	267	258	210	235

a. Make a scatter plot of the data using price as the independent variable.

b. Find the correlation coefficient and the equation of the line of best fit. Draw the line of best fit on your scatter plot.

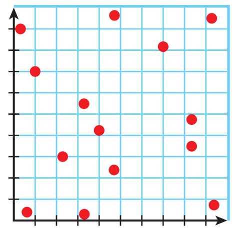
c. Predict the attendance at a concert where the price of tickets is $9. How accurate do you think your prediction is?

8. Make a scatter plot for this data set. Estimate to find the equation of the line of best fit.

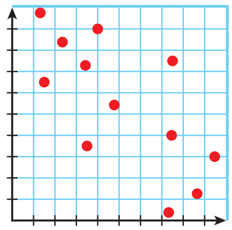
x	2	8	15	21	24	30	33	37
y	71	63	64	194	160	267	258	210

Estimation Estimate the value of r for each scatter plot.

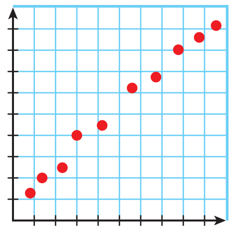
9.



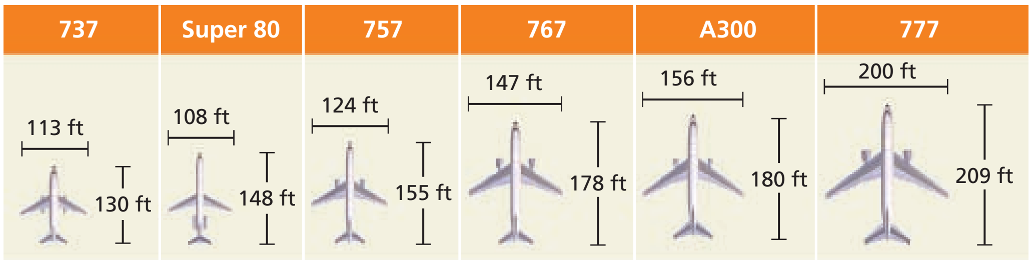
10.



11.



12. Aviation Make a scatter plot for the lengths and wingspans of planes in the American Airlines fleet. Sketch a line of best fit with length as the independent variable, and find its equation.



CONCEPT CONNECTION

13. This problem will prepare you for the Concept Connection on page 164.

The table gives the scores of the first 10 entries in a livestock show competition.

a. What equation could you use to estimate the score from the place? Graph the equation.

b. Suppose each score is increased by 5. How would this affect the equation and graph of the line?

Competition Results

Place	Score	Place	Score
1	95	6	90
2	93	7	89
3	92	8	87
4	91	9	86
5	90	10	85

14. **Athletics** Use the data set relating the number of steps per second to speed for a group of top female runners at different speeds.

Steps Taken by Distance Runners

Speed (ft/s)	15.86	16.88	17.5	18.62	19.97	21.06	22.11
Steps per second	3.05	3.12	3.17	3.25	3.36	3.46	3.55

Make a scatter plot of the data using speed as the independent variable. Find the correlation coefficient and the line of best fit, and draw it on your scatter plot. Use your equation to predict the number of steps per second taken by a runner going 18 feet per second. How accurate is your prediction? Explain.

15. **Paleontology** The table below shows the lengths of the femur, a leg bone, and the humerus, an arm bone, for five fossil specimens of the archaeopteryx, an extinct animal that had feathers and characteristics of a reptile.

Archaeopteryx Bone Lengths

Femur Length (cm)	38	56	59	64	74
Humerus Length (cm)	41	63	70	72	84

a. Make a scatter plot of the data using femur length as the independent variable. Find the correlation coefficient and the line of best fit. Draw the line of best fit on your scatter plot.

b. What does the slope of your line mean for the archaeopteryx?

c. Use your equation to predict the length of the femur of an archaeopteryx whose humerus is 50 cm long. How accurate do you think your prediction is?

16. **Critical Thinking** Does a strong linear relationship between two variables mean that one causes the other (for example, if higher daily bee stings correspond to higher ice cream sales)? Explain.

17. **Data Collection** Use a graphing calculator and a motion detector. Stand in a doorway and measure the distance to a person as the person walks from the opposite side of the room toward the motion detector. Is a linear model a good model for distance versus time? Explain.

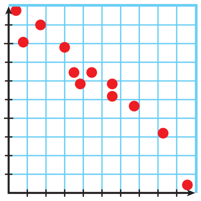
18. **Write About It** Describe the process of finding a line of best fit.

STANDARDIZED TEST PREP

19. The equation of the line of best fit for a set of data is $y = 1.05x - 1.3$. Which of the following could be the correlation coefficient for the set of data?

Ⓐ $r = -1.3$ Ⓑ $r = -0.7$ Ⓒ $r = 0.8$ Ⓓ $r = 1.05$

20. Which of the following best describes the correlation shown?



Ⓕ Strong positive Ⓗ Strong negative

Ⓖ Weak positive Ⓙ Weak negative

21. Which of the following relationships would likely have a negative correlation coefficient for an automobile?

Ⓐ Age and total miles

Ⓑ Age and resale value

Ⓒ Length and width

Ⓓ Highway mileage and city mileage

CHALLENGE AND EXTEND

Are the data linear? Are the data related? Explain.

22.

x	2	7	13	15	22
y	4	4	4	4	4

23.

x	35	45	55	65	75
y	30	34	36	34	30

24. The following data sets were developed by statistician Frank Anscombe. Make a scatter plot of each set of data, and find r and a line of best fit. Why is it important to plot the data before using a linear model to make predictions?

x	10	8	13	9	11	14	6	4	12	7	5
y	9.14	8.14	8.74	8.77	9.29	8.1	6.13	3.1	9.13	7.26	4.74

x	10	8	13	9	11	14	6	4	12	7	5
y	7.46	6.77	12.74	7.11	7.81	8.84	6.08	5.39	8.15	6.42	5.73

SPIRAL REVIEW

Simplify each expression. *(Lesson 1-4)*

25. $3(x^2 - 2) + 4xy - 10x^2y + 5x^2$

26. $-a^4 + 3ab + (2a^2)^2$

27. $-3g^2 + 3(g - 4) - 2(g - g^2)$

28. $n(4t^2 - t) - 10nt^2 + nt$

Solve and graph. *(Lesson 2-1)*

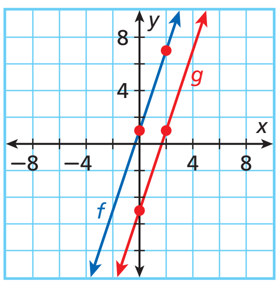
29. $3x < x - 12$

30. $44 + 6x > -5x$

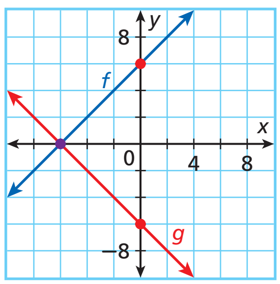
31. $-2(q - 4) + 3q \le 1 + q$

Write the equation for each function graphed. Describe $g(x)$ as a transformation of $f(x)$. *(Lesson 2-6)*

32.



33.



2-8 Solving Absolute-Value Equations and Inequalities

Objectives
Solve compound inequalities.

Write and solve absolute-value equations and inequalities.

Vocabulary
disjunction
conjunction
absolute value

California Standards

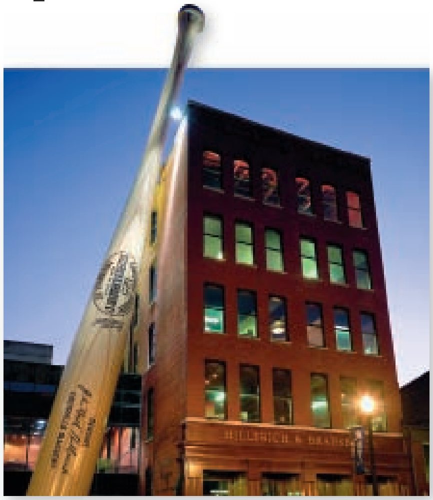
1.0 Students solve equations and inequalities involving absolute value.

Reading Math

Dis- means "apart." Disjunctions have two separate pieces. *Con-* means "together." Conjunctions represent one piece.

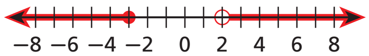
Who uses this?

Absolute value can be used to represent the acceptable ranges for the dimensions of baseball bats classified by length or weight. (See Exercise 43.)



A compound statement is made up of more than one equation or inequality.

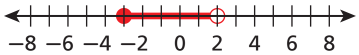
A **disjunction** is a compound statement that uses the word *or.*



Disjunction: $x \le -3$ **OR** $x > 2$ Set builder notation: $\{x \mid x \le -3 \cup x > 2\}$

A disjunction is true if and only if at least one of its parts is true.

A **conjunction** is a compound statement that uses the word *and.*



Conjunction: $x \ge -3$ **AND** $x < 2$ Set builder notation: $\{x \mid x \ge -3 \cap x < 2\}$
A conjunction is true if and only if all of its parts are true. Conjunctions can be written as a single statement as shown.

$$x \ge -3 \textit{ and } x < 2 \rightarrow -3 \le x < 2$$

EXAMPLE 1 Solving Compound Inequalities

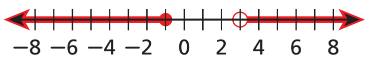
Solve each compound inequality. Then graph the solution set.

A $x + 3 \le 2$ OR $3x > 9$

Solve both inequalities for x.

$x + 3 \le 2$	*or*	$3x > 9$
$x \le -1$		$x > 3$

The solution set is all points that satisfy $\{x \mid x \le -1 \textit{ or } x > 3\}$.



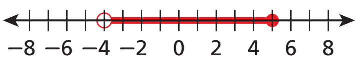
$(-\infty, -1] \cup (3, \infty)$

B $-2x < 8$ AND $x - 3 \le 2$

Solve both inequalities for x.

$-2x < 8$	*and*	$x - 3 \le 2$
$x > -4$		$x \le 5$

The solution set is the set of points that satisfy both $x > -4$ and $x \le 5$, $\{x \mid -4 < x \le 5\}$



$(-4, 5]$

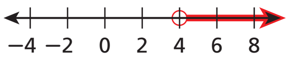
Solve each compound inequality. Then graph the solution set.

C $x + 3 > 7$ **OR** $3x \geq 18$

Solve both inequalities for x.

$$x + 3 > 7 \qquad \textit{or} \qquad 3x \geq 18$$
$$x > 4 \qquad \qquad \qquad x \geq 6$$

Because every point that satisfies $x \geq 6$ also satisfies $x > 4$, the solution set is $\{x \mid x > 4\}$.



$(4, \infty)$

CHECK IT OUT! **Solve each compound inequality. Then graph the solution set.**

1a. $x - 2 < 1$ or $5x \geq 30$ **1b.** $2x \geq -6$ and $-x > -4$

1c. $x - 5 < 12$ or $6x \leq 12$ **1d.** $-3x < -12$ and $x + 4 \leq 12$

Recall that the **absolute value** of a number x, written $|x|$, is the distance from x to zero on the number line. Because absolute value represents distance without regard to direction, the absolute value of any real number is nonnegative.

Know it! Note

Absolute Value

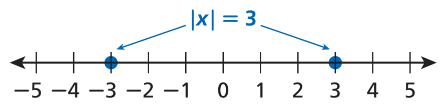
WORDS	NUMBERS	ALGEBRA
The absolute value of a real number x, $\lvert x\rvert$, is equal to its distance from zero on a number line.	$\lvert 5\rvert = 5$ $\lvert -5\rvert = 5$	$\lvert x\rvert = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

Absolute-value equations and inequalities can be represented by compound statements. Consider the equation $|x| = 3$.

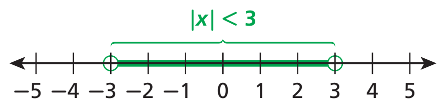
Helpful Hint

Think: Greater**or** inequalities involving $>$ or $\geq$ symbols are disjunctions.
Think: Less th**and** inequalities involving $<$ or $\leq$ symbols are conjunctions.

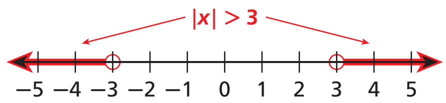
The solutions of $|x| = 3$ are the two points that are 3 units from zero. The solution is a disjunction: $x = -3$ or $x = 3$.



The solutions of $|x| < 3$ are the points that are less than 3 units from zero. The solution is a conjunction: $-3 < x < 3$.



The solutions of $|x| > 3$ are the points that are more than 3 units from zero. The solution is a disjunction: $x < -3$ or $x > 3$.



Know it! Note

Absolute-Value Equations and Inequalities

For all real numbers x and all positive real numbers a:

$\lvert x\rvert = a$	$\lvert x\rvert < a$	$\lvert x\rvert > a$
$x = -a$ OR $x = a$	$x > -a$ AND $x < a$ $-a < x < a$	$x < -a$ OR $x > a$

Note: The symbol $\leq$ can replace $<$, and the rules still apply. The symbol $\geq$ can replace $>$, and the rules still apply.

EXAMPLE 2 Solving Absolute-Value Equations

Solve each equation.

A $|x - 7| = 5$ — *This can be read as "the distance from x to 7 is 5."*

$x - 7 = 5$ or $x - 7 = -5$ — *Rewrite the absolute value as a disjunction.*

$x = 12$ or $x = 2$ — *Add 7 to both sides of each equation.*

B $|3x| + 5 = 14$

$|3x| = 9$ — *Isolate the absolute-value expression.*

$3x = 9$ or $3x = -9$ — *Rewrite the absolute value as a disjunction.*

$x = 3$ or $x = -3$ — *Divide both sides of each equation by 3.*

CHECK IT OUT! **Solve each equation.**

2a. $|x + 9| = 13$ **2b.** $|6x| - 8 = 22$

You can solve absolute-value inequalities using the same methods that are used to solve an absolute-value equation.

Know it! Note

Solving an Absolute-value Inequality
1. Isolate the absolute-value expression, if necessary.
2. Rewrite the absolute-value expression as a compound inequality.
3. Solve each part of the compound inequality for *x*.

EXAMPLE 3 Solving Absolute-Value Inequalities with Disjunctions

Solve each inequality. Then graph the solution set.

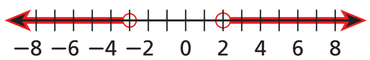
A $|2x + 1| > 5$

$2x + 1 > 5$ or $2x + 1 < -5$ — *Rewrite the absolute value as a disjunction.*

$2x > 4$ or $2x < -6$ — *Subtract 1 from both sides of each inequality.*

$x > 2$ or $x < -3$ — *Divide both sides of each inequality by 2.*

$\{x \mid x > 2 \cup x < -3\}$



$(-\infty, -3) \cup (2, \infty)$

To check, you can test a point in each of the three regions.

$|2(-4) + 1| > 5$ — $|-7| > 5$ ✔

$|2(0) + 1| > 5$ — $|1| > 5$ ✘

$|2(5) + 1| > 5$ — $|11| > 5$ ✔

Helpful Hint

In Example 3B, if you recognize that

$|\text{expression}| > -8$

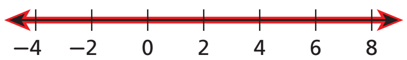
is always true, you will know the solution immediately.

B $|4x| + 16 > 8$

$|4x| > -8$ — *Isolate the absolute-value expression.*

$4x > -8$ or $4x < 8$ — *Rewrite the absolute value as a disjunction.*

$x > -2$ or $x < 2$ — *Divide both sides of each inequality by 4.*



$(-\infty, \infty)$

The solution set is *all real numbers*, $\mathbb{R}$.

CHECK IT OUT! **Solve each inequality. Then graph the solution set.**

3a. $|4x - 8| > 12$ **3b.** $|3x| + 36 > 12$

EXAMPLE 4 Solving Absolute-Value Inequalities with Conjunctions

Solve each inequality. Then graph the solution set.

A $\frac{|3x-9|}{2} \le 12$

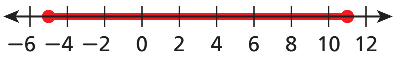
$|3x-9| \le 24$ — *Multiply both sides by 2.*

$3x - 9 \le 24$ and $3x - 9 \ge -24$ — *Rewrite the absolute value as a conjunction.*

$3x \le 33$ and $3x \ge -15$ — *Add 9 to both sides of each inequality.*

$x \le 11$ and $x \ge -5$ — *Divide both sides of each inequality by 3.*

The solution set is $\{x \mid -5 \le x \le 11\}$.



B $-4|x+3| \ge 8$

$|x+3| \le -2$ — *Divide both sides by −4, and reverse the inequality symbol.*

$x + 3 \le -2$ and $x + 3 \ge 2$ — *Rewrite the absolute value as a conjunction.*

$x \le -5$ and $x \ge -1$ — *Subtract 3 from both sides of each inequality.*

Because no real number satisfies both $x \le -5$ and $x \ge -1$, there is *no solution*. The solution set is $\varnothing$.

Helpful Hint

In Example 4B, if you recognize that

$|\text{expression}| \le -2$

is never true, you will know the solution immediately.

CHECK IT OUT! **Solve each inequality. Then graph the solution set.**

4a. $\frac{|x-5|}{2} \le 4$ **4b.** $-2|x+5| > 10$

THINK AND DISCUSS

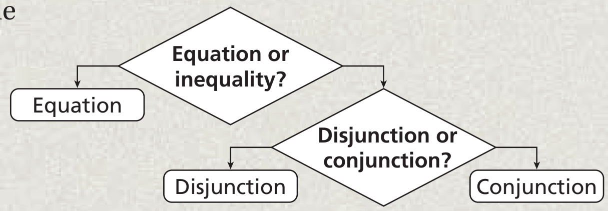
1. Explain why the solution set to $|7x| > -1$ is all real numbers.

2. Explain why there is no solution to $|x+3| \le -2$. Give another example of an absolute-value equation that has no solution.

3. Write an absolute-value inequality to model "the distance between x and 5 is greater than 10."

Know it! Note

4. GET ORGANIZED Copy and complete the graphic organizer. Use the flowchart to explain the decisions and steps needed to solve an absolute-value equation or inequality.



2-8 Exercises

California Standards
1.0; Preparation for 25.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 2-8
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** A graph of an inequality on a number line with two parts is a __?__. (*conjunction, disjunction*)

SEE EXAMPLE 1 p. 150

Solve each compound inequality. Then graph the solution set.

2. $x - 7 > -3$ OR $5x \leq -15$
3. $3x \leq 18$ AND $x + 4 > 2$
4. $x - 2 > -5$ OR $5x \geq 25$

SEE EXAMPLE 2 p. 152

Solve each equation.

5. $|x + 5| = 2$
6. $|2x| - 6 = 4$
7. $|-x| + 4 = 7$

SEE EXAMPLE 3 p. 152

Solve each inequality. Then graph the solution set.

8. $|2x - 3| \geq 5$
9. $2|x - 3| > 8$
10. $|3x| + 8 > 5$

SEE EXAMPLE 4 p. 153

11. $\dfrac{|4x + 8|}{3} < 8$
12. $|9 - 3x| \leq 6$
13. $-5|x - 3| \geq 15$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–15	1
16–19	2
20–23	3
24–27	4

Extra Practice

Skills Practice p. S7
Application Practice p. S33

Solve each compound inequality. Then graph the solution set.

14. $2x - 3 \geq 7$ OR $x + 5 < 2$
15. $3x + 6 \leq 21$ AND $4x - 2 \geq -6$

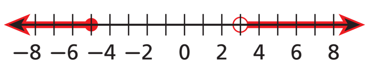
Solve each equation.

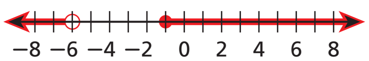
16. $|-3x| = 9$
17. $|x + 7| = 2$
18. $|3x - 9| = 6$
19. $5|2x| - 6 = 24$

Solve each inequality. Then graph the solution set.

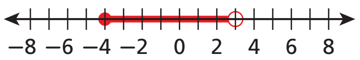
20. $|-2x| < 2$
21. $|x + 5| \geq 2$
22. $|8x| + 56 \geq 40$
23. $|7x + 14| \geq 35$
24. $|-0.5x| > 1$
25. $6|2x + 5| > 66$
26. $-8|x + 4| > 48$
27. $\dfrac{|8x + 4|}{6} < 10$

Write a compound inequality for each graph.

28. 

29. 

30. 

31. 

Solve and graph.

32. $5x - 9 > 11$ AND $7x + 12 \leq 61$
33. $7x + 4 \leq 3x - 12$ OR $\dfrac{9x - 15}{5} > 6$
34. $4(3 - 2x) < -20$ AND $\dfrac{3}{2}x - 4 < 5$
35. $5x + 12 > 2x - 3$ OR $3 - 5x < -17$

36. **///ERROR ANALYSIS///** Find and explain the error in one solution below.

A

$|3x - 6| < 12$

$3x - 6 < -12$ and $3x - 6 > 12$

$3x < -6$ and $3x > 18$

$x < -2$ and $x > 6$

B

$|3x - 6| < 12$

$3x - 6 > -12$ and $3x - 6 < 12$

$3x > -6$ and $3x < 18$

$x > -2$ and $x < 6$

Solve and graph.

37. $|5x - 8| = 27$ **38.** $8|3x - 10| - 12 = 20$ **39.** $\left|4(2x - 5)\right| \geq 4$

40. $\left|\frac{2x + 1}{5}\right| < 3$ **41.** $\frac{|4x + 5|}{3} + 9 > 15$ **42.** $|5 - 6x| - 10 \leq 8$

43. Estimation The table shows a sample of baseball bats considered to be within and outside the 32.5-inch-length class by the National Collegiate Athletics Association (NCAA). Write a possible absolute-value inequality to represent the bat lengths considered within the 32.5 inch class of bats.

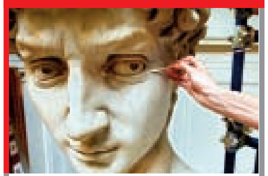
Bat Lengths (in.)

32.5-Inch Class	Outside 32.5-Inch Class
32.60	32.18
32.48	32.90
32.36	32.77
32.74	32.24

44. Psychology The IQ scores for the middle 50% of the population can be written as $\left|\frac{x - 100}{15}\right| \leq \frac{2}{3}$, where x is a person's IQ. Write and solve a compound inequality to find an interval for the IQ scores for the middle 50% of the population.

LINK

Sculpture



Michelangelo's *David* was sculpted from a single block of Carrara marble. It is nearly 18 feet tall and weighs well over 9 tons.

45. Geology Twenty cubic feet of marble can weigh 3400 pounds, plus or minus 100 pounds. Write and solve an absolute-value inequality for the possible weights of a cubic foot of marble.

46. Business A grocery scale is accurate to within 1 ounce. Write the error in the price when weighing an item that costs \$9 per pound as an absolute value expression.

47. Critical Thinking Is $c|a + b| = |ca + cb|$ always, sometimes, or never true? Justify your answer.

48. Manufacturing The acceptable tolerance of a machine part is 1 foot $\pm \frac{3}{64}$ in. Write the tolerance as an absolute-value equation in feet.

The solutions of an absolute-value equation are given. What is the equation?

49. $x = 2 \pm 3$ **50.** $x = -\frac{5}{2} \pm \frac{9}{2}$ **51.** $x = b \pm 2a$

52. Astronomy During 2007, Earth will travel around the Sun along a path that is not a perfect circle. Earth will be closest to the Sun on January 20, at a distance of 91.4 million miles, and farthest on July 7, at a distance of 94.5 million miles. Write and solve an absolute-value inequality for the distance between Earth and the Sun throughout the year.

53. Write About It When is $|x| = |-x|$? When is $|x| = -|x|$? Explain.

CONCEPT CONNECTION

54. This problem will help prepare you for the Concept Connection on page 164.

For a livestock competition, the weight classes for goats are shown in this table.

a. What is the center of each weight class?

b. How would you express each weight class as an absolute-value expression?

c. Is there exactly one class for any goat in the weight range shown in the table? Explain.

d. What if...? Suppose just the upper range of the heavy class were increased by 1 lb. How would the absolute-value expression change to reflect the increase?

Goat Weight Classes

Class	Weight Range (lb)
Light	40–50
Medium	50–60
Heavy	60–73

STANDARDIZED TEST PREP

55. Which statement is equivalent to $|x - y|$?

Ⓐ $|x + y|$ Ⓑ $|y - x|$ Ⓒ $x + y$ Ⓓ $y - x$

56. Which of the following is NOT a solution of $|x - 8| \leq 12$?

Ⓕ $x = 20$ Ⓖ $x = 3$ Ⓗ $x = -2$ Ⓙ $x = -10$

57. How many solutions does $-5|3x + 5| - 6 = 4$ have?

Ⓐ An infinite number Ⓑ 2 Ⓒ 1 Ⓓ 0

58. A thermometer measures 5 body temperatures accurately to within ± 0.15°F. Which of the following is an expression for the actual temperature t of a person if this thermometer measures the person's temperature as 98.5°F?

Ⓕ $|t - 98.5| \leq 0.15$ Ⓗ $|t - 98.5| \geq 0.15$

Ⓖ $|t + 98.5| \leq 0.15$ Ⓙ $|t + 98.5| \geq 0.15$

CHALLENGE AND EXTEND

59. Solve $|3x - 8| = 5x$.

60. Solve $|5x + 2| + 3x \leq 8$.

61. If x is an integer, which statement is equivalent to $|x - 3| < 16$? Explain.

a. $|x - 3| \leq 16$ **b.** $|x - 3| \leq 15$ **c.** $|x - 3| \leq 17$ **d.** $|x - 2| \leq 16$

62. Are the solution sets of $|x + a| = b$ and $|x| + a = b$ the same? Explain.

63. Consider the equation $(a + b) + c = a + (b + c)$.

a. What property of real numbers does this demonstrate?

b. Is $|a + b| + c = a + |b + c|$ a true statement? Support your answer.

c. What can you conclude about this property with respect to absolute value?

64. Technology A binary search repeatedly divides records of a sorted file in half until the correct record is found. For example, to find data in record 6 of an 8-record file, the binary search will examine records 1–8, then it would narrow the search to records 5–8, then 5–6, then locate the data in record 6. Write absolute-value statements for the records searched in the first three search intervals.

SPIRAL REVIEW

65. Travel Pamela filled her 15 gal gas tank before a trip. She added 13 gal after driving 385 mi and 14 gal after another 412 mi. Estimate the number of mi/gal her car got on this trip. *(Previous course)*

Determine the value of n. Identify the property demonstrated. *(Lesson 1-2)*

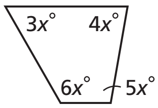
66. $7 \cdot n = 1$

67. $24 + 16 = (n + 4)4$

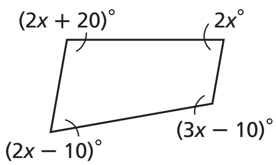
68. $(2 + 3) + n = 0$

Geometry Find the measure of each angle in the quadrilaterals below. (Hint: The sum of the angle measures in a quadrilateral is 360°.) *(Lesson 2-1)*

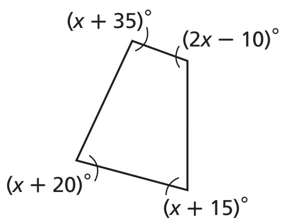
69.



70.



71.



2-9 Technology LAB

Solving Absolute-Value Equations

Use with Lesson 2-9

A graphing calculator is helpful for visualizing solutions of absolute value equations.

California Standards

1.0 Students solve equations and inequalities involving absolute value.

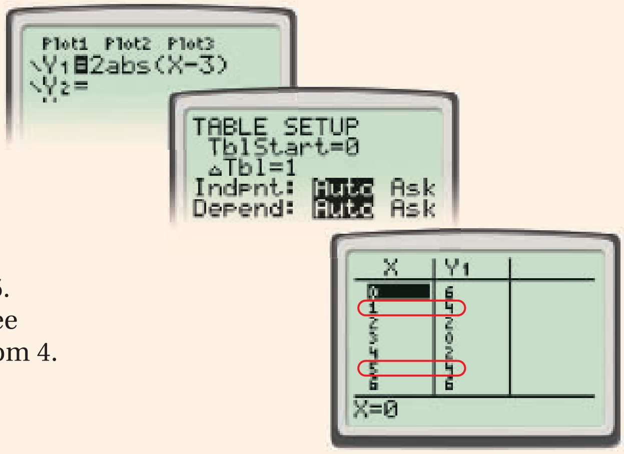
go.hrw.com
Lab Resources Online
KEYWORD: MB7 Lab2

Activity

1 **Use a table to solve $2|x - 3| = 4$.**

Enter the left side of expression in the Y= editor. Press MATH and use the **NUM** menu for **ABS(**.

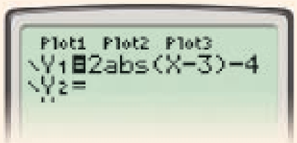
Use the defaults for 2nd WINDOW (TBLSET), and then select 2nd GRAPH (TABLE) to see values for $2|x - 3|$ when $x = 0, 1, 2, 3, \ldots$



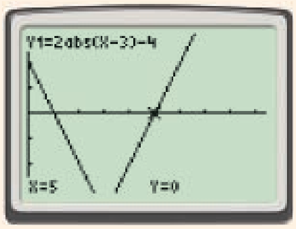
Notice that **Y1** $= 4$ when $x = 1$ and when $x = 5$. If you scroll up and down the table, you will see that the values of **Y1** get farther and farther from 4. The solution set is $\{1, 5\}$.

2 **Use a graph to solve $2|x - 3| = 4$.**

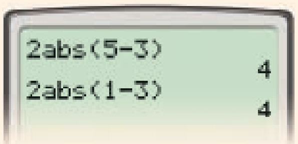
First get 0 on one side by adding -4 to both sides of the equation, obtaining the equation $2|x - 3| - 4 = 0$



Enter the left side of the equation as **Y1**, and graph in the friendly window $[0, 9.4]$ by $[-3.2, 3.2]$. TRACE to the solutions $x = 1$ and $x = 5$.



You can test $x = 1$ and $x = 5$ back in the original equation on the home screen.



You should check algebraically.

$$\begin{array}{r|l} 2|x-3| & = 4 \\ \hline 2|1-3| & 4 \\ 2(2) & 4 \\ 4 & 4\ \checkmark \end{array} \qquad \begin{array}{r|l} 2|x-3| & = 4 \\ \hline 2|5-3| & 4 \\ 2(2) & 4 \\ 4 & 4\ \checkmark \end{array}$$

Try This

1. Solve $3|x - 1| = 6$ by using a table of values. Then solve by graphing.
2. Solve $5|x + 3| = 0$ by using a table of values. Then solve by graphing.
3. What happens when you solve $2|x + 1| = -4$ by using a table of values? What happens when you solve by graphing?

2-9

Absolute-Value Functions

Objective
Graph and transform absolute-value functions.

Vocabulary
absolute-value function

California Standards
1.0 Students solve equations and inequalities involving absolute value.

Who uses this?
Park rangers can use absolute value to monitor the movement of an animal as it passes a specific location. (See Exercise 30.)

An **absolute-value function** is a function whose rule contains an absolute-value expression. The graph of the parent absolute-value function $f(x) = |x|$ has a V shape with a minimum point or vertex at $(0, 0)$.

Know it! Note

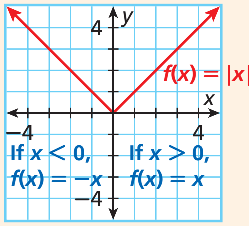
The Absolute-Value Parent Function $f(x) = |x|$

Domain: all real numbers

Range: nonnegative real numbers

Vertex: (0,0)

x	$y = \|x\|$
−10	10
−5	5
0	0
5	5
10	10



The absolute-value parent function is composed of two linear pieces, one with a slope of −1 and one with a slope of 1. In Lesson 2-6, you transformed linear functions. You can also transform absolute-value functions.

EXAMPLE 1

Translating Absolute-Value Functions

Let $g(x)$ be the indicated transformation of $f(x) = |x|$. Write the rule for $g(x)$ and graph the function.

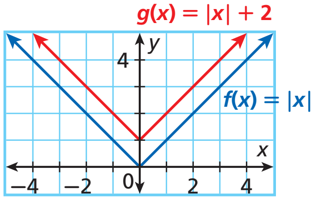
A 2 units up

$f(x) = x$

$g(x) = f(x) + k$

$g(x) = x + 2$ *Substitute.*

The graph of $g(x) = |x| + 2$ is the graph of $f(x) = |x|$ after a vertical shift of 2 units up. The vertex of $g(x)$ is $(0, 2)$.



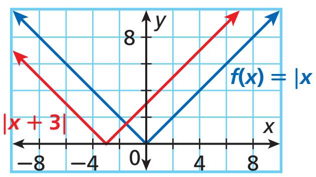
B 3 units left

$f(x) = |x|$

$g(x) = f(x - h)$

$g(x) = |x - (-3)| = |x + 3|$ *Substitute.*

The graph of $g(x) = |x + 3|$ is the graph of $f(x) = |x|$ after a horizontal shift of 3 units left. The vertex of $g(x)$ is $(-3, 0)$.



Remember!
The general forms for translations are
Vertical:
$g(x) = f(x) + k$

Horizontal:
$g(x) = f(x - h)$

CHECK IT OUT! Let $g(x)$ be the indicated transformation of $f(x) = |x|$. Write the rule for $g(x)$ and graph the function.

1a. 4 units down **1b.** 2 units right

Because the entire graph moves when shifted, the shift from $f(x) = |x|$ determines the vertex of an absolute-value graph.

Know it! Note

Vertex of an Absolute-Value Function

The graph of $g(x) = |x - h| + k$ is the image of $f(x) = |x|$ after a horizontal shift of h units and a vertical shift of k units so that the vertex is at (h, k).

EXAMPLE 2 Translations of an Absolute-Value Function

Translate $f(x) = |x|$ so that the vertex is at $(-5, 3)$. Then graph.

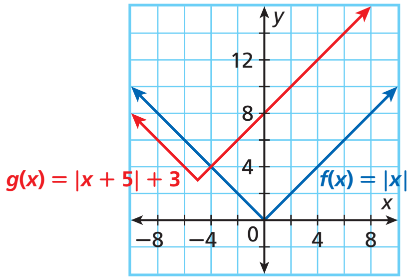
$g(x) = |x - h| + k$

$g(x) = |x - (-5)| + 3$ *Substitute.*

$g(x) = |x + 5| + 3$

The graph of $g(x) = |x + 5| + 3$ is the graph of $f(x) = |x|$ after a vertical shift up 3 units and a horizontal shift left 5 units.

The graph confirms that the vertex is $(-5, 3)$.



Math Builders

For more on transformations, see the Transformation Builder on page MB2.

CHECK IT OUT! **2.** Translate $f(x) = |x|$ so that the vertex is at $(4, -2)$. Then graph.

Absolute-value functions can also be stretched, compressed, and reflected.

EXAMPLE 3 Transforming Absolute-Value Functions

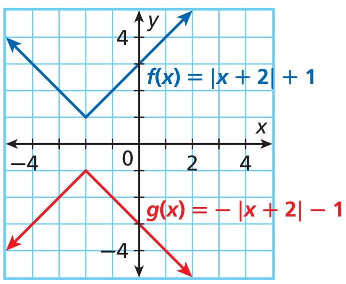
Perform each transformation. Then graph.

A **Reflect the graph of $f(x) = |x + 2| + 1$ across the x-axis.**

$g(x) = -f(x)$ *Take the opposite of the entire function.*

$g(x) = -(|x + 2| + 1)$ *Distribute the negative sign.*

The vertex of the graph of $g(x) = -|x + 2| - 1$ is $(-2, -1)$. The graph is reflected across the x-axis.



Remember!

Reflection across x-axis:
$g(x) = -f(x)$

Reflection across y-axis:
$g(x) = f(-x)$

Perform each transformation. Then graph.

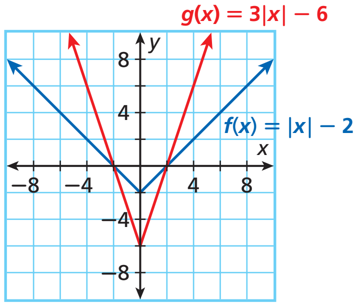
B **Stretch the graph of $f(x) = |x| - 2$ vertically by a factor of 3.**

$g(x) = af(x)$

$g(x) = 3(|x| - 2)$ *Multiply the entire function by 3.*

$g(x) = 3|x| - 6$

The graph of $g(x) = 3|x| - 6$ is the graph of $f(x) = |x| - 2$ after a vertical stretch by a factor of 3. The vertex of g is at $(0, -6)$.



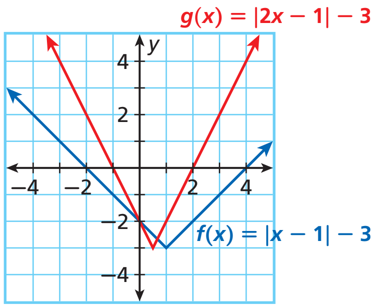
C **Compress the graph of $f(x) = |x - 1| - 3$ horizontally by a factor of 0.5.**

$g(x) = f\left(\frac{1}{b}x\right)$

$g(x) = \left|\frac{1}{0.5}x - 1\right| - 3$ *Substitute 0.5 for b.*

$g(x) = |2x - 1| - 3$ *Simplify.*

The graph of $g(x) = |2x - 1| - 3$ is the graph of $f(x) = |x - 1| - 3$ after a horizontal compression by a factor of 0.5. The vertex of g is $\left(\frac{1}{2}, -3\right)$.



Remember!

Vertical stretch and compression:
$g(x) = af(x)$

Horizontal stretch and compression:
$g(x) = f\left(\frac{1}{b}x\right)$

CHECK IT OUT!

Perform each transformation. Then graph.

3a. Reflect the graph of $f(x) = -|x - 4| + 3$ across the y-axis.

3b. Compress the graph of $f(x) = |x| + 1$ vertically by a factor of $\frac{1}{2}$.

3c. Stretch the graph of $f(x) = |4x| - 3$ horizontally by a factor of 2.

THINK AND DISCUSS

1. Explain why the vertex of $f(x) = |x|$ stays the same when the graph is stretched but not when the graph is shifted.

2. Tell what the graph of $y = |-x|$ looks like.

3. **GET ORGANIZED** Copy and complete the graphic organizer. Fill in the table with examples of absolute-value transformations.

Know it! Note

Transformation	Absolute-Value Function	Transformed Function	Graph
Vertical translation			
Horizontal translation			
(h, k) translation			
Stretch			
Compression			
Reflection			

2-9 Exercises

California Standards
1.0; Review of 1A2.0, 1A4.0, 1A5.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 2-9
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** Explain the reason for the shape of the graph of an *absolute-value function.*

SEE EXAMPLE 1 p. 158

Let $g(x)$ be the indicated transformation of $f(x) = |x|$. Write the rule for $g(x)$ and graph the function.

2. 5 units down
3. 4 units left

SEE EXAMPLE 2 p. 159

Translate $f(x) = |x|$ so that the vertex is at the given point. Then graph.

4. $(-4, -5)$
5. $(1, 6)$

SEE EXAMPLE 3 p. 159

Perform each transformation. Then graph.

6. Reflect the graph of $f(x) = |2x + 3| - 4$ across the y-axis.
7. Stretch $f(x) = |x + 3|$ vertically by a factor of 2.
8. Compress $f(x) = |x + 3|$ horizontally by a factor of $\frac{2}{3}$.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–11	1
12–14	2
15–17	3

Extra Practice
Skills Practice p. S7
Application Practice p. S33

Let $g(x)$ be the indicated transformation of $f(x) = |x|$. Write the rule for $g(x)$ and graph the function.

9. 2 units right
10. 1 unit down
11. 4 units left

Translate $f(x) = |x|$ so that the vertex is at the given point. Then graph.

12. $(8, 0.5)$
13. $(1.5, 4.5)$
14. $(-2.5, 3)$

Perform each transformation. Then graph.

15. Reflect $f(x) = |x - 5| + 2$ across the x-axis.
16. Compress $f(x) = |2x| - 3$ vertically by a factor of $\frac{1}{4}$.
17. Stretch $f(x) = |2x| - 3$ horizontally by a factor of $\frac{3}{2}$.
18. **Football** Yard lines of a football field have the relationship shown in the table below (0 yard lines are the goal lines).

Football Field Yard Lines

Distance from One End Zone (yd)	0	10	20	30	40	50	60	70	80	90	100
Marked Yard Line	0	10	20	30	40	50	40	30	20	10	0

a. Write an absolute-value function to find the marked yard line for a given distance from the end zone. (*Hint:* Graph the ordered pairs to find the transformation from $f(x) = |x|$.)

b. What yard line is 195 feet from the end zone?

c. **What if...?** Suppose the absolute-value function is based on the distance from the end zone *in feet*. How would this relationship affect the function?

State the transformation from the graph of $f(x) = |x|$. Then graph the transformed function.

19. $g(x) = |x| - 6$ **20.** $g(x) = |x - 6|$ **21.** $g(x) = 2|x - 1|$

Find the vertex of the graph of each function.

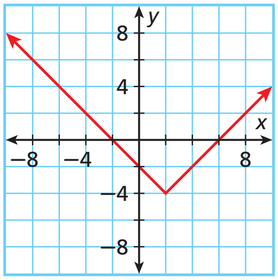
22. $g(x) = |x - 12| + 8$ **23.** $g(x) = |x + 5| + 9$ **24.** $g(x) = 6 + |x - 7|$

25. Write About It How do the slopes of the two parts of an absolute-value function compare? Justify your answer and give examples.

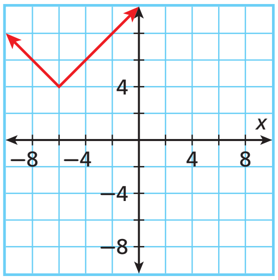
26. Critical Thinking Name two different transformations that move $f(x) = |x|$ 4 units up.

Find an absolute-value function for each graph.

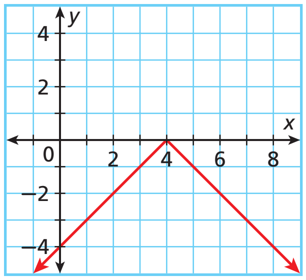
27.



28.



29.



30. Zoology Park rangers track a panther by using a radio transmitter. The panther's distance from the ranger station can be modeled by the function $d = \left|760 - \frac{4}{3}t\right| + 10$, where d is distance in meters and t is the time in seconds since the rangers started timing.

a. How fast is the panther walking along the path?

b. Find the vertex of the function. How long will it take the panther to reach its closest point to the ranger station?

c. How long will the panther be within 200 meters of the ranger station?

d. How far from the ranger station will the panther be after 15 minutes?

31. Critical Thinking Compare vertical stretch to horizontal compression. How are they different? How are they the same?

CONCEPT CONNECTION

32. This problem will prepare you for the Concept Connection on page 164.

For a livestock competition, the weight classes for hogs are shown in this table.

a. What is the center of each weight class?

b. Write functions in terms of y for the range of each weight class. Specify the domain for each.

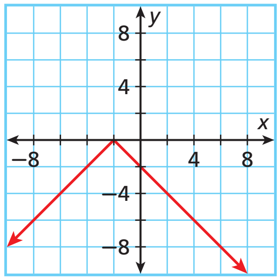
c. Graph the functions on the same coordinate plane for the relevant domain.

d. Where would the functions overlap without the domain restrictions?

Hog Weight Classes

Class	Weight Range (lb)
Light	200–230
Heavy	230–250

STANDARDIZED TEST PREP



33. Which function best describes the graph shown?

Ⓐ $y = |x - 2|$ Ⓒ $y = -|x| - 2$

Ⓑ $y = |-x| + 2$ Ⓓ $y = -|x + 2|$

34. The graph of which function is the same as the graph of $f(x) = |x|$?

Ⓕ $g(x) = \left|\frac{1}{x}\right|$ Ⓗ $g(x) = |-x|$

Ⓖ $g(x) = -\left|\frac{1}{x}\right|$ Ⓙ $g(x) = -|x|$

35. At which of the following points are the x-intercepts found for the graph of $f(x) = |3x| - 9$?

Ⓐ $(9, 0)$ and $(-9, 0)$ Ⓒ $(0, 9)$ and $(0, -9)$

Ⓑ $(3, 0)$ and $(-3, 0)$ Ⓓ $(0, 3)$ and $(0, -3)$

36. For which function does y correspond to a nonnegative real number?

Ⓕ $y = |x| - 3$ Ⓖ $y = |x - 3|$ Ⓗ $y = -|3 - x|$ Ⓙ $y + 3 = |x + 3|$

37. If $g(x)$ is the reflection of $f(x) = |x + 1| - 2$ across the y-axis, at which of the following points does the graph of $g(x)$ cross the x-axis?

Ⓐ $(-3, 0)$ and $(-1, 0)$ Ⓒ $(-3, 0)$ and $(1, 0)$

Ⓑ $(3, 0)$ and $(-1, 0)$ Ⓓ $(3, 0)$ and $(1, 0)$

CHALLENGE AND EXTEND

38. Graph $y < |x + 2|$.

39. Graph $|y| \leq 4$ on a coordinate plane.

40. **Geometry** Graph the triangle that is above the x-axis below the graph of $f(x) = -|2x| + 8$. Find the area.

41. Write an absolute-value equation for $f(x) = \begin{cases} 2x + 6 \text{ if } x \geq -3 \\ -2x - 6 \text{ if } x < -3 \end{cases}$. Then graph.

42. Graph $x = |y|$. Is the graph a function? Explain.

SPIRAL REVIEW

Perform the indicated operation. Write each answer in scientific notation. *(Lesson 1-5)*

43. $(1.5 \times 10^{-4})(5.0 \times 10^{13})$

44. $(9.8 \times 10^{7})(8.9 \times 10^{-7})$

45. $(8.1 \times 10^{3})^2$

46. $\dfrac{6.2 \times 10^{7}}{3.1 \times 10^{-4}}$

47. $\dfrac{1.9 \times 10^{-6}}{9.5 \times 10^{18}}$

48. $\dfrac{2 \times 10^{-3}}{5 \times 10^{-3}}$

Perform the given transformation on the point $(3, -5)$, and give the coordinates of the translated point. *(Lesson 1-8)*

49. 2 units left, 6 units up

50. 10 units down

51. 3 units right

52. reflected across the x-axis

53. 1 unit right, 5 units down

54. reflected across the y-axis

Solve. *(Lesson 2-1)*

55. $-2x + 3(1 - x) = -\dfrac{10x}{2}$

56. $0.75(-4x - 12) = -3(3 + x)$

CHAPTER 2

SECTION 2B

CONCEPT CONNECTION

Applying Linear Functions

Data Dilemma The Livestock Show and Rodeo School Art Program is an annual competition for students. Participants in grades ranging from kindergarten through 12 must submit an original art project based on Western culture, history, or heritage. Projects are judged by the show's School Art Committee. Each school district selects the top 20 students to compete in this annual citywide competition. The scores for the top entries in the East District are shown in the table.

The Art Committee guidelines state that the top score awarded in district competitions should be 100. The East District judges have decided to add 5 points to each score in order to comply with the competition guidelines.

Competition Results

Entry	Score
1	95
2	93
3	92
4	91
5	90
6	90
7	89
8	87
9	86
10	85
11	84
12	83
13	82
14	81
15	80
16	79
17	77
18	74
19	71
20	65

1. Create a table to show the new scores. Compare the mean and median of the original scores with those of the modified scores.
2. Graph the original scores using the entry number as the x-coordinate and the score as the y-coordinate. Describe the parent function to which this graph belongs.
3. Predict how the graph of the modified scores will compare with the graph of the original scores. Graph the modified scores on the same graph as the original scores to check your prediction.
4. If $y = f(x)$ represents the function rule for the original scores, determine a function rule for the modified scores. Explain.
5. One judge suggested that the original scores should be multiplied by a factor that would make the highest score 100 points. What factor should be used?
6. Make a table showing the new scores. Compare the mean and median of the original scores with those of these new scores.
7. Graph the newest set of scores on the same graph as the original scores, and describe the transformation.
8. Which method do you think the judges should use to adjust the scores? Explain your answer.

READY TO GO ON?

Quiz for Lessons 2-6 Through 2-9

2-6 Transforming Linear Functions

Let $g(x)$ be the indicated transformation(s) of $f(x)$. Write the rule for $g(x)$.

1. $f(x) = x$; horizontal translation 5 units right
2. $f(x) = 2x$; vertical stretch by a factor of 5
3. $f(x) = x + 6$; vertical compression by a factor of $\frac{1}{3}$ followed by a horizontal translation left 4 units
4. $f(x) = 3x - 5$; vertical translation 6 units up followed by a horizontal stretch by a factor of $\frac{3}{2}$

2-7 Curve Fitting with Linear Models

5. Lea keeps track of the number of hours she works in a week and her income for the week. Here are the results from a randomly selected sample of weeks.

Hours	8	23	18	30	12	28
Income ($)	152	465	315	530	240	525

   a. Draw a scatter plot of the data using hours as the independent variable.
   b. Use your graphing calculator to find the correlation coefficient and the equation of the line of best fit for the data. What does the slope of the line of best fit mean for Lea?
   c. Use your equation to predict how much Lea would make in a 40-hour week.

2-8 Solving Absolute-Value Equations and Inequalities

Solve each equation.

6. $|9 - 2x| = 15$
7. $2|x| - 12 = 16$
8. $\frac{|3x - 4|}{-5} = 6$
9. $|2x - 5| = x + 3$

Solve each inequality. Then graph the solution.

10. $|5x + 15| > 20$
11. $\left|\frac{x - 2}{4}\right| \leq 5$
12. $-3|5x - 8| - 5 \geq 6$
13. $|12 - 4x| - 4 > 20$

2-9 Absolute-Value Functions

Translate $f(x) = |x|$ so that the vertex is at the given point. Then graph.

14. $(0, -4)$
15. $(2, 7)$
16. $(-2, 0)$
17. A food order at a restaurant is paid for with a $10 bill.
   a. What function represents the difference between the cost of the food and the change returned? Assume that this difference is nonnegative.
   b. Graph the function.

CHAPTER 2

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. If there are no values that make an equation true, then the equation is a(n) ___?___ .

2. The equation $y - 5 = 2(x - 1)$ is in ___?___ .

3. ___?___ is the strength and direction of the linear relationship between two variables.

2-1 Solving Linear Equations and Inequalities *(pp. 90–96)*

EXAMPLES

Solve.

■ $5(x + 4) = 3x - 2$

$5x + 20 = 3x - 2$ *Use the Distributive Property.*

$2x + 20 = -2$ *Subtract 3x from both sides.*

$2x = -22$ *Subtract 20 from both sides.*

$x = -11$ *Divide both sides by 2.*

■ $\frac{15 - 3x}{2} < 12$

$15 - 3x < 24$ *Multiply both sides by 2.*

$-3x < 9$ *Subtract 15 from both sides.*

$x > -3$ *Divide both sides by –3x, and reverse the inequality.*

EXERCISES

Review of 1A4.0, 1A5.0

Solve.

4. $35 = 7(2x - 8)$

5. $3x + 12 - 9x = 12 - 6x$

6. $4(3x + 5) = 12 - 2x$

7. $3x - 5(x + 3) = 16 - 4x$

8. $\frac{5}{2}\left(3x - \frac{3}{2}\right) - \frac{3}{4} = \frac{2}{3}x + 4$

9. Magnets cost \$10 plus \$1.25 each to produce. You sell them for \$1.75. How many magnets were sold if you made a profit of \$60?

10. $24 \geq 6x - 18$

11. $8x + 12 < 5x - 20$

12. $\frac{13 - 5x}{8} \geq -4$

Write an equation or inequality, and solve.

13. Ali's health club membership costs \$19.95 per month. Ali pays \$2.75 each time he works out. If Ali wants to spend less than \$50 per month at the health club, how often can he visit?

2-2 Proportional Reasoning *(pp. 97–103)*

Review of 1A5.0, 1A15.0

EXAMPLE

Solve the proportion.

- $\dfrac{x+2}{12} = \dfrac{15}{20}$

$20(x+2) = (12)(15)$ *Set cross products equal.*

$20x + 40 = 180$

$20x = 140$

$x = 7$

EXERCISES

Solve each proportion.

14. $\dfrac{12}{x} = \dfrac{4}{11}$

15. $\dfrac{-9}{4} = \dfrac{3x}{20}$

16. $\dfrac{x-3}{4} = -\dfrac{5}{3}$

17. $\dfrac{4}{5-2x} = \dfrac{3}{3x-1}$

18. If a flagpole that is 20 feet tall casts a 6 foot shadow, how long a shadow would a building that is 15 feet tall cast at the same time of day?

2-3 Graphing Linear Functions *(pp. 105–112)*

Review of 1A6.0

EXAMPLES

Find the intercepts. Then graph.

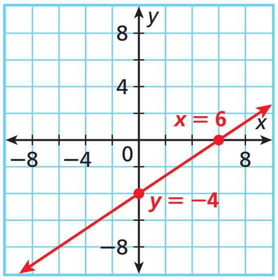
- $2x - 3y = 12$

$2x = 12$ *Set y equal to 0 to find the x-intercept.*

$x = 6$

$-3y = 12$ *Set x equal to 0 to find the y-intercept.*

$y = -4$

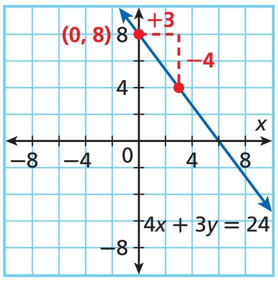


Write each function in slope-intercept form. Then graph.

- $4x + 3y = 24$

$3y = -4x + 24$ *Isolate the y-term.*

$y = -\dfrac{4}{3}x + 8$ *Divide both sides by 3.*



EXERCISES

Determine whether the data set could represent a linear function.

19.

x	1	4	7	10
$f(x)$	3	−2	−7	−12

Find the intercepts. Then graph.

20. $2x + 5y = 10$

21. $-6x + 9y = -18$

22. $8x = 12y - 18$

23. $y = 6 - 4x$

Write each function in slope-intercept form. Then graph.

24. $6x + 3y = 15$

25. $5x - 3y = -9$

26. $9x = 12 - 6y$

27. $\dfrac{8}{9}x + \dfrac{4}{3}y = 12$

Determine whether each line is vertical or horizontal. Then graph.

28. $-3 = x$

29. $y = \dfrac{5}{2}$

30. A rock climber is descending down a 500-ft-tall cliff. After 8 min, the rock climber has descended to a height of 280 ft. Find the height as a linear function of the time, and graph the function.

2-4 Writing Linear Functions *(pp. 115–123)*

Review of 1A7.0, 1A8.0

EXAMPLE

- Write the equation of the line through $(3, 4)$ and $(5, 10)$ in slope-intercept form.

  Find the slope $m = \frac{10 - 4}{5 - 3} = 3$

  Write an equation:

Method 1	Method 2	
$y - y_1 = m(x - x_1)$	$y = mx + b$	
$y - 4 = 3(x - 3)$	$y = 3x + b$	
$y - 4 = 3x - 9$	$4 = 3(3) + b$	
$y = 3x - 9 + 4$	$-5 = b$	*−5 is the*
$y = 3x - 5$	$y = 3x - 5$	*y-intercept*

EXERCISES

Write the equation of each line in slope-intercept form.

31. passing through $(4, 6)$ with slope $\frac{1}{2}$

32. passing through $(2, 6)$ and $(3, 9)$

33. through $(4, -2)$ and parallel to $y = \frac{3}{2}x + 9$

34. through $(-3, 4)$ and perpendicular to $y = \frac{3}{2}x + 9$

2-5 Linear Inequalities in Two Variables *(pp. 124–131)*

Review of 1A6.0

EXAMPLE

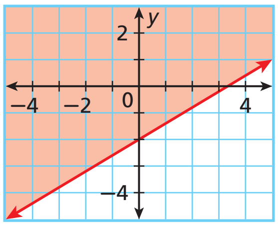
Solve for *y*. Graph the solution.

- $3x - 5y \leq 10$

  $-5y \leq -3x + 10$

  $y \geq \frac{3}{5}x - 2$

  Use a solid boundary line and shade the region above the boundary.



EXERCISES

Solve for *y*. Graph the solution.

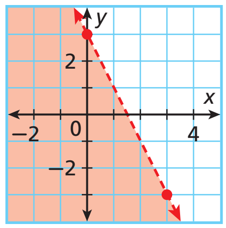
35. $y > -3$

36. $y \leq x + 3$

37. $2x + 4y > -12$

38. $6x - 2y > 8$

39. Write an inequality for the graph.



40. A gallery offers a limited-access ticket for \$12 and a standard ticket for \$21. More than \$2520 in tickets were sold. Write and graph an inequality for the numbers of each type of ticket sold.

2-6 Transforming Linear Functions *(pp. 134–140)*

Prep for 9.0

EXAMPLE

Let $g(x)$ be the indicated transformation of $f(x) = x$. Write the rule for $g(x)$.

- horizontal shift 5 units left followed by a horizontal stretch by a factor of 3

  Translating $f(x)$ 5 units left replaces each x with $(x + 5)$.

  Let $h(x) = f(x + 5)$

  Replace each x with $\left(\frac{x}{3}\right)$.

  $g(x) = h\left(\frac{x}{3}\right) = \frac{x}{3} + 5$

EXERCISES

Let $g(x)$ be the indicated transformation of $f(x) = x$. Write the rule for $g(x)$.

41. horizontal shift 8 units right

42. vertical shift 5 units up followed by a vertical stretch by a factor of 3

43. horizontal shift 3 units left followed by a vertical shift down 7 units

44. vertical shift 5 units up followed by a reflection across the x-axis

45. horizontal shift 12 units right followed by a reflection across the y-axis

2-7 Curve Fitting with Linear Models *(pp. 142–149)*

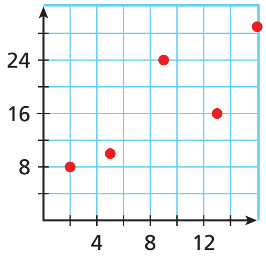
Ext. of 1A7.0

EXAMPLE

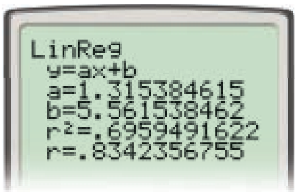
- **Make a scatter plot of the data. Find the correlation coefficient *r* and the equation of the line of best fit.**

x	2	5	9	13	16
y	8	10	24	16	29

The scatter plot is shown at right



Use **LinReg** on your graphing calculator.



$r \approx 0.834$. The equation of the line of best fit is $y \approx 1.32x + 5.56$.

EXERCISES

46. Find the following for this set of data on median income and median home price.

a. Make a scatter plot of the data using median income as the independent variable.

b. Find the correlation coefficient r and the line of best fit for these data.

Median Income (thousands)	Median Home Price (thousands)
69.5	130.2
46.3	94.5
56.7	115.5
65.2	106.4
54.7	98.6
59.6	115.5

2-8 Solving Absolute-Value Equations and Inequalities *(pp. 150–156)*

1.0

EXAMPLE

Solve the inequality. Then graph the solution set.

- $|2x + 8| - 10 \le 2$

$|2x + 8| \le 12$

$2x + 8 \le 12$ and $2x + 8 \ge -12$ *Conjunction*

$2x \le 4$ and $2x \ge -20$

$x \le 2$ and $x \ge -10$

The solution set is $\{x \mid -10 \le x \le 2\}$

EXERCISES

Solve.

47. $|x - 8| = 20$

48. $\left|\frac{x-6}{5}\right| = 12$

49. $4|3x - 8| + 16 = 2$

Solve each inequality. Then graph the solution.

50. $3x + 6 > 15$ or $5x + 13 < -12$

51. $2(3x + 6) \le 32 + 2x$ AND $5x + 15 \ge 2x + 9$

52. $|4x - 8| < 4$

53. $|5x + 10| \ge 30$

2-9 Absolute Value Functions *(pp. 158–163)*

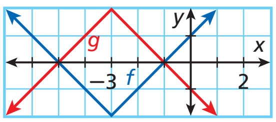
1.0

EXAMPLE

- Reflect the graph of $f(x) = |x + 3| - 2$ across the x-axis, and graph the function.

$g(x) = -(|x + 3| - 2)$ *$g(x) = -f(x)$*

$g(x) = -|x + 3| + 2$



EXERCISES

Translate $f(x) = |x|$ so the vertex is at the given point.

54. $(-5, 7)$

55. $(6, -9)$

Perform each transformation. Then graph.

56. $f(x) = |x - 4| + 1$ reflected across the y-axis

57. $f(x) = |3x + 1|$ compressed vertically by $\frac{1}{3}$

58. $f(x) = |x - 3| + 5$ reflected across the x-axis

CHAPTER 2

Chapter Test

Solve.

1. $5(3x - 4) - 12 = 73$
2. $2x + 12 - 8x = 9 - x - 5x$
3. $4(3 - 3x) - 8x = 15 - 2(5x + 8)$
4. $\frac{-5}{4} = \frac{12}{x}$
5. $\frac{3x - 9}{15} = \frac{18}{12}$
6. $\frac{2}{2x - 5} = \frac{3}{x + 1}$
7. Tim and Kim took 4.6 hours to complete a 25.3 mile kayaking trip. If they want to paddle for 3 hours on their next trip, how far should they plan to go?

Graph.

8. $y = \frac{5}{3}x - 4$
9. $6x + 8y = 24$
10. $6x + 2y < 10$

Write the equation of each line in slope-intercept form.

11. passing through $(9, 12)$ and $(7, 2)$
12. parallel to $9x - 5y = 8$ and through $(-10, 2)$
13. perpendicular to $y = -\frac{2}{7}x + 3$ and through $(6, 4)$
14. The Spanish Club is selling T-shirts and hats and would like to raise at least \$2400. It sells T-shirts for \$15 and hats for \$8. Write and graph an inequality representing the number of T-shirts and hats the club must sell to meet its goal.

Let $g(x)$ be the indicated transformation(s) of $f(x) = x$. Write the rule for $g(x)$.

15. vertical stretch by a factor of 4
16. horizontal translation 6 units right
17. horizontal compression by a factor of $\frac{1}{6}$ followed by a vertical shift 4 units down
18. A consumer group is studying how hospitals are staffed. Here are the results from eight randomly selected hospitals in a state.

Full-Time Hospital Employees								
Hospital Beds	23	29	35	42	46	54	64	76
Full-Time Employees	69	95	118	126	123	178	156	176

   a. Make a scatter plot of the data with hospital beds as the independent variable.
   b. Find the correlation coefficient and the equation of the line of best fit. Draw the line of best fit on your scatter plot.
   c. Predict the number of beds in a hospital with 80 full-time employees.

19. Solve $|12 + 4x| - 6 = 26$.

Solve and graph.

20. $16 \le \frac{24 - 8x}{5}$
21. $|3x - 9| > 12$
22. $3|12 - 4x| + 4 \le 28$
23. A pollster predicts the actual percent p of a population that favors a political candidate by using a sample percent s plus or minus 3%. Write an absolute-value inequality for p.
24. Translate $f(x) = |x|$ so that its vertex is at $(4, -2)$. Then graph.
25. Find $g(x)$ if $f(x) = |2x| - 3$ is stretched horizontally by a factor of 3 and reflected across the x-axis.

COLLEGE ENTRANCE EXAM PRACTICE

FOCUS ON ACT

The ACT measures college-preparedness by testing skills in English, mathematics, reading, and science. The Mathematics Test is a 60-minute test with 60 multiple-choice questions. There is no penalty for incorrect answers.

You may want to time yourself as you take this practice test. It should take you about 5 minutes to complete.

HOT TIP! All questions on the ACT Mathematics Test can be answered without using a calculator, but you are allowed to use one. If you bring a calculator to the test center, make sure it is one of the types of calculators approved for the test, as many types are prohibited.

1. In a school choir, the ratio of boys to girls is 3 : 5. If there are a total of 24 singers in the choir, how many girls are in the choir?

(A) 6

(B) 9

(C) 14

(D) 15

(E) 40

2. If $12 - 3(x + 2) = x + 8$, then what is the value of x?

(A) $-\frac{5}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

(E) $\frac{5}{2}$

3. What are the values of x where $2|x + 4| < 6$?

(A) $x < -1$ and $x < -7$

(B) $x > -1$ or $x < -7$

(C) $x < -1$ or $x > -7$

(D) $x > -1$ and $x < -7$

(E) $x < -1$ and $x > -7$

4. Line ℓ passes through $(1, -3)$ and is perpendicular to $y = \frac{1}{5}x - 7$. What is the equation of line ℓ?

(A) $y = -5x + 2$

(B) $y = -5x - 2$

(C) $y = \frac{1}{5}x - \frac{14}{5}$

(D) $y = -\frac{1}{5}x - \frac{14}{5}$

(E) $y = 5x + 2$

5. Which of the following inequalities is equivalent to $-3y - 5x \le 15$?

(A) $y \ge \frac{5}{3}x + 5$

(B) $y \le -\frac{5}{3}x - 5$

(C) $y \ge -\frac{5}{3}x - 5$

(D) $y \ge \frac{5}{3}x - 5$

(E) $y \le -\frac{5}{3}x + 5$

6. In a state park, any trout caught that weighs less than 10 oz or greater than 30 oz must be returned to the water. Which of the following represents the weights of trout that may be kept?

(A) $|x - 20| \le 10$

(B) $|x - 10| \le 10$

(C) $|x - 10| \ge 20$

(D) $|x - 30| \ge 10$

(E) $|x - 20| \le 30$

CHAPTER 2

Strategies for Success

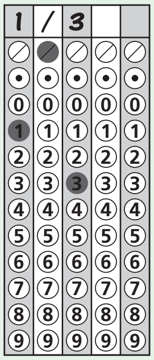
Gridded Response: Write Gridded Responses

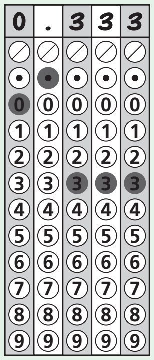
To answer a gridded-response test item, you must write your answer correctly in the top of the provided grid and fill in the bubbles accurately, or the item will be marked as incorrect. Answers may be gridded using several correct formats.

The answer to a gridded-response item is always a *whole number,* a *fraction,* or a *decimal.* Non-numerical signs and symbols, such as units of measure, the percent sign, the degree sign, the negative sign, variables, and commas, cannot be gridded.

EXAMPLE 1

Gridded Response: Solve the equation. $25 - 3(5x - 4) = 32$





$$25 - 3(5x - 4) = 32$$
$$25 - 15x + 12 = 32$$
$$-15x = -5$$
$$x = \frac{5}{15} = \frac{1}{3}$$

Grid $\frac{1}{3}$ or its rounded decimal equivalent 0.333 or .3333:

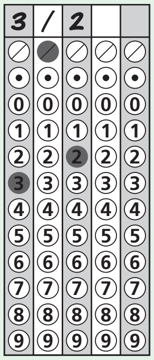
Write your answer in the boxes at the top of the grid.
Put only a digit, the fraction bar, or the decimal point in each box.

Put the first digit of your answer in the box on the left OR put the last digit of your answer in the box on the right. Do not leave a blank box in the middle of an answer.

Shade the bubble of each digit or symbol in its corresponding column.

EXAMPLE 2

Gridded Response: Find the slope of the line that passes through $(-2, -5)$ and $(8, 10)$.



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-(-5)}{8-(-2)} = \frac{15}{10} = 1\frac{1}{2}$$

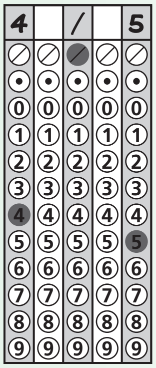
The slope of the line is $1\frac{1}{2}$, but a mixed number must be converted to either a decimal or an improper fraction before the answer can be written on the grid.

Grid the answer 1.5 or $\frac{3}{2}$ following the instructions in Example 1.

Read each statement, and then answer the questions that follow.

Sample A

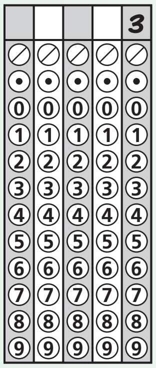
A student solved a proportion for x and got $\frac{4}{5}$ as a result. He then gridded his answer as shown.



1. Is it possible to grid fractional answers? Explain.
2. If the student solved the proportion correctly, why was the answer marked as incorrect?
3. Describe one way to correctly grid the response $\frac{4}{5}$.

Sample B

What is the x-intercept of the linear function $6x + 9y = 18$?
Wyatt found that the x-intercept point occurs at $(3, 0)$, and then he filled out the grid.

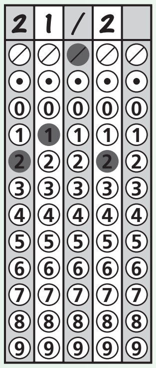


4. Will Wyatt's answer be marked as correct? Explain.
5. Anita got the same answer as Wyatt, but her answer was marked as correct. She did not place the 3 in the last column. Describe Anita's grid.

HOT TIP! When filling out a grid, be sure to completely fill in the bubbles, and be careful not to rip the paper.

Sample C

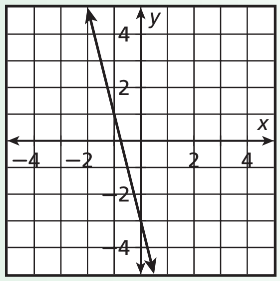
For a gridded-response test item, Jill had to determine the slope of a linear function. She correctly determined the slope to be $2\frac{1}{2}$ and then gridded her answer as shown.



6. Read the number in the answer box grid. What number is recorded in the grid?
7. Why does gridding a mixed number result in an incorrect response?
8. Write a decimal equivalent for $2\frac{1}{2}$, and then write $2\frac{1}{2}$ as an improper fraction. Explain how to correctly grid these values.

Sample D

Daniel is taking an exam where he has to determine the y-intercept of the function shown below.



9. Is the y-intercept a positive or negative value?
10. Explain why the answer to this test item cannot be recorded on an answer grid.

CHAPTER 2

STANDARDIZED TEST PREP

go.hrw.com
Test Practice Online
KEYWORD: MB7 TestPrep

CUMULATIVE ASSESSMENT, CHAPTERS 1–2

Multiple Choice

1. For which function is $g(-3) > g(5)$?

Ⓐ $g(x) = 5x - 9$
Ⓑ $g(x) = x^2 - 12$
Ⓒ $g(x) = (x + 5)^2$
Ⓓ $g(x) = (x - 9)^2$

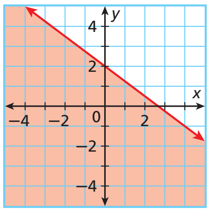
2. A television commercial claims that 4 out of every 5 dentists surveyed preferred Freshen toothpaste to the leading brand. If 120 dentists in the survey preferred Freshen, how many dentists participated in the survey?

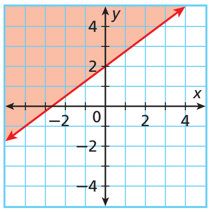
Ⓕ 30 Ⓗ 150
Ⓖ 96 Ⓙ 180

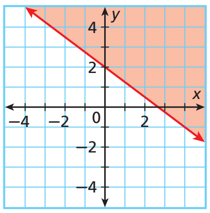
3. Which is an equation of a line with a slope of -3 that passes through $(-2, 7)$?

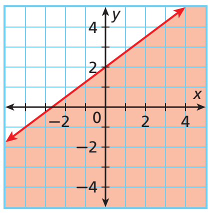
Ⓐ $y = -3x - 1$
Ⓑ $y = -3x + 1$
Ⓒ $y = -3x + 13$
Ⓓ $y = -\frac{1}{3}x + 1$

4. Which of the following shows the graph of $y + \frac{3}{4}x \geq 2$?

Ⓕ


Ⓗ


Ⓖ


Ⓙ


5. In which of the following number sets does -3 NOT belong?

Ⓐ Integers Ⓒ Real numbers
Ⓑ Rational numbers Ⓓ Whole numbers

6. What is a reasonable slope of the line of best fit of the salary data for teachers in a New York school district, as shown in the table below?

Salaries of Teachers

Years of Experience	Salary
0	\$33,407
2	\$34,273
5	\$37,882
8	\$40,185
10	\$42,977
12	\$45,864
15	\$53,811

Ⓕ 450 Ⓗ 1275
Ⓖ 750 Ⓙ 2650

7. Which shows a reflection across the x-axis and a vertical translation of 3 units down of the parent function $y = |x|$?

Ⓐ $y = -|x - 3|$ Ⓒ $y = -|x| - 3$
Ⓑ $y = |x| - 3$ Ⓓ $y = |x - 3|$

8. Simplify the expression $4\sqrt{50} + 3\sqrt{72}$.

Ⓕ $4\sqrt{7}$ Ⓗ $12\sqrt{5}$
Ⓖ $7\sqrt{112}$ Ⓙ $38\sqrt{2}$

9. Find the slope of the line $-3y = 6x + 12$.

Ⓐ -4 Ⓒ $-\frac{1}{2}$
Ⓑ -2 Ⓓ $-\frac{1}{4}$

HOT TIP! When a word problem contains information about dimensions used to solve a problem, you might find it useful to draw a diagram. The diagram should be clearly labeled and sketched close to scale.

10. A lamppost casts a shadow that is 24 feet long. Tad, who is 6 feet tall, is standing directly next to the lamppost. His shadow is 15 feet long. About how tall is the lamppost?

(F) 10 feet

(G) 15 feet

(H) 33 feet

(J) 60 feet

11. What is the effect on the graph of $y = 2x + 2$ when it is changed to $y = 2x - 2$?

(A) The slope of the line becomes steeper.

(B) The line slants down and right instead of up and right.

(C) The y-intercept is translated 4 units down.

(D) The line is reflected across the y-axis.

12. The cost of renting a moving van is \$39.95 plus \$0.40 per mile. Which equation best represents the relationship between cost c and the number of miles driven m?

(F) $c = 39.95 + 0.40$

(G) $c = 39.95m + 0.40$

(H) $c = 39.95 + 0.40m$

(J) $c = 39.95m + 0.40m$

Gridded Response

13. The baseball statistic "total bases" is calculated by adding the number of singles, twice the number of doubles, three times the number of triples, and four times the number of home runs. In 2001, a player collected 411 total bases, including 49 singles, 32 doubles and 2 triples. How many home runs did the player hit that year?

14. The function $g(x)$ is the reflection across the y-axis of $f(x) = -\frac{2}{3}x - 5$. What is the slope, to the nearest hundredth, of $g(x)$?

15. Evaluate $h^2 - hk + 2k^3 - 2$ for $h = 4$ and $k = -1$.

16. Write the product of $(1.2 \times 10^{-8})(6.8 \times 10^{10})$ in standard form.

17. What is the y-intercept of $3x + 4y = 24$?

Short Response

18. Consider the inequality $|5x + 6| \geq 11$.

a. Solve the inequality.

b. Graph your solution on a number line.

19. The city would like to construct a community amphitheater in the park. The stage of the amphitheater should be 25 feet across and 12 feet deep. The production group that uses the facility has anticipated that at least 5 feet of space should be a sufficient amount for each row. The area allotted for placement of the amphitheater's stage and seating is 2000 ft^2.

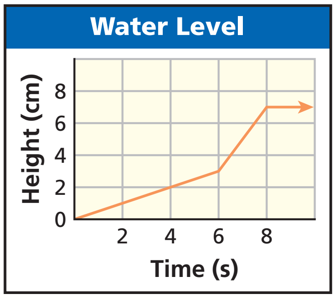
a. Write an equation that can be used to determine the maximum number of rows that can be constructed.

b. Determine the maximum number of rows that can be constructed.

c. Suppose the city would like to construct a fence at least 200 feet away from the stage and all of the seats. Find the perimeter of fencing needed.

Extended Response

20. A container is filled with water at a constant rate. The water level over time is shown in the graph.



a. What does the flat portion of the graph represent?

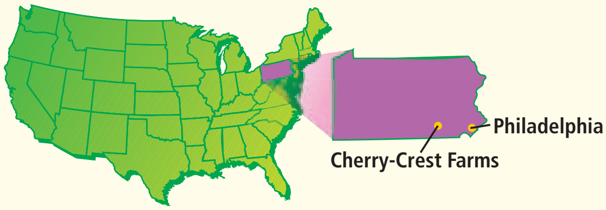
b. Sketch a possible shape for the container.

c. Suppose the container is filled twice as fast. Sketch a graph to represent the situation, and identify the transformation of the original graph that it represents.

d. Suppose the container initially contains 2 cm of water. Would the new graph be a vertical translation of the original graph? Justify your answer.

Problem Solving on Location

PENNSYLVANIA



The Philly Cheese Steak Sandwich

Philadelphia's best-known sandwich was created in 1930 when a hot dog vendor tossed some steak and onions onto the grill and then served them on a hot dog bun. Cheese was soon added to the recipe, and the Philly cheese steak sandwich has been a local specialty ever since.

Choose one or more strategies to solve each problem.

1. At Geno's Steaks, the busiest shift of the week is on Saturday from 11:00 A.M. to 7:00 P.M. During that time Geno's makes an average of 1.5 cheese steak sandwiches a minute. Use the recipe below. How many pounds of steak are needed to get through this shift?

2. At Pat's King of Steaks, a plain steak sandwich costs $5.75 and a cheese steak sandwich costs $6.00. A tour group bought 32 sandwiches for a total of $189.00. How many of each type did they buy?

3. In 1930, the first steak sandwich sold for 2 cents. In 2004, a cheese steak sandwich cost $6. Assuming that cost is a linear function of time, predict the cost of a cheese steak sandwich in 2011.

4. You can expect to find a line at many cheese steak stands, but service is quick. Once an order is placed, the sandwich is made and served in 1 minute and 15 seconds. Suppose it takes 18 seconds for each person in line to place an order. What is the maximum number of people who can be in line ahead of you if you want to have your sandwich in less than 10 minutes from the time you get in line?

Philly Cheese Steak Recipe

5 oz steak
2 1/2 oz. American cheese
Fried onions
9 1/2 in. roll

Thinly slice steak and fry on grill. Just before it's done, cover with cheese and cook until melted. Serve on roll, topped with onions.

The Amazing Maize Maze

Cherry-Crest Farm, located in the heart of Pennsylvania Dutch Country, welcomes visitors by telling them to get lost—in an enormous cornfield maze! The design of the maze changes from year to year, but it always includes bridges and tunnels. There are also clues to discover along the way.

Choose one or more strategies to solve each problem.

1. Corn is usually planted at 30,000 plants per acre. The Amazing Maize Maze measures 360 feet by 660 feet. Assuming that 60% of that area is covered with corn plants, about how many plants form the maze? $\left(\textit{Hint: } 1 \text{ acre} = 43{,}560 \text{ ft}^2\right)$

2. Admission to Cherry-Crest Farm is \$11 for adults and \$9 for children. One group of visitors paid \$213 for admission, and there were more adults than children in the group. How many of each were in the group?

For 3, use the table.

3. Getting through the maze depends on two things: the speed at which you walk and your luck in choosing the right path. The table shows the average walking speeds of eight visitors and the time it took them to exit the maze. Predict the time it would take you to exit the maze if you walked at an average speed of 3.5 mi/h.

Average Walking Speed (mi/h)	2.5	3.0	4.0	3.1	2.8	3.9	4.0	2.7
Time to Exit Maze (min)	72	61	45	58	66	50	51	69

4. For most visitors, the time in minutes that it takes to exit the maze satisfies the inequality $|t - 60| \leq 15$.

   However, people who have already been through the maze usually improve their time by about 7 minutes. What are the minimum and maximum times it takes to exit the maze for repeat visitors?