CHAPTER 7

Exponential and Logarithmic Functions

7A Exponential Functions and Logarithms

7-1 Exponential Functions, Growth, and Decay

Lab Explore Inverses of Functions

7-2 Inverses of Relations and Functions

7-3 Logarithmic Functions

7-4 Properties of Logarithms

CONCEPT CONNECTION

7B Applying Exponential and Logarithmic Functions

7-5 Exponential and Logarithmic Equations and Inequalities

Lab Prove Laws of Logarithms

7-6 The Natural Base, *e*

7-7 Transforming Exponential and Logarithmic Functions

7-8 Curve Fitting with Exponential and Logarithmic Models

CONCEPT CONNECTION

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KEYWORD: MB7 ChProj

Exponential and logarithmic functions are used to express the magnitudes of earthquakes that occur on the San Andreas Fault.

Carrizo Plain National Monument
near Bakersfield, CA

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. exponent
2. function
3. relation
4. variable

A. a symbol used to represent one or more numbers
B. the set of counting numbers and their opposites
C. a relation with at most one y-value for each x-value
D. the number of times the base of a power is used as a factor
E. a set of ordered pairs

Properties of Exponents

Simplify each expression.

5. $x^2(x^3)(x)$ **6.** $3y^{-1}(5x^2y^2)$ **7.** $\dfrac{a^8}{a^2}$ **8.** $y^{15} \div y^{10}$

9. $\dfrac{x^2y^5}{xy^6}$ **10.** $\left(\dfrac{x}{3}\right)^{-3}$ **11.** $(3x)^2(4x^3)$ **12.** $\dfrac{a^{-2}b^3}{a^4b^{-1}}$

Simple Interest

Use the simple interest formula, $I = Prt$, where I is the interest, P is the initial amount (the principal), and r is the interest rate for Problems 13–15.

13. Find the simple interest on an investment of \$3000 at 3% for 2 years.

14. A savings account of \$2000 earned \$90 simple interest in 3 years. Find the interest rate.

15. Jeri got a loan at 6% simple interest for 3 years. She paid back a total of \$5310. How much was the loan?

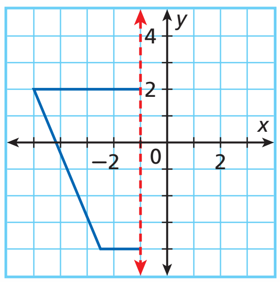
Solve for a Variable

Solve each equation for x.

16. $3x - y = 4$ **17.** $y = -7x + 3$ **18.** $\dfrac{x}{2} = 3y - 4$ **19.** $y = \dfrac{3}{4}x - \dfrac{1}{2}$

Symmetry

20. Copy the graph, and use the line of symmetry to complete the figure.



Scientific Notation

Write in scientific notation.

21. 7,000,000,000 **22.** 0.0000000093 **23.** 16.75

Write in standard notation.

24. 9.4×10^{-6} **25.** 4.7×10^5 **26.** 7.8×10^4

CHAPTER 7

Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
11.0 Students prove simple laws of logarithms. (Lessons 7-3, 7-4; Lab 7-6)	**prove** to use logical reasoning to show that a statement is true **law** a property	You demonstrate that laws of logarithms are true.
11.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. (Lessons 7-3, 7-4, 7-5, 7-6) (Connecting)	**inverse** something that is opposite **relationship** a connection or association	You solve problems by working back and forth between exponents and logarithms.
11.2 Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step. (Lesson 7-4; Lab 7-6)	**validity** truth **according to** depending on	You prove laws of logarithms and determine whether given proofs are correct or incorrect.
12.0 Students know the laws of fractional exponents, **understand exponential functions, and use these functions in problems involving exponential growth and decay.** (Lessons 7-1, 7-6, 7-7, 7-8)	**involving** using or containing **growth** an increase **decay** a decrease	You apply exponents to functional situations to solve real-world problems.
13.0 Students use the definition of logarithms to translate between logarithms in any base. (Lessons 7-4, 7-6)	**translate** to change from one form to another	You translate between logarithms with different bases.
14.0 Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values. (Lessons 7-3, 7-4, 7-5) (Connecting)	**simplify** (simplification) to make things easier **numeric** involving numbers **identify** to locate or to be able to name	You simplify expressions that include logarithms. You also find the value of logarithms. ***Example:*** $\log_3 81 = 4$

Standards 15.0 and 24.0 are also covered in this chapter. To see these standards unpacked, go to Chapter 8, p. 566, and Chapter 9, p. 652.

Reading and Writing Math

CHAPTER 7

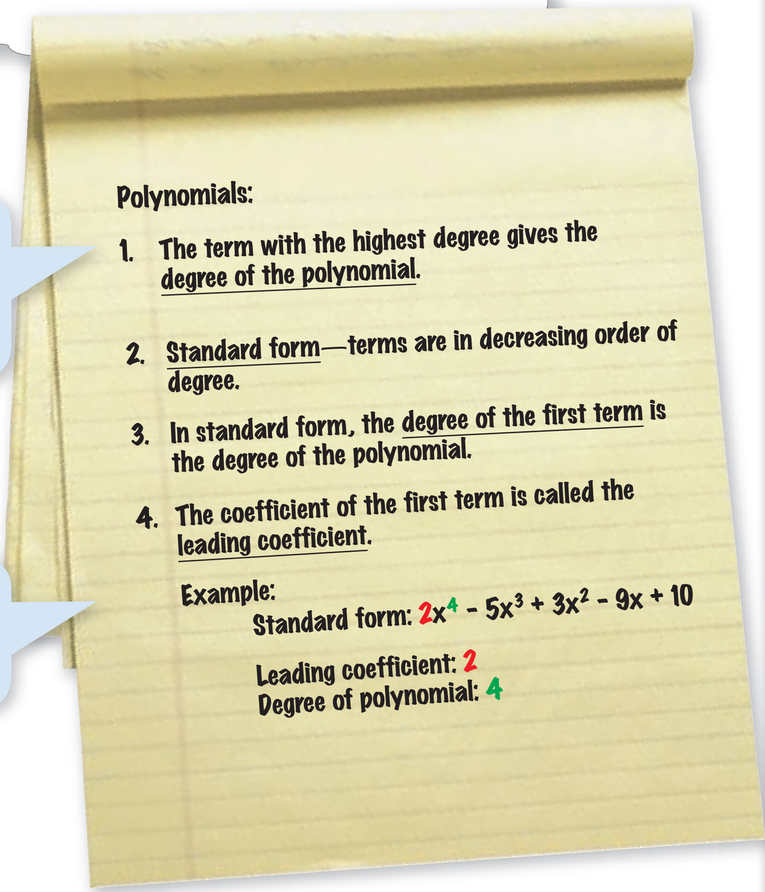
Writing Strategy: Use Your Own Words

When studying a difficult mathematical concept, rewrite the concept using your own words so that you can better comprehend the material. You may also find it helpful to provide your own example.

The **degree of a polynomial** is given by the term with the greatest degree. A polynomial with one variable is in standard form when its terms are written in descending order by degree. So, in standard form, the degree of the first term indicates the degree of the polynomial, and the **leading coefficient** is the coefficient of the first term.

REWRITE the above paragraph with short phrases and sentences to clarify important concepts about polynomials.

INCLUDE an example to connect the words and the mathematics.



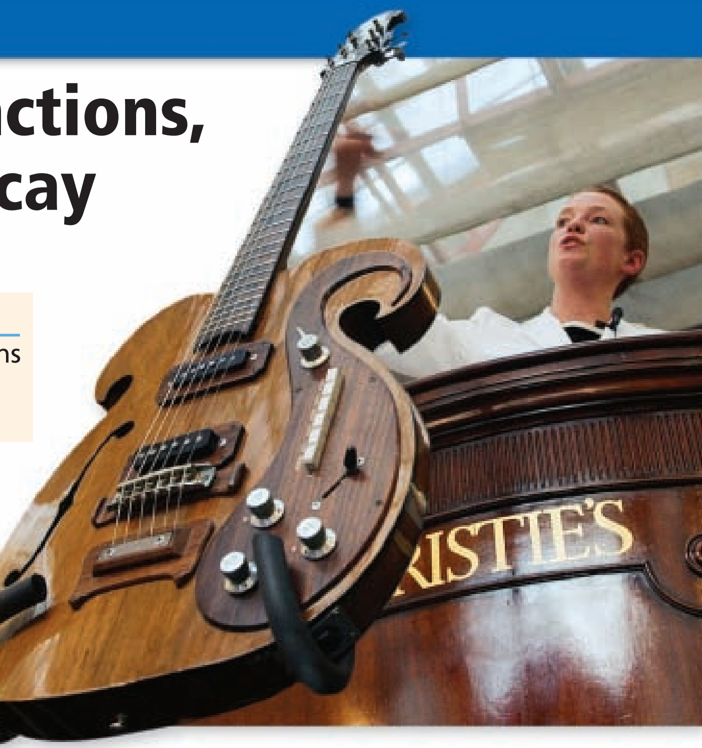
Try This

Read the following paragraph from Lesson 6-5, and rewrite it using your own words.

The Irrational Root Theorem states that irrational roots come in conjugate pairs. For example, if you know that $1 + \sqrt{2}$ is a root of $x^3 - x^2 - 3x - 1 = 0$, then you know that $1 - \sqrt{2}$ is also a root.

Recall that the real numbers are made up of the rational and the irrational numbers. You can use the Rational Root Theorem and the Irrational Root Theorem together to find *all* of the real roots of $P(x) = 0$.

7-1 Exponential Functions, Growth, and Decay



Objective
Write and evaluate exponential expressions to model growth and decay situations.

Vocabulary
exponential function
base
asymptote
exponential growth
exponential decay

California Standards
12.0 Students know the laws of fractional exponents, **understand exponential functions, and use these functions in problems involving exponential growth and decay.**

Who uses this?
Collectors can use exponential functions to model the value of rare musical instruments. (See Example 2.)

Moore's law, a rule used in the computer industry, states that the number of transistors per integrated circuit (the processing power) doubles every year. Beginning in the early days of integrated circuits, the growth in capacity may be approximated by this table.

Transistors per Integrated Chip

Year	1965	1966	1967	1968	1969	1970	1971
Transistors	60	120	240	480	960	1920	3840

×2 ×2 ×2 ×2 ×2 ×2

Growth that doubles every year can be modeled by using a function with a variable as an exponent. This function is known as an *exponential function*. The parent **exponential function** is $f(x) = b^x$, where the **base** b is a constant and the exponent x is the independent variable.

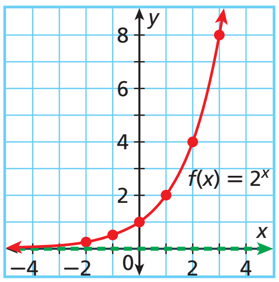
Remember!
In the function $y = b^x$, y is a function of x because the value of y *depends* on the value of x.

Base Exponent

$$f(x) = b^x, \text{ where } b > 0, b \neq 1$$

The graph of the parent function $f(x) = 2^x$ is shown. The domain is all real numbers and the range is $\{y \mid y > 0\}$.

x	−2	−1	0	1	2	3
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



Notice that as the x-values decrease, the graph of the function gets closer and closer to the x-axis. The function never reaches the x-axis because the value of 2^x cannot be zero. In this case, the x-axis is an *asymptote*. An **asymptote** is a line that a graphed function approaches as the value of x gets very large or very small.

A function of the form $f(x) = ab^x$, with $a > 0$ and $b > 1$, is an **exponential growth** function, which increases as x increases. When $0 < b < 1$, the function is called an **exponential decay** function, which decreases as x increases.

EXAMPLE 1 Graphing Exponential Functions

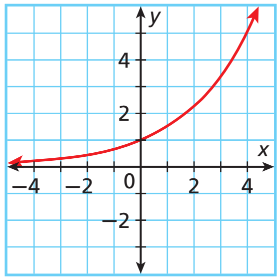
Tell whether the function shows growth or decay. Then graph.

A $f(x) = 1.5^x$

Step 1 Find the value of the base.

$f(x) = 1.5^x$ *The base, 1.5, is greater than 1. This is an exponential growth function.*

Step 2 Graph the function by using a table of values.



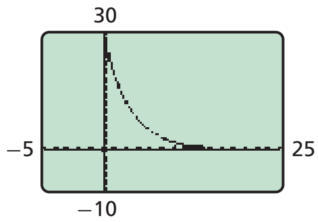
x	−2	−1	0	1	2	3	4
$f(x)$	0.4	0.7	1	1.5	2.3	3.4	5.1

B $g(x) = 30(0.8^x)$

Step 1 Find the value of the base.

$g(x) = 30(0.8^x)$ *The base, 0.8, is less than 1. This is an exponential decay function.*

Step 2 Graph the function by using a graphing calculator.



Remember!

Negative exponents indicate a reciprocal. For example:

$$x^{-2} = \frac{1}{x^2}$$

CHECK IT OUT! **1.** Tell whether the function $p(x) = 5(1.2^x)$ shows growth or decay. Then graph.

You can model growth or decay by a constant percent increase or decrease with the following formula:

$$A(t) = a(1 \pm r)^t$$

Initial amount: a; Number of time periods: t; Final amount: $A(t)$; Rate of increase: r

In the formula, the base of the exponential expression, $1 + r$, is called the *growth factor*. Similarly, $1 - r$ is the *decay factor*.

Student to Student: *Growth and Decay*

Angela Jones,
Independence
High School

*When a function **increases** by a constant rate, such as 7%, this is the same as multiplying by 100% + 7%, or 107% .*

In decimal form, I would multiply by 1 + 0.07, or 1.07.

*When a function **decreases** by a constant rate, such as 12%, this is the same as multiplying by 100% − 12%, or 88% .*

In decimal form, it's (1 − 0.12), or 0.88.

EXAMPLE 2 Economics Application

Tony purchased a rare 1959 Gibson Les Paul guitar in 2000 for $12,000. Experts estimate that its value will increase by 14% per year. Use a graph to find when the value of the guitar will be $60,000.

Step 1 Write a function to model the growth in value for this guitar.

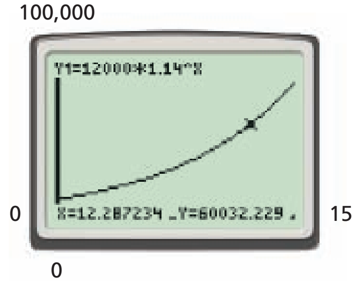
$f(t) = a(1 + r)^t$	*Exponential growth function*
$= 12{,}000(1 + 0.14)^t$	*Substitute 12,000 for a and 0.14 for r.*
$= 12{,}000(1.14)^t$	

Step 2 Graph the function.

When graphing exponential functions in an appropriate domain, you may need to adjust the range a few times to show the key points.

Step 3 Use the graph to predict when the value of the guitar will reach $60,000.

Use the TRACE feature to find the t-value where $f(t) \approx 60{,}000$.



The function value is approximately 60,000 when $t \approx 12.29$. The guitar will be worth $60,000 about 12.29 years after it is purchased, or sometime in 2012.

Helpful Hint

X is used on the graphing calculator for the variable t:
Y1=12000*1.14^X

CHECK IT OUT!

2. In 1981, the Australian humpback whale population was 350 and has increased at a rate of about 14% each year since then. Write a function to model population growth. Use a graph to predict when the population will reach 20,000.

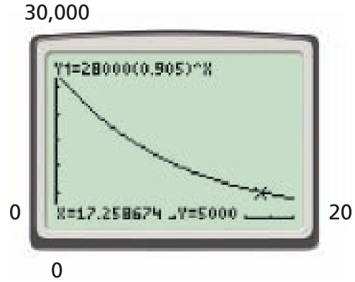
EXAMPLE 3 Depreciation Application

The value of a truck bought new for $28,000 decreases 9.5% each year. Write an exponential function, and graph the function. Use the graph to predict when the value will fall to $5000.

Write a function to model the growth in value for this truck.

$f(x) = a(1 - r)^t$	*Exponential decay function*
$= 28{,}000(1 - 0.095)^t$	*Substitute 28,000 for a and 0.095 for r.*
$= 28{,}000(0.905)^t$	*Simplify.*

Graph the function. Use TRACE to find when the value of the truck will fall below $5000.



It will take about 17.3 years for the value to drop to $5000.

CHECK IT OUT!

3. A motor scooter purchased for $1000 depreciates at an annual rate of 15%. Write an exponential function, and graph the function. Use the graph to predict when the value will fall below $100.

THINK AND DISCUSS

1. Use a calculator to compare the values of 1.01^{500} and 0.99^{500}. Explain the results.
2. Discuss the differences between the graph of $f(x) = 1.1^x$ and the graph of $g(x) = 0.9^x$. What happens in each when $x = 0$?
3. Describe the function $f(t) = a(1 - r)^t$ when $0 < r < 1$ and $t > 0$. Describe the function when $-1 < r < 0$ and $t > 0$.

Know it! Note

4. **GET ORGANIZED** Copy and complete the graphic organizer. Compare exponential growth and decay.

$f(x) = ab^x$, where $a > 0$	Growth	Decay
Value of b		
General shape of the graph		
What happens to $f(x)$ as x increases?		
What happens to $f(x)$ as x decreases?		

7-1 Exercises

California Standards 2.0, 12.0

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KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** When the base in an exponential function is between 0 and 1, the function shows __?__ . (*exponential growth* or *exponential decay*)

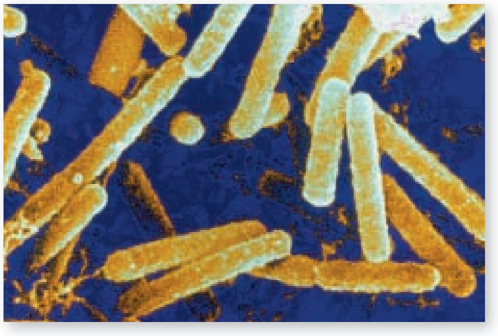
SEE EXAMPLE 1 p. 491

Tell whether the function shows growth or decay. Then graph.

2. $f(x) = 32(0.5^x)$
3. $f(x) = 0.5(1.2^x)$
4. $f(x) = 0.4\left(\frac{3}{4}\right)^x$

SEE EXAMPLE 2 p. 492

5. **Biology** An acidophilus culture containing 150 bacteria doubles in population every hour. Predict the number of bacteria after 12 hours.
   a. Write a function representing the bacteria population for every hour that passes.
   b. Graph the function.
   c. Use the graph to predict the number of bacteria after 12 hours.



SEE EXAMPLE 3 p. 492

6. **Physics** A new softball dropped onto a hard surface from a height of 25 inches rebounds to about $\frac{2}{5}$ the height on each successive bounce.
   a. Write a function representing the rebound height for each bounce.
   b. Graph the function.
   c. After how many bounces would a new softball rebound less than 1 inch?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
7–9	1
10	2
11	3

Extra Practice

Skills Practice p. S16
Application Practice p. S38

Tell whether the function shows growth or decay. Then graph.

7. $f(x) = \left(\frac{1}{3}\right)^x$ **8.** $f(x) = \left(\frac{1}{3}\right)(1.3)^x$ **9.** $f(x) = 10(2.7)^x$

10. Railroads The amount of freight transported by rail in the United States was about 580 billion *ton-miles* in 1960 and has been increasing at a rate of 2.32% per year since then.

a. Write a function representing the amount of freight, in billions of ton-miles, transported annually (1960 = year 0).

b. Graph the function.

c. In what year would you predict that the number of ton-miles would have exceeded or would exceed 1 trillion (1000 billion)?

11. Medicine A quantity of insulin used to regulate sugar in the bloodstream breaks down by about 5% each minute. A body-weight adjusted dose is generally 10 units.

a. Write a function representing the amount of the dose that remains.

b. Use a calculator to graph the function.

c. About how much insulin remains after 10 minutes?

d. About how long does it take for half of the dose to remain?

LINK

History

The name *Manhattan* is probably a combination of two Native American words, the Delaware word *mannah,* "island," and the Algonquian word *hatin*, "hills." So the name *Manhattan* means "hilly island."

Explain whether each function is exponential.

12. $f(x) = 2x^{10}$ **13.** $f(x) = 0^x$ **14.** $f(x) = 1 \cdot 0.5^x$

15. History In 1626, the Dutch bought Manhattan Island, now part of New York City, for $24 worth of merchandise. Suppose that, instead, $24 had been invested in an account that paid 3.5% interest each year. Find the balance in 2008.

16. Technology The quantity of new information stored electronically in 2002 was about 5 *exabytes,* or 5×10^{18} bytes. Researchers estimate that this is double what was stored in 1999. Suppose this trend continues. Write and graph a function to predict the pattern of growth beginning in 1999.

17. Business On federal income tax returns, self-employed people can depreciate the value of business equipment. Suppose a computer valued at $2765 depreciates at a rate of 30% per year. Estimate the number of years it will take for the computer's value to be less than $350.

Complete the table for each function. Round each value to the nearest hundredth.

	x	–3	–2	–1	0	1	2	3	4	5
18.	$f(x) = 2.2^x$	■	■	■	■	■	■	■	■	■
19.	$g(x) = 0.4^x$	■	■	■	■	■	■	■	■	■

CONCEPT CONNECTION

20. This problem will prepare you for the Concept Connection on page 520.

For a certain credit card, the total amount A that you owe after n months is given by $A = P(1.015)^n$, where P is the starting balance.

a. Suppose that you begin with a debt of $1000. Graph the function for the amount that you owe.

b. How much will you owe after one year?

c. How long will it take for the total amount that you owe to reach $1300?

21. **Collectibles** At the peak of a beanbag animal fad, one sales representative sold 12,000 of the animals in one month. Each month after that, the rep sold about 20% fewer animals.

  a. About how many beanbag animals did the rep sell in the 6th month after the peak?

  b. In which month did the rep first sell fewer than 1000 animals?

22. **Banking** The compound interest formula is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the amount earned, P is the principal, r is the annual interest rate, t is the time in years, and n is the number of compounding periods per year. Harry invested \$5000 at 5% interest compounded quarterly (4 times per year).

  a. How much will the investment be worth after 5 years?

  b. When will the investment be worth more than \$10,000?

  c. **What if...?** Harry could have invested the same amount in an account that paid 5% interest compounded monthly (12 times per year). How much more would his investment have been worth after 5 years?

23. **Critical Thinking** What are the coordinates of the point that is common to the graph of $f(x) = \left(\frac{2}{3}\right)^x$ and the graph of $f(x) = \left(\frac{3}{2}\right)^x$?

Find the range of each function for the domain $(0, 10]$.

24. $f(x) = 3^x - 2^x$

25. $f(x) = 100(0.9)^x$

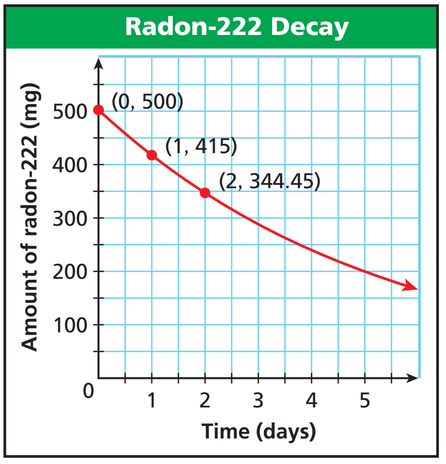
26. $f(x) = \frac{3}{4}(2)^x$

27. **Geology** Radon-222 is a gas that escapes from rocks and soil. It can accumulate in buildings and can be dangerous for people who breathe it. Radon-222 decays to polonium and eventually to lead.

  a. Find the percent decrease in the amount of radon-222 each day.

  b. Write an exponential decay function for the amount of a 500 mg sample of radon-222 remaining after t days.

  c. How much of the radon-222 sample would remain after 14 days?



28. **Estimation** According to the Population Reference Bureau, the world population in 2000 was 6.1 billion and increasing at an annual rate of 1.4%. Estimate the world population in 2020. Then write and evaluate an exponential function to predict the actual population, and compare it to your estimate.

29. **Critical Thinking** Which grows faster as x increases, x^3 or 3^x? Explain.

30. **Write About It** Describe a situation that could be modeled by an exponential function. Give the function and describe the meanings of several function values.

Standardized Test Prep

31. Which function represents exponential decay?

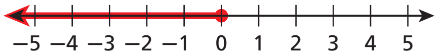
  Ⓐ $f(x) = 0.9(1.001^x)$

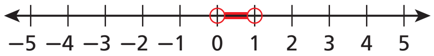
  Ⓑ $f(x) = 1.5\left(\frac{10}{11}\right)^x$

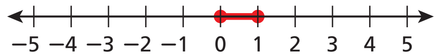
  Ⓒ $f(x) = 0.5(2^x)$

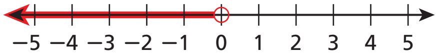
  Ⓓ $f(x) = \left(\frac{1}{0.5}\right)^x$

32. Which number line represents the values of b in $y = ab^x$ for an exponential decay function?

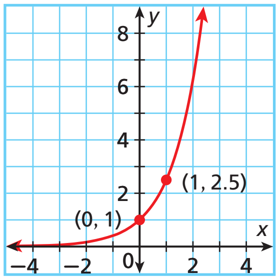
Ⓕ 

Ⓗ 

Ⓖ 

Ⓙ 

33. **Short Response** What are the values of a and b in $f(x) = ab^x$ for the graph shown?



34. The population of a town was 89,443 in 1990 and has increased at a rate of 0.6% per year since then. Which function represents the town's population t years after 1990?

Ⓐ $89{,}443(1.6)^t$ Ⓒ $89{,}443(1.06)^t$

Ⓑ $89{,}443(1.006)^t$ Ⓓ $89{,}443(1.0006)^t$

CHALLENGE AND EXTEND

35. **Critical Thinking** Recall that polynomials are classified by degree. Why doesn't an exponential function have a degree?

Solve by graphing. Write the answer to the nearest hundredth.

36. $1.15^x \geq 3$ 37. $0.97^x < 0.5$ 38. $5 < 1.5^x < 6$

39. Compare the graphs of $y = 2^x$ and $y = x^2$, where $-10 < x < 10$. How many points of intersection are there? Give the coordinates of these points.

40. **Biology** Researchers found that the number of mosquitoes per acre of wetland after a frost is about 10 to the power $\frac{1}{2}d + 2$, where d is the number of days since the frost. How many mosquitoes per acre are there at the time of the frost? How long after the frost does it take for the population to quadruple?

41. In $f(x) = b^x$, why is the domain of b restricted $(b > 0,\ b \neq 1)$ for exponential functions?

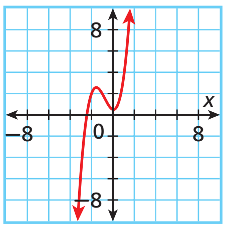
SPIRAL REVIEW

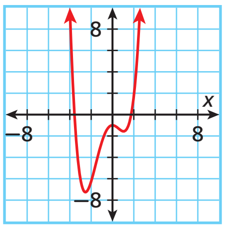
Graph each function with a graphing calculator. Identify the domain and range of the function, and describe the transformation from its parent function. *(Lesson 1-9)*

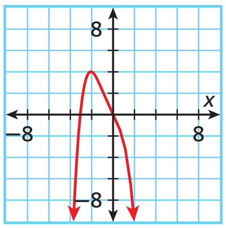
42. $f(x) = \sqrt{x - 3}$ 43. $f(x) = -x^2 + 1$ 44. $f(x) = 2x^3$ 45. $f(x) = x - 4$

46. **Entertainment** Fred and Katrina are buying video games. Fred bought 3 new video games and 2 old video games for \$235. Katrina bought 1 new video game and 4 old video games for \$195. Find the cost of each type of video game. *(Lesson 3-2)*

Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient. *(Lesson 6-7)*

47. 

48. 

49. 

7-2 Technology LAB

Use with Lesson 7-2

Explore Inverses of Functions

You can use a graphing calculator to explore inverse functions and their relationship to the linear parent function $f(x) = x$.

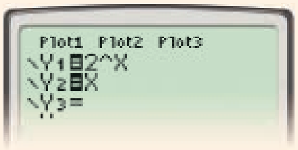
California Standards

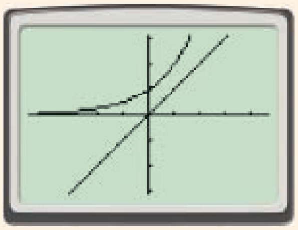
24.0 Students solve problems involving functional concepts, such as composition, **defining the inverse function** and performing arithmetic operations on functions.

Activity

Graph the function $f(x) = 2^x$ and its inverse.

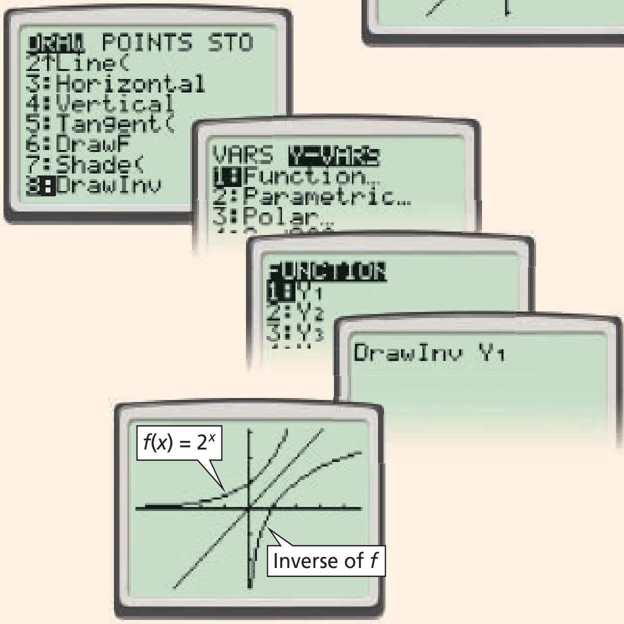
1 Graph the function $f(x) = 2^x$ and the linear parent $f(x) = x$ in the decimal window. Enter the functions, and then press ZOOM and select **4:ZDecimal**.





2 Use the **DrawInv** feature to graph the inverse of **Y1**. Enter the DRAW menu by pressing 2nd DRAW PRGM. Then select **8:DrawInv**.

To select **Y1**, press VARS. Use the arrow keys to move to the **Y-VARS** submenu. Select **1:Function**, and then select **1:Y1** and press ENTER.



The graph shows the original function $f(x) = 2^x$, its inverse, and the linear parent $f(x) = x$. Notice that the inverse appears to be a function. Its domain is $\{x | x > 0\}$, and its range is $\mathbb{R}$.

Try This

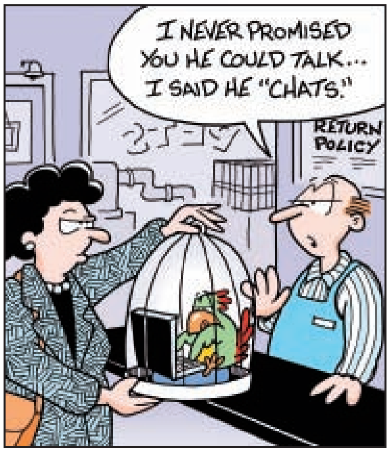
Graph $f(x) = x^2$, its inverse, and $f(x) = x$.

1. Compare the domain and range of $f(x) = x^2$ with the domain and range of its inverse. Is the inverse of $f(x) = x^2$ a function? Explain why or why not.

Graph $f(x) = x^3$, its inverse, and $f(x) = x$.

2. Compare the domain and range of $f(x) = x^3$ with the domain and range of its inverse. Is the inverse of $f(x) = x^3$ a function? Explain why or why not.
3. **Make a Conjecture** Make a conjecture about the relationship between the domain and range of a function and its inverse.
4. **Make a Conjecture** Make a conjecture about the relationship of a function and its inverse to the line $f(x) = x$.

7-2 Inverses of Relations and Functions



Objectives
Graph and recognize inverses of relations and functions.

Find inverses of functions.

Vocabulary
inverse relation
inverse function

California Standards

24.0 Students solve problems involving functional concepts, such as composition, **defining the inverse function** and performing arithmetic operations on functions.

Why learn this?
Inverse functions can be used to find prices before taxes, discounts, and extra charges. (See Example 5.)

You have seen the word *inverse* used in various ways.

The additive inverse of 3 is -3.

The multiplicative inverse of 5 is $\frac{1}{5}$.

The multiplicative inverse matrix of $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} 1 & -0.5 \\ -2 & 1.5 \end{bmatrix}$.

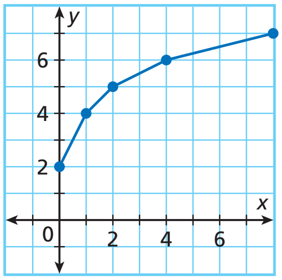
You can also find and apply inverses to relations and functions. To graph the **inverse relation**, you can reflect each point across the line $y = x$. This is equivalent to switching the x- and y-values in each ordered pair of the relation.

EXAMPLE 1 **Graphing Inverse Relations**

Graph the relation and connect the points. Then graph the inverse. Identify the domain and range of each relation.

x	0	1	2	4	8
y	2	4	5	6	7

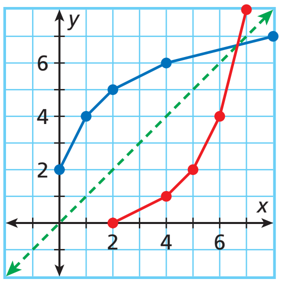
Graph each ordered pair and connect them.



Switch the x- and y-values in each ordered pair.

x	2	4	5	6	7
y	0	1	2	4	8

Reflect each point across $y = x$, and connect them. Make sure the points match those in the table.



Domain: $\{x \mid 0 \le x \le 8\}$ Range: $\{y \mid 2 \le y \le 7\}$

Domain: $\{x \mid 2 \le x \le 7\}$ Range: $\{y \mid 0 \le y \le 8\}$

Remember!
A *relation* is a set of ordered pairs. A *function* is a relation in which each x-value has, at most, one y-value paired with it.

CHECK IT OUT!

1. Graph the relation and connect the points. Then graph the inverse. Identify the domain and range of each relation.

x	1	3	4	5	6
y	0	1	2	3	5

When the relation is also a function, you can write the inverse of the function $f(x)$ as $f^{-1}(x)$. This notation does *not* indicate a reciprocal.

Functions that undo each other are **inverse functions**.

Input 3	→	Function $f(x) = x + 6$	→	Output 9
Input 9	→	Inverse function $f^{-1}(x) = x - 6$	→	Output 3

To find the inverse function, use the inverse operation. In the example above, 6 is added to x in $f(x)$, so 6 is subtracted to find $f^{-1}(x)$.

EXAMPLE 2 Writing Inverse Functions by Using Inverse Operations

Use inverse operations to write the inverse of $f(x) = 2x$.

$f(x) = 2x$ *The variable, x, is multiplied by 2.*

$f^{-1}(x) = \frac{x}{2}$ *Divide x by 2 to write the inverse.*

Check Use the input $x = 7$ in $f(x)$.

$$f(x) = 2x$$

$$f(7) = 2(7)$$ *Substitute 7 for x.*

$$= 14$$

Substitute the result into $f^{-1}(x)$.

$$f^{-1}(x) = \frac{x}{2}$$

$$f^{-1}(14) = \frac{14}{2}$$ *Substitute 14 for x.*

$$= 7$$

The inverse function *does* undo the original function. ✔

CHECK IT OUT! **Use inverse operations to write the inverse of each function.**

2a. $f(x) = \frac{x}{3}$ **2b.** $f(x) = x + \frac{2}{3}$

Undo operations in the opposite order of the order of operations.

EXAMPLE 3 Writing Inverses of Multi-Step Functions

Use inverse operations to write the inverse of $f(x) = \frac{x}{4} - 5$.

$f(x) = \frac{x}{4} - 5$ *The variable x is divided by 4, then 5 is subtracted.*

$f^{-1}(x) = 4(x + 5)$ *First, undo the subtraction by adding 5 to x. Then, undo the division by multiplying by 4.*

Check Use a sample input.

$f(40) = \frac{40}{4} - 5 = 10 - 5 = 5$ $f^{-1}(5) = 4(5 + 5) = 4(10) = 40$ ✔

Helpful Hint

The *reverse* order of operations:
Addition or Subtraction
Multiplication or Division
Exponents
Parentheses

CHECK IT OUT! **3.** Use inverse operations to write the inverse of $f(x) = 5x - 7$.

You can also find the inverse function by writing the original function with x and y switched and then solving for y.

EXAMPLE 4 Writing and Graphing Inverse Functions

Graph $f(x) = 3x + 6$. Then write and graph the inverse.

$y = 3x + 6$ *Set $y = f(x)$ and graph f.*

$x = 3y + 6$ *Switch x and y.*

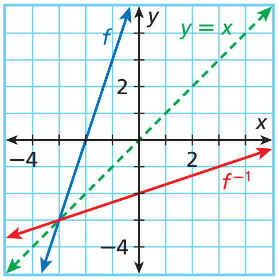
$x - 6 = 3y$ *Solve for y.*

$\frac{x - 6}{3} = y$

$y = \frac{x - 6}{3}$ *Write in $y =$ format.*

$f^{-1}(x) = \frac{x - 6}{3}$ *Set $y = f(x)$.*

$= \frac{1}{3}x - 2$ *Simplify. Then graph f^{-1}.*



CHECK IT OUT! **4.** Graph $f(x) = \frac{2}{3}x + 2$. Then write the inverse and graph.

Any time you need to undo an operation or work backward from a result to the original input, you can apply inverse functions.

EXAMPLE 5 *Retailing Application*

A clerk needs to price a digital camera returned by a customer. The customer paid a total of \$103.14, which included a gift-wrapping charge of \$3 and 8% sales tax. What price should the clerk mark on the tag?

Step 1 Write an equation for the total cost as a function of price.

$c = 1.08(p + 3)$ *Cost c is a function of price p.*

Step 2 Find the inverse function that models price as a function of cost.

$c = 1.08(p + 3)$

$c = 1.08p + 3.24$ *Distribute.*

$c - 3.24 = 1.08p$ *Subtract 3.24 from both sides.*

$\frac{c - 3.24}{1.08} = p$ *Divide to isolate p.*

Step 3 Evaluate the inverse function for $c = \$103.14$.

$p = \frac{103.14 - 3.24}{1.08} = 92.50$

The clerk should mark the tag as \$92.50.

Check $c = 1.08(92.50 + 3)$ *Substitute.*

$= 1.08(95.50)$

$= 103.14$ ✔

Remember!

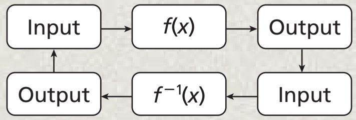
In a real-world situation, don't switch the variables, because they are named for specific quantities.

CHECK IT OUT! **5.** To make tea, use $\frac{1}{6}$ teaspoon of tea per ounce of water plus a teaspoon for the pot. Use the inverse to find the number of ounces of water needed if 7 teaspoons of tea are used.

THINK AND DISCUSS

1. Explain the result of interchanging x and y to find the inverse function of $f(x) = x$. How could you have predicted this from the graph of $f(x)$?
2. Give an example of a function whose inverse is a function. Give an example of a function whose inverse is not a function.
3. Tell what happens when you take the inverse of the inverse of a function. Is the result necessarily a function? Explain.
4. **GET ORGANIZED** Copy and complete the graphic organizer. Show a possible input value, inverse function, and output value for a function $f(x)$.

Know it! Note



7-2 Exercises

California Standards
12.0, 24.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 7-2
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** When switching x and y, the result is always an *inverse* ___?___. (*relation* or *function*)

SEE EXAMPLE 1
p. 498

Graph the relation and connect the points. Then graph the inverse. Identify the domain and range of each relation.

2.

x	1	2	3	4
y	1	2	4	8

3.

x	3	4	1	–1
y	–1	–2	–4	–4

SEE EXAMPLE 2
p. 499

Use inverse operations to write the inverse of each function.

4. $f(x) = x + 3$
5. $f(x) = 4x$
6. $f(x) = \frac{x}{2}$
7. $f(x) = x - 2\frac{1}{2}$

SEE EXAMPLE 3
p. 499

8. $f(x) = 5x - 1$
9. $f(x) = \frac{x}{2} + 3$
10. $f(x) = 3 - \frac{1}{2}x$
11. $f(x) = \frac{1}{2}(3 - 3x)$
12. $f(x) = 4(x + 1)$
13. $f(x) = \frac{3x - 5}{2}$

SEE EXAMPLE 4
p. 500

Graph each function. Then write and graph its inverse.

14. $f(x) = 5 - 2x$
15. $f(x) = \frac{x}{4} + 2$
16. $f(x) = 10 + 0.6x$

SEE EXAMPLE 5
p. 500

17. **Meteorology** The formula $C = \frac{5}{9}(F - 32)$ gives degrees Celsius as a function of degrees Fahrenheit. Find the inverse of this function to convert degrees Celsius to Fahrenheit and use it to find 16°C in degrees Fahrenheit.

PRACTICE AND PROBLEM SOLVING

Graph the relation and connect the points. Then graph the inverse. Identify the domain and range of each relation.

18.

x	–1	2	3	5
y	1	3	5	5

19.

x	–4	–2	0	2	4
y	–2	–1	0	1	2

Independent Practice

For Exercises	See Example
18–19	1
20–22	2
23–25	3
26–28	4
29	5

Extra Practice

Skills Practice p. S16

Application Practice p. S38

LINK

Physics

Mountain climbers at very high altitudes can drink tea while it's boiling with bubbles because it's cool enough not to burn them.

Use inverse operations to write the inverse of each function.

20. $f(x) = 0.825x$ **21.** $f(x) = x - 1\frac{3}{4}$ **22.** $f(x) = \frac{x}{0.25}$

23. $f(x) = 21 - 32x$ **24.** $f(x) = 145 + 12.5x$ **25.** $f(x) = \frac{1}{5}x + 12$

Graph each function. Then write and graph its inverse.

26. $f(x) = \frac{4}{5}(x - 15)$ **27.** $f(x) = 2 - \frac{x}{3}$ **28.** $f(x) = 1.21x$

29. Education A linear model projects that the number of bachelor's degrees awarded in the United States will increase by 19,500 each year. In 2001, 1.28 million bachelor's degrees were awarded. Use the inverse function to predict the number of years after 2001 that 1.7 million bachelor's degrees will be awarded. *Source:* nces.ed.gov

30. Critical Thinking Graph the line that passes through (2, 9) and (3, 4).

a. What is the slope of this line?

b. What is the slope of the line that is the inverse of the original line?

31. Physics At sea level, the boiling point of water is 212°F. At x thousand feet, the boiling point of water is given by the function $f(x) = 212 - 1.85x$.

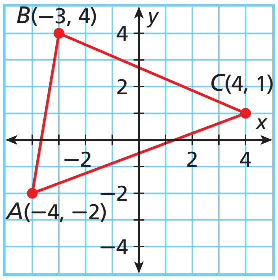
a. Write the inverse function.

b. Above what altitude, to the nearest 500 feet, does the boiling point of water fall below 200°F?

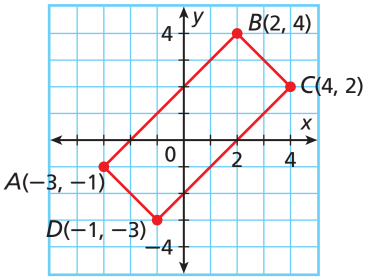
c. At the summit of Nepal's Lhotse Mountain, water boils at 160.3°F. What is the mountain peak's altitude?

Geometry **Find the coordinates of the vertices of the inverse for each figure.**

32.



33.



34. Critical Thinking What is the inverse of $f(x) = 3$? (Hint: Write this function as $y = 0x + 3$.) Is the inverse a function? Explain.

35. Animals In 1999, Warhol the albino ferret ran a 10 m tube race in 12.59 s. Assume that he ran at a constant rate. Write a function that gives distance as a function of time. Write and use the inverse function to find the time it would take Warhol to complete a 25 m race at the same speed.

CONCEPT CONNECTION

36. This problem will prepare you for the Concept Connection on page 520.

A theater sells tickets for $22. If you pay by credit card, the theater adds a service charge of $3.50 to the entire order.

a. Write a function that gives the amount billed to the credit card as a function of the number of tickets purchased.

b. Write the inverse function, and use it to find the number of tickets purchased when the credit card bill is $157.50.

c. Is it possible to have a total of $332.50 billed to your credit card for these tickets? Why or why not?

37. **ERROR ANALYSIS** Two students found the inverse of $f(x) = \frac{1}{2}x + 1$. Which is incorrect? Explain the error.

A

$f(x) = \frac{1}{2}x + 1$

$f^{-1}(x) = 2(x - 1)$

B

$f(x) = \frac{1}{2}x + 1$

$f^{-1}(x) = 2x - 1$

38. **Write About It** Explain the effect on a function and its graph when you switch the coordinates of the ordered pairs.

39. **Critical Thinking** Can the inverse of a relation that is not a function be a function itself? Explain your answer by using an example.

40. **Clothing** Hat size is a linear function of head circumference. A person with a head circumference of $21\frac{1}{2}$ in. has a hat size of $6\frac{7}{8}$, while a person with a head circumference of $21\frac{7}{8}$ in. has a hat size of 7.
   a. Find hat size as a function of head circumference.
   b. Find the inverse. Is it a function? What does the inverse represent?
   c. A hat was found with a size of $7\frac{3}{8}$. What is the head circumference of the owner?

Tell whether each statement is sometimes, always, or never true.

41. The inverse of an ordered pair on a graph is its reflection across the line $y = x$.

42. The inverse of a linear function is a linear function.

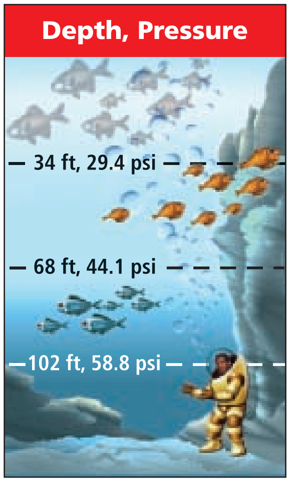
43. The inverse of a line with positive slope is a line with negative slope.

44. The inverse of a line with a slope greater than 1 is a line with slope less than 1.

45. The inverse of the inverse of a point (x, y) is the original point.

46. The line $y = k$, where k is a constant, has an inverse.

47. **Diving** Scuba divers must know that the deeper the dive, the greater the water pressure in pounds per square inch (psi) for fresh water diving, as shown in the diagram.
   a. Write the pressure as a function of depth.
   b. Identify a reasonable domain and range of the pressure function.
   c. Find the inverse of the function from part **a.** What does the inverse function represent?
   d. The point $(25.9, 25.9)$ is an approximate solution to both the function from part **a** and its inverse. What does this point mean in the context of the problem?



STANDARDIZED TEST PREP

48. Which function is the inverse of $f(x) = 4x - \frac{3}{4}$?

Ⓐ $f^{-1}(x) = \frac{1}{4}x + \frac{3}{16}$

Ⓑ $f^{-1}(x) = -\frac{1}{4}x + 3$

Ⓒ $f^{-1}(x) = \frac{1}{4}x + 3$

Ⓓ $f^{-1}(x) = -\frac{1}{4}x + \frac{3}{16}$

49. Eliza's auto repair bill includes \$175 for parts and \$35 per hour for labor. The bill can be expressed as a function of hours x with the function $f(x) = 175 + 35x$. Which statement explains the meaning of the inverse of the function?

(F) Number of hours as a function of the total bill

(G) Total bill as a function of the number of hours

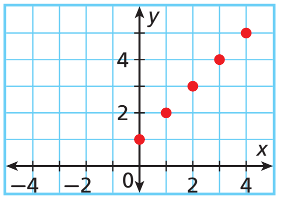
(H) Cost per hour as a function of the total bill

(J) Total bill as a function of the cost per hour

50. The inverse of a point is $(5, -2)$. What point is this the inverse of?

(A) $(-5, 2)$ (B) $(5, 2)$ (C) $(-2, 5)$ (D) $(2, -5)$

51. **Short Response** Make a table to show the inverse of the relation shown in the graph.



CHALLENGE AND EXTEND

Give the inverse of each linear function, where $y = f(x)$.

52. $y = mx + b$ **53.** $ax + by = c$ **54.** $y - y_1 = m(x - x_1)$

55. Graph the relation given by the points in the table. Then reflect each point across the line $y = x$ to see the graph of the inverse relation. If the equation of the relation is $f(x) = x^2$, verify algebraically that the equation of the inverse relation is $x = y^2$.

x	y
–3	9
–2	4
–1	1
0	0
1	1
2	4
3	9

56. **Critical Thinking** A linear function and its inverse have the same slope. What must be true of these functions?

Graph each function and its inverse.

57. $y = 3$ **58.** $y = x^3$ **59.** $y = 2^x$

SPIRAL REVIEW

60. **Business** A stock was purchased for \$45.18 per share. The change in value is shown in the table. *(Lesson 1-1)*

a. Order the stock values from least to greatest. Include the purchase day as day 0.

b. Use set-builder notation to represent the range of the stock value.

Stock Market Value

Day	Change in Value (\$)
1	–0.23
2	+2.58
3	–0.64
4	+1.27
5	–2.12

Write the polynomial equation of least degree with the given roots and leading coefficient of 2. *(Lesson 6-6)*

61. $-3, 2, 1$ **62.** $\sqrt{5}, -\sqrt{5}$

63. $1 - i, 2$ **64.** $-3, 8, 9$

Tell whether the function shows growth or decay. Then graph. *(Lesson 7-1)*

65. $f(x) = 15\left(\frac{89}{100}\right)^x$ **66.** $f(x) = \frac{1}{25}(0.5^x)$

67. $f(x) = 2(1.1^x)$ **68.** $f(x) = 0.01(1.9^x)$

7-3 Logarithmic Functions

Objectives

Write equivalent forms for exponential and logarithmic functions.

Write, evaluate, and graph logarithmic functions.

Vocabulary

logarithm
common logarithm
logarithmic function

California Standards

11.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Also covered: **14.0, 11.0**

Reading Math

Read $\log_b a = x$, as "the log base b of a is x." Notice that the log is the exponent.

Why learn this?

A logarithmic scale is used to measure the acidity, or pH, of water. (See Example 5.)

How many times would you have to double \$1 before you had \$8? You could use an exponential equation to model this situation. $1(2^x) = 8$. You may be able to solve this equation by using mental math if you know that $2^3 = 8$. So you would have to double the dollar 3 times to have \$8.

How many times would you have to double \$1 to have \$512? You could solve this problem if you could solve $2^x = 8$ by using an inverse operation that undoes raising a base to an exponent. This operation is called finding the logarithm. A **logarithm** is the exponent to which a specified base is raised to obtain a given value.

You can write an exponential equation as a logarithmic equation and vice versa.

Exponential Equation → **Logarithmic Equation**

$$b^x = a \qquad \log_b a = x$$

$b > 0,\ b \neq 1$

EXAMPLE 1 Converting from Exponential to Logarithmic Form

Write each exponential equation in logarithmic form.

	Exponential Equation	Logarithmic Form	
a.	$2^6 = 64$	$\log_2 64 = 6$	*The base of the exponent becomes the base of the logarithm.*
b.	$4^1 = 4$	$\log_4 4 = 1$	*The exponent is the logarithm.*
c.	$5^0 = 1$	$\log_5 1 = 0$	*Any nonzero base to the 0 power is 1.*
d.	$5^{-2} = 0.04$	$\log_5 0.04 = -2$	*An exponent (or log) can be negative.*
e.	$3^x = 81$	$\log_3 81 = x$	*The log (and the exponent) can be a variable.*

CHECK IT OUT! **Write each exponential equation in logarithmic form.**

1a. $9^2 = 81$ **1b.** $3^3 = 27$ **1c.** $x^0 = 1 (x \neq 0)$

EXAMPLE 2 Converting from Logarithmic to Exponential Form

Write each logarithmic equation in exponential form.

	Logarithmic Equation	Exponential Form	
a.	$\log_{10} 100 = 2$	$10^2 = 100$	*The base of the logarithm becomes the base of the power.*
b.	$\log_7 49 = 2$	$7^2 = 49$	*The logarithm is the exponent.*
c.	$\log_8 0.125 = -1$	$8^{-1} = 0.125$	*A logarithm can be a negative number.*
d.	$\log_5 5 = 1$	$5^1 = 5$	
e.	$\log_{12} 1 = 0$	$12^0 = 1$	

CHECK IT OUT! **Write each logarithmic equation in exponential form.**

2a. $\log_{10} 10 = 1$ **2b.** $\log_{12} 144 = 2$ **2c.** $\log_{\frac{1}{2}} 8 = -3$

A logarithm is an exponent, so the rules for exponents also apply to logarithms. You may have noticed the following properties in the last example.

Know it! Note

Special Properties of Logarithms

For any base b such that $b > 0$ and $b \neq 1$,

LOGARITHMIC FORM	EXPONENTIAL FORM	EXAMPLE
Logarithm of Base b		
$\log_b b = 1$	$b^1 = b$	$\log_{10} 10 = 1$ $10^1 = 10$
Logarithm of 1		
$\log_b 1 = 0$	$b^0 = 1$	$\log_{10} 1 = 0$ $10^0 = 1$

A logarithm with base 10 is called a **common logarithm**. If no base is written for a logarithm, the base is assumed to be 10. For example, $\log 5 = \log_{10} 5$.

You can use mental math to evaluate some logarithms.

EXAMPLE 3 Evaluating Logarithms by Using Mental Math

Evaluate by using mental math.

A $\log 1000$

$10^? = 1000$ *The log is the exponent.*

$10^3 = 1000$ *Think: What power of the base is the value?*

$\log 1000 = 3$

B $\log_4 \frac{1}{4}$

$4^? = \frac{1}{4}$

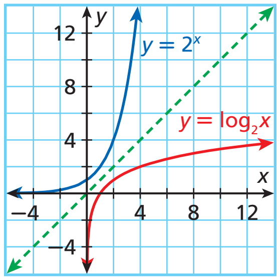
$4^{-1} = \frac{1}{4}$

$\log_4 \frac{1}{4} = -1$

CHECK IT OUT! **Evaluate by using mental math.**

3a. $\log 0.00001$ **3b.** $\log_{25} 0.04$

Because logarithms are the inverses of exponents, the inverse of an exponential function, such as $y = 2^x$, is a **logarithmic function**, such as $y = \log_2 x$.



You may notice that the domain and range of each function are switched.

The domain of $y = 2^x$ is all real numbers ($\mathbb{R}$), and the range is $\{y \mid y > 0\}$. The domain of $y = \log_2 x$ is $\{x \mid x > 0\}$, and the range is all real numbers ($\mathbb{R}$).

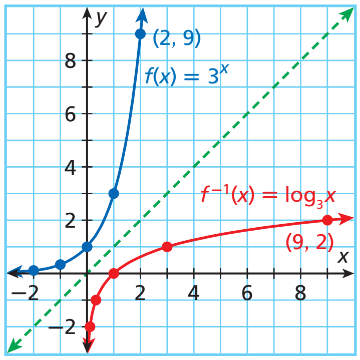
EXAMPLE 4 Graphing Logarithmic Functions

Use the given *x*-values to graph each function. Then graph its inverse. Describe the domain and range of the inverse function.

A $f(x) = 3^x$; $x = -2, -1, 0, 1,$ and 2

Graph $f(x) = 3^x$ by using a table of values.

x	−2	−1	0	1	2
$f(x) = 3^x$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9



To graph the inverse, $f^{-1}(x) = \log_3 x$, reverse each ordered pair.

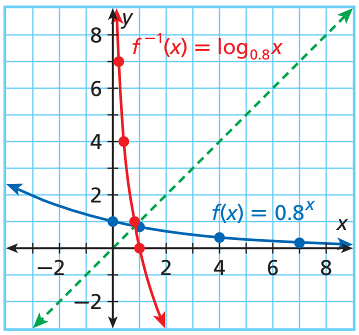
x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
$f^{-1}(x) = \log_3 x$	−2	−1	0	1	2

The domain of $f^{-1}(x)$ is $\{x \mid x > 0\}$, and the range is $\mathbb{R}$.

B $f(x) = 0.8^x$; $x = -3, 0, 1, 4,$ and 7

Graph $f(x) = 0.8^x$ by using a table of values. Round the output values to the nearest tenth, if necessary.

x	−3	0	1	4	7
$f(x) = 0.8^x$	2	1	0.8	0.4	0.2



To graph $f^{-1}(x) = \log_{0.8} x$, reverse each ordered pair.

x	2	1	0.8	0.4	0.2
$f^{-1}(x) = \log_{0.8} x$	−3	0	1	4	7

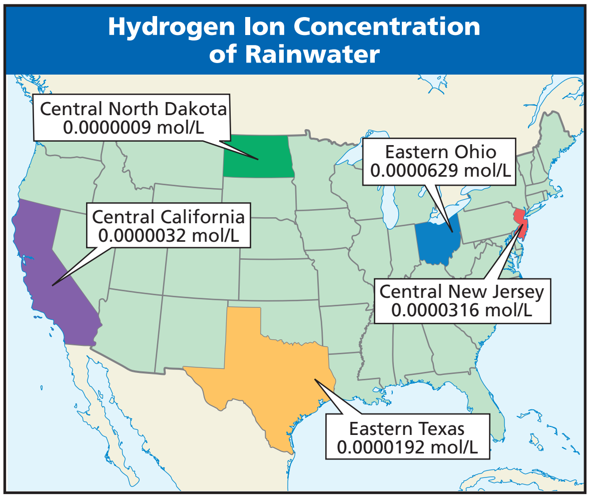
The domain of $f^{-1}(x)$ is $\{x \mid x > 0\}$, and the range is $\mathbb{R}$.

CHECK IT OUT! **4.** Use $x = -2, -1, 1, 2,$ and 3 to graph $f(x) = \left(\frac{3}{4}\right)^x$. Then graph its inverse. Describe the domain and range of the inverse function.

EXAMPLE 5 *Environmental Application*

Chemists regularly test rain samples to determine the rain's acidity, or concentration of hydrogen ions (H^+). Acidity is measured in pH, as given by the function pH $= -\log[H^+]$, where $[H^+]$ represents the hydrogen ion concentration in moles per liter.

Find the pH of rainwater from each location.



> **Helpful Hint**
>
> The LOG key is used to evaluate logarithms in base 10. 2nd LOG (10^x) is used to find 10^x, the inverse of log.

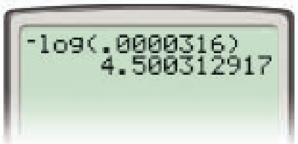
A **Central New Jersey**

The hydrogen ion concentration is 0.0000316 moles per liter.

$\text{pH} = -\log[H^+]$

$\text{pH} = -\log(0.0000316)$ *Substitute the known values in the function.*

Use a calculator to find the value of the logarithm in base 10. Press the LOG key.



The rainwater has a pH of about 4.5.

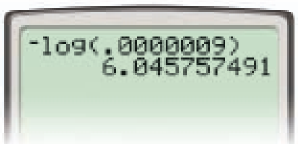
B **Central North Dakota**

The hydrogen ion concentration is 0.0000009 moles per liter.

$\text{pH} = -\log[H^+]$

$\text{pH} = -\log(0.0000009)$ *Substitute the known values in the function.*

Use a calculator to find the value of the logarithm in base 10. Press the LOG key.



The rainwater has a pH of about 6.0.

CHECK IT OUT! **5.** What is the pH of iced tea with a hydrogen ion concentration of 0.000158 moles per liter?

THINK AND DISCUSS

1. Explain why $\log_b b$ is always equal to 1 for $b > 0$ and $b \neq 1$.

2. Explain whether $\log_b a$ is the same as $\log_a b$. Support your answer.

3. **GET ORGANIZED** Copy and complete the graphic organizer. Use your own words to explain a logarithmic function.

Definition	Characteristics
Logarithmic Function	
Examples	Nonexamples

Know it! Note

7-3 Exercises

California Standards
8.0, 11.0, 11.1, 11.2, 14.0

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GUIDED PRACTICE

1. **Vocabulary** In the exponential equation $a^x = b$, the logarithm is ___?___. (a, x, or b)

SEE EXAMPLE 1 p. 505

Write each exponential equation in logarithmic form.

2. $2.4^0 = 1$
3. $4^{1.5} = 8$
4. $10^{-2} = 0.01$
5. $3^x = 243$

SEE EXAMPLE 2 p. 506

Write each logarithmic equation in exponential form.

6. $\log_4 0.0625 = -2$
7. $\log_x(-16) = 3$
8. $\log_{0.9} 0.81 = 2$
9. $\log_6 x = 3$

SEE EXAMPLE 3 p. 506

Evaluate by using mental math.

10. $\log_7 343$
11. $\log_3\left(\frac{1}{9}\right)$
12. $\log_{0.5} 0.25$
13. $\log_{1.2} 1.44$

SEE EXAMPLE 4 p. 507

Use the given x-values to graph each function. Then graph its inverse. Describe the domain and range of each function.

14. $f(x) = 5^x$; $x = -2, -1, 0, 1, 1.5$
15. $f(x) = 0.5^x$; $x = -2, -1, 0, 1, 2$

SEE EXAMPLE 5 p. 508

16. **Chemistry** The acid potential of a solution is given by pOH, where $\text{pOH} = -\log[\text{OH}^-]$, and OH^- represents the concentration of hydroxide ions in moles per liter. The water in one sample contains a hydroxide ion concentration of 0.000000004. What is the pOH of the water?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–20	1
21–24	2
25–28	3
29–30	4
31	5

Extra Practice
Skills Practice p. S16
Application Practice p. S38

Write each exponential equation in logarithmic form.

17. $x^{2.5} = 32$
18. $6^x = 216$
19. $1.2^0 = 1$
20. $4^{-1} = 0.25$

Write each logarithmic equation in exponential form.

21. $\log_5 625 = 4$
22. $\log_2 x = 6$
23. $\log_{4.5} 1 = 0$
24. $\log_\pi \pi = 1$

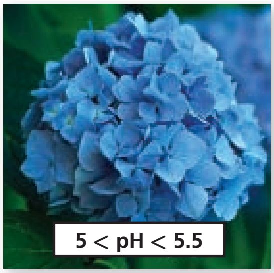
Evaluate by using mental math.

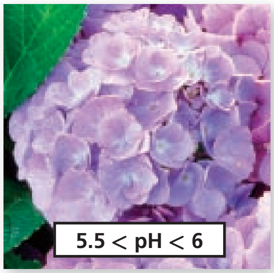
25. $\log_2 1$
26. $\log 0.001$
27. $\log_4 64$
28. $\log_{0.1} 100$

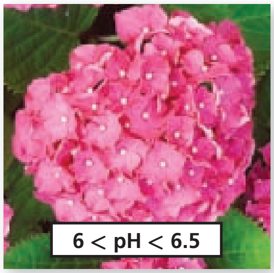
Use the given x-values to graph each function. Then graph its inverse. Describe the domain and range of each function.

29. $f(x) = \left(\frac{4}{5}\right)^x$; $x = -2, -1, 0, 1, 2, 3$
30. $f(x) = \left(\frac{4}{3}\right)^x$; $x = -2, -1, 0, 1, 2, 3$

31. **Gardening** The flower color of bigleaf hydrangeas is determined by the soil pH. A gardener growing blue hydrangeas believes that lime may be leaching out of a nearby sidewalk and increasing the pH of the soil. The gardener measures the hydrogen ion concentration and finds it to be 0.0000006 moles per liter. Is the soil still good for growing blue flowers? Explain.


$5 < \text{pH} < 5.5$


$5.5 < \text{pH} < 6$


$6 < \text{pH} < 6.5$

CONCEPT CONNECTION

32. This problem will prepare you for the Concept Connection on page 520.

For a certain credit card, given a starting balance of P and an ending balance of A, the function $n = \frac{\log A - \log P}{\log(1.0175)}$ gives the number of months that have passed, assuming that there were no payments or additional purchases during that time.

a. You started with a debt of \$1000 and now owe \$1210.26. For how many months has the debt been building? Use a calculator.

b. How many additional months will it take until the debt exceeds \$1420?

c. What do you notice from the results of parts **a** and **b**?

33. Critical Thinking If $\log_a b = 0$, what is the value of b? Explain.

34. Sound The loudness of sound is measured on a logarithmic scale according to the formula $L = 10\log\left(\frac{I}{I_0}\right)$, where L is the loudness of sound in decibels (dB), I is the intensity of sound, and I_0 is the intensity of the softest audible sound.

a. Find the loudness in decibels of each sound listed in the table.

b. The sound at a rock concert is found to have a loudness of 110 decibels. Where should this sound be placed in the table in order to keep the sound intensities in order from least to greatest?

c. What if...? A decibel is $\frac{1}{10}$ of a *bel*. Is a jet plane louder than a sound that measures 20 *bels*? Explain.

Sound	Intensity
Jet takeoff	$10^{15}I_0$
Jackhammer	$10^{12}I_0$
Hair dryer	$10^{7}I_0$
Whisper	$10^{3}I_0$
Leaves rustling	$10^{2}I_0$
Softest audible sound	I_0

35. Critical Thinking If n is an integer, and 10^n is written in expanded form, can you find $\log 10^n$ by counting the number of zeros in 10^n? Support your answer with an example or counterexample.

36. Estimation Given that $\log 100 = 2$ and $\log 1000 = 3$, estimate the values of $\log 200$ and $\log 500$.

37. Food The hydrogen ion concentrations of three juice samples are given. Identify the type of juice in each sample.

a. 0.00014 moles per liter

b. 0.0081 moles per liter

c. 0.00074 moles per liter

Juice	pH Range
Lemon	2.0–2.6
Grapefruit	2.9–3.2
Orange	3.3–4.1
Tomato	4.1–4.6

38. Write About It Explain why $\log_0 3$ and $\log_1 3$ do not exist.

STANDARDIZED TEST PREP

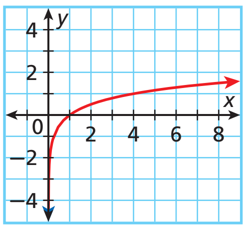
39. The graph of which function is shown?

(A) $f(x) = \log x$

(B) $f(x) = \log_2 x$

(C) $f(x) = \log_4 x$

(D) $f(x) = 2^x$



40. Which logarithmic equation is equivalent to $2^7 = 128$?

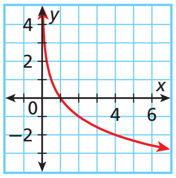
(F) $\log_7 2 = 128$ (H) $\log_2 7 = 128$

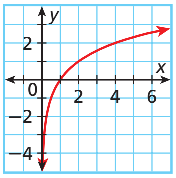
(G) $\log_2 128 = 7$ (J) $\log_7 128 = 2$

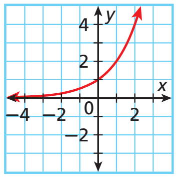
41. Which is the best estimate of $\log 50$?

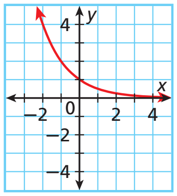
(A) 1.7 (B) 2.5 (C) 5 (D) 10

42. Which graph is the best representation of $f(x) = \log_{0.5} x$?

(F)


(G)


(H)


(J)


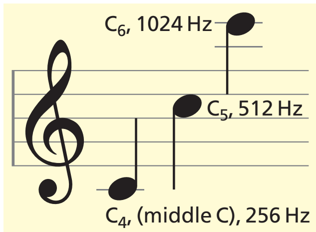
43. Gridded Response Evaluate $\log_2 64$.

CHALLENGE AND EXTEND

44. Graph $\log_7 x$ and $\log_{0.7} x$. Describe the difference between the two functions in terms of their graphs.

45. Evaluate $\log_3 9$, $\log_3 27$, and $\log_3 243$. Make a statement about the relationship between the three logarithms. Generalize the result by using variables.

46. Prove that $\log_7 7^{2x+1} = 2x + 1$, giving a reason for each step.



47. Music Musical scales are logarithmic. One scale uses a pitch standard called "scientific pitch." In this scale, the frequency of each C note, in vibrations per second, or Hz, can be expressed as a power of 2, as shown.

a. Express the frequency of the note C_7 in exponential form and in logarithmic form.

b. If the frequency of one note C is 32 vibrations per second, how many octaves higher or lower than middle C is this note? Explain by using logarithms.

SPIRAL REVIEW

Simplify each expression. Assume that all variables are nonzero. *(Lesson 1-5)*

48. $\left[(2a^4)(5b^2)\right]^2$

49. $\dfrac{8s^2t^6}{4st^8}$

50. $-2t^2(5st^{-1})$

51. $7a^{-2}b^3(3ab + 4a^{-1}b^2)$

52. Construction A brick fell at a construction site from a height of 25 feet. Use $h(t) = h_0 - 16t^2$, where h is the height in feet and t is the time in seconds, to determine the time that it took for the brick to hit the ground. *(Lesson 5-3)*

Complete the table of values for each function. Round to the nearest hundredth. *(Lesson 7-1)*

	x	−2	−1	0	1	2
53.	$f(x) = 1.7^x$					
54.	$f(x) = 0.6^x$					
55.	$f(x) = 0.3^x$					

7-4

Properties of Logarithms

Objectives

Use properties to simplify logarithmic expressions.

Translate between logarithms in any base.

Who uses this?

Seismologists use properties of logarithms to calculate the energy released by earthquakes. (See Example 6.)

California Standards

14.0 Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

Also covered: **15.0, 13.0, 11.0, 11.1, 11.2**

The logarithmic function for pH that you saw in the previous lesson, $\text{pH} = -\log[\text{H}^+]$, can also be expressed in exponential form, as $10^{-\text{pH}} = [\text{H}^+]$. Because logarithms are exponents, you can derive the properties of logarithms from the properties of exponents.

Remember that to *multiply* powers with the same base, you *add* exponents.

$$b^m b^n = b^{m+n}$$

Know it! Note

Product Property of Logarithms

For any positive numbers m, n, and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a product is equal to the sum of the logarithms of its factors.	$\log_3 1000 = \log_3(10 \cdot 100)$ $= \log_3 10 + \log_3 100$	$\log_b mn = \log_b m + \log_b n$

Helpful Hint

Think:
$\log j + \log a + \log m$
$= \log jam$

The property above can be used in reverse to write a sum of logarithms (exponents) as a single logarithm, which can often be simplified.

EXAMPLE 1 Adding Logarithms

Express as a single logarithm. Simplify, if possible.

A $\log_4 2 + \log_4 32$

$\log_4(2 \cdot 32)$	*To add the logarithms, multiply the numbers.*
$\log_4 64$	*Simplify.*
3	*Think: $4^? = 64$*

CHECK IT OUT! **Express as a single logarithm. Simplify, if possible.**

1a. $\log_5 625 + \log_5 25$ **1b.** $\log_{\frac{1}{3}} 27 + \log_{\frac{1}{3}} \frac{1}{9}$

Remember that to *divide* powers with the same base, you *subtract* exponents.

$$\frac{b^m}{b^n} = b^{m-n}$$

Because logarithms are exponents, subtracting logarithms with the same base is the same as finding the logarithm of the quotient with that base.

Know it! Note

Quotient Property of Logarithms

For any positive numbers m, n, and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a quotient is the logarithm of the dividend minus the logarithm of the divisor.	$\log_5\left(\frac{16}{2}\right) = \log_5 16 - \log_5 2$	$\log_b \frac{m}{n} = \log_b m - \log_b n$

Caution!

Just as a^5b^3 cannot be simplified, logarithms must have the *same* base to be simplified.

The property above can also be used in reverse.

EXAMPLE 2 Subtracting Logarithms

Express $\log_2 32 - \log_2 4$ as a single logarithm. Simplify, if possible.

$\log_2 32 - \log_2 4$	
$\log_2 (32 \div 4)$	*To subtract the logarithms, divide the numbers.*
$\log_2 8$	*Simplify.*
3	*Think: $2^? = 8$*

CHECK IT OUT!

2. Express $\log_7 49 - \log_7 7$ as a single logarithm. Simplify, if possible.

Because you can multiply logarithms, you can also take powers of logarithms.

Know it! Note

Power Property of Logarithms

For any real number p and positive numbers a and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a power is the product of the exponent and the logarithm of the base.	$\log 10^3$ $\log (10 \cdot 10 \cdot 10)$ $\log 10 + \log 10 + \log 10$ $3 \log 10$	$\log_b a^p = p \log_b a$

EXAMPLE 3 Simplifying Logarithms with Exponents

Express as a product. Simplify, if possible.

A $\log_3 81^2$

$2 \log_3 81$

$2(4) = 8$ *Because $3^4 = 81$, $\log_3 81 = 4$.*

B $\log_5 \left(\frac{1}{5}\right)^3$

$3 \log_5 \frac{1}{5}$

$3(-1) = -3$ *$5^{-1} = \frac{1}{5}$*

CHECK IT OUT!

Express as a product. Simplify, if possible.

3a. $\log 10^4$ **3b.** $\log_5 25^2$ **3c.** $\log_2 \left(\frac{1}{2}\right)^5$

Exponential and logarithmic operations undo each other since they are inverse operations.

Know it! Note

Inverse Properties of Logarithms and Exponents

For any base b such that $b > 0$ and $b \neq 1$,

ALGEBRA	EXAMPLE
$\log_b b^x = x$	$\log_{10} 10^7 = 7$
$b^{\log_b x} = x$	$10^{\log_{10} 2} = 2$

EXAMPLE 4 Recognizing Inverses

Simplify each expression.

A $\log_8 8^{3x+1}$

$\log_8 8^{3x+1}$

$3x + 1$

B $\log_5 125$

$\log_5 (5 \cdot 5 \cdot 5)$

$\log_5 5^3$

3

C $2^{\log_2 27}$

$2^{\log_2 27}$

27

CHECK IT OUT! **4a.** Simplify $\log 10^{0.9}$. **4b.** Simplify $2^{\log_2 (8x)}$.

Most calculators calculate logarithms only in base 10 or base e (see Lesson 7-6). You can change a logarithm in one base to a logarithm in another base with the following formula.

Know it! Note

Change of Base Formula

For $a > 0$ and $a \neq 1$ and any base b such that $b > 0$ and $b \neq 1$,

ALGEBRA	EXAMPLE
$\log_b x = \dfrac{\log_a x}{\log_a b}$	$\log_4 8 = \dfrac{\log_2 8}{\log_2 4}$

EXAMPLE 5 Changing the Base of a Logarithm

Evaluate $\log_4 8$.

Method 1 Change to base 10.

$$\log_4 8 = \frac{\log 8}{\log 4}$$

$$\approx \frac{0.0903}{0.602} \quad \textit{Use a calculator.}$$

$$= 1.5 \quad \textit{Divide.}$$

Method 2 Change to base 2, because both 4 and 8 are powers of 2.

$$\log_4 8 = \frac{\log_2 8}{\log_2 4} = \frac{3}{2}$$

$$= 1.5$$

CHECK IT OUT! **5a.** Evaluate $\log_9 27$. **5b.** Evaluate $\log_8 16$.

Logarithmic scales are useful for measuring quantities that have a very wide range of values, such as the intensity (loudness) of a sound or the energy released by an earthquake.

EXAMPLE 6 Geology Application

Seismologists use the Richter scale to express the energy, or magnitude, of an earthquake. The Richter magnitude of an earthquake, *M*, is related to the energy released in ergs *E* shown by the formula $M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$.

In 1964, an earthquake centered at Prince William Sound, Alaska, registered a magnitude of 9.2 on the Richter scale. Find the energy released by the earthquake.

> **Helpful Hint**
>
> The Richter scale is logarithmic, so an increase of 1 corresponds to a release of 10 times as much energy.

$9.2 = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$ *Substitute 9.2 for M.*

$\left(\frac{3}{2}\right)9.2 = \log\left(\frac{E}{10^{11.8}}\right)$ *Multiply both sides by $\frac{3}{2}$.*

$13.8 = \log\left(\frac{E}{10^{11.8}}\right)$ *Simplify.*

$13.8 = \log E - \log 10^{11.8}$ *Apply the Quotient Property of Logarithms.*

$13.8 = \log E - 11.8$ *Apply the Inverse Properties of Logarithms and Exponents.*

$25.6 = \log E$

$10^{25.6} = E$ *Given the definition of a logarithm, the logarithm is the exponent.*

$3.98 \times 10^{25} = E$ *Use a calculator to evaluate.*

The energy released by an earthquake with a magnitude of 9.2 is 3.98×10^{25} ergs.

CHECK IT OUT!

6. How many times as much energy is released by an earthquake with a magnitude of 9.2 than by an earthquake with a magnitude of 8?

THINK AND DISCUSS

1. Explain how to graph $y = \log_5 x$ on a calculator.

2. Tell how you could find $10^{25.6}$ in Example 6 by applying a law of exponents.

3. Describe what happens when you use the change-of-base formula, $\log_b x = \frac{\log_a x}{\log_a b}$, when $x = a$.

Know it! Note

4. GET ORGANIZED Copy and complete the graphic organizer. Use your own words to show related properties of exponents and logarithms.

Property of Exponents	Property of Logarithms

7-4 Exercises

California Standards 5.0, 11.0, 11.1, 11.2, 13.0, 14.0, 15.0

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 512

Express as a single logarithm. Simplify, if possible.

1. $\log_5 50 + \log_5 62.5$
2. $\log 100 + \log 1000$
3. $\log_3 3 + \log_3 27$

SEE EXAMPLE 2 p. 513

Express as a single logarithm. Simplify, if possible.

4. $\log_4 320 - \log_4 5$
5. $\log 5.4 - \log 0.054$
6. $\log_6 496.8 - \log_6 2.3$

SEE EXAMPLE 3 p. 513

Simplify, if possible.

7. $\log_8 8^2$
8. $\log_3 3^5$
9. $\log_7 49^3$
10. $\log_{\frac{1}{2}}(0.25)^4$

SEE EXAMPLE 4 p. 514

11. $\log_2 2^{\frac{x}{2}+5}$
12. $2.5^{\log_{2.5} 19}$
13. $\log_4 1024$
14. $\log_2(0.5)^4$

SEE EXAMPLE 5 p. 514

Evaluate.

15. $\log_9\left(\frac{1}{27}\right)$
16. $\log_8 32$
17. $\log_5 10$
18. $\log_2 27$

SEE EXAMPLE 6 p. 515

19. **Geology** The Richter magnitude M of an earthquake is related to the energy released in ergs E shown by the formula $M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$. How many times as much energy was released by the 1811 New Madrid, Missouri, earthquake than by the Fort Tejon, California, earthquake?

Largest Earthquakes in Continental U.S.

Location	Year	*M*
New Madrid, MO	1811	8.1
New Madrid, MO	1812	8.0
Fort Tejon, CA	1957	7.9
San Francisco, CA	1906	7.8
Imperial Valley, CA	1892	7.8

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–22	1
23–25	2
26–28	3
29–31	4
32–34	5
35	6

Extra Practice
Skills Practice p. S16
Application Practice p. S38

Express as a single logarithm. Simplify, if possible.

20. $\log_8 4 + \log_8 16$
21. $\log 2 + \log 5$
22. $\log_{2.5} 3.125 + \log_{2.5} 5$
23. $\log 1000 - \log 100$
24. $\log_2 16 - \log_2 2$
25. $\log_{1.5} 6.75 - \log_{1.5} 2$

Simplify, if possible.

26. $\log_2 16^3$
27. $\log(100)^{0.1}$
28. $\log_5 125^{\frac{1}{3}}$
29. $\log_3 3^{7+x}$
30. $3^{\log_3 4.52}$
31. $\log_9 6561$

Evaluate.

32. $\log_{\frac{1}{2}} 16$
33. $\log_{25} 125$
34. $\log_4 9$

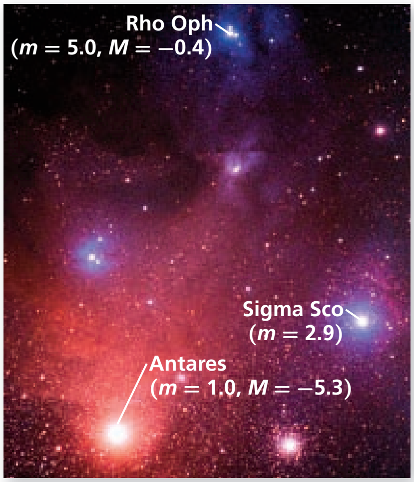
35. **Sound** After some complaints, it was found that the music from an outdoor concert was 5 decibels louder than the city's allowable level of 100 decibels. The loudness L of sound in decibels is given by $L = 10\log\left(\frac{I}{I_0}\right)$, where I is the intensity of sound and I_0 is the intensity of the softest audible sound. How many times more intense is the concert sound than the allowable level?

36. **Astronomy** The difference between the apparent magnitude (brightness) m of a star, and its absolute magnitude M is given by the formula $m - M = 5\log\frac{d}{10}$, where d is the distance of the star from Earth, measured in parsecs.

a. Find the distance d of Antares from Earth.

b. Sigma Sco is 225 parsecs from Earth. Find its absolute magnitude.

c. How many times as great is the distance to Antares as the distance to Rho Oph?



Write the equivalent logarithmic form for each equation.

37. $b^{m+n} = b^m b^n$

38. $b^{m-n} = \dfrac{b^m}{b^n}$

39. $(b^m)^n = b^{mn}$

Simplify, if possible.

40. $\log_2 32 - \log_2 128$

41. $\log 0.1 + \log 1 + \log 10$

42. $2 - \log_{11} 121$

43. $\log_{\frac{1}{2}} 2 + \log_{\frac{1}{2}} 2^{\frac{1}{2}}$

44. $7^{\log_7 7} - \log_7 7^7$

45. $\dfrac{10^{\log 10}}{\log 10^{10}}$

46. **Critical Thinking** Use the properties of logarithms with the fact that $\log 2 \approx 0.301$ to evaluate.

a. $\log 20$ **b.** $\log 200$ **c.** $\log 2000$

47. **Chemistry** Most swimming pool experts recommend a pH of between 7.0 and 7.6 for water in a swimming pool. Use $\text{pH} = -\log[\text{H}^+]$, and write an expression for the difference in hydrogen ion concentration over this pH range.

48. **Multi-Step** Suppose that the population of one endangered species decreases at a rate of 4% per year. In one habitat, the current population of the species is 143.

a. Write an exponential function for the population by year.

b. Write a logarithmic function for the time based upon population.

c. Write the keystrokes necessary to enter the logarithmic function on a calculator.

d. After how long will the population drop below 30, to the nearest year?

49. **Finance** A stock priced at $40 increases at a rate of 8% per year. Write and evaluate a logarithmic expression for the number of years that it will take for the value of the stock to reach $50. (*Hint:* Write the expression in exponential form first.)

LINK

Math History

Scottish mathematician John Napier (1550–1617) invented logarithms and named them by joining the Greek words *logos* (ratio) and *arithmos* (number).

CONCEPT CONNECTION

50. This problem will prepare you for the Concept Connection on page 520.

For a certain credit card with 19.2% annual interest compounded monthly, the total amount A that you owe after n months is given by $A = P(1.016)^n$, where P is the starting balance.

a. You start with a balance of $500. Write and solve a logarithmic expression for the number of months it will take for the debt to double.

b. How many additional months will it take for the debt to double again?

c. Does the amount of time that it takes the debt to double depend on the starting balance?

Graphing Calculator **Use the change of base formula and a graphing calculator to graph.**

51. $y = \log_3 x$ **52.** $y = 2\log_5 x$ **53.** $y = \dfrac{\log_{12} x}{3}$

54. **Write About It** Explain how to graph a logarithm in a base other than 10 on a calculator.

55. **Critical Thinking** Given $\log_{12} 20 \approx 1.2$ and $\log_{12} 33 \approx 1.4$, find each approximate value.

a. $\log_{12} 1.65$ **b.** $\log_{12} 660$ **c.** $\log_{12} 400$

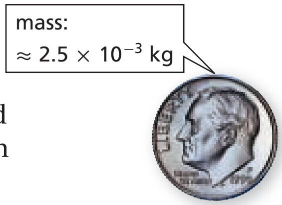
56. **Critical Thinking** There is an interesting relationship between logarithms and scientific notation.

a. Find the logarithm of 2.5.

b. Find the logarithm of the mass of the *Titanic*. Compare it to your answer from part **a.**

c. **Make a Conjecture** A lion has a mass of 2.5×10^2 kg. Find the logarithm of this number. Use your answers and the answers to parts **a** and **b,** to explain how to find the base 10 logarithm of a number written in scientific notation.

d. Use your conjecture to find the logarithm of the mass of a dime. Does your conjecture hold for scientific notation with negative exponents?



Assume $b > 0$ and $b \neq 1$. Tell whether each statement is sometimes, always, or never true.

57. A logarithm with base b can be changed to another rational-number base.

58. The logarithm with base 6 of 6 raised to an expression is equal to the expression.

59. Subtracting log base b of 1 from a number is just the number itself.

60. The base of a logarithm can be a negative number.

61. The logarithm of the square of a number is equal to twice the logarithm of the number.

62. Logarithms with different bases can be added without changing a base.

63. $\dfrac{\log_b 16}{\log_b 8}$ can be simplified.

64. A logarithm of a logarithm of a number is the number.

65. **ERROR ANALYSIS** Two simplifications of $\log 80 + \log 20$ are shown. Which of these is incorrect? Explain.

A

$\log 80 + \log 20 = \log(80 \cdot 20)$
$= \log(1600)$
$= \log(16 \cdot 10^2)$
$= \log 16 + \log 10^2$
$= \log 16 + 2$

B

$\log 80 + \log 20 = \log(80 + 20)$
$= \log 100$
$= \log 10^2$
$= 2\log 10$
$= 2$

STANDARDIZED TEST PREP

66. Which statement is NOT true?

Ⓐ $\log 140 - \log 35 = \log 4$

Ⓑ $\dfrac{\log 140}{\log 35} = \log 4$

Ⓒ $\log 35 + \log 4 = \log 140$

Ⓓ $\log \dfrac{140}{35} = \log 4$

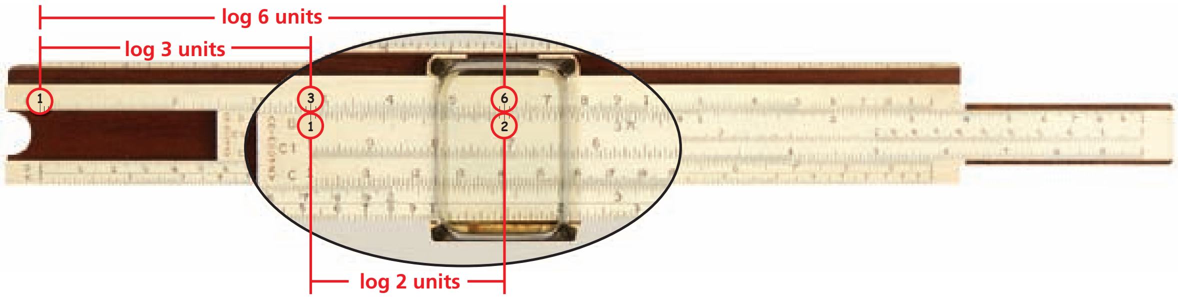
67. Simplify $\log_9 x^2 + \log_9 x$.

Ⓕ $\log_9 (x^2 + x)$ Ⓖ $\log_9 3x$ Ⓗ $3\log_9 x$ Ⓙ $3(x^2 + x)$

68. Which logarithmic expression is equal to $\log 6$?

Ⓐ $\log 3 + \log 2$ Ⓑ $\log 3 + \log 3$ Ⓒ $(\log 3)(\log 2)$ Ⓓ $(\log 3)(\log 3)$

CHALLENGE AND EXTEND



69. **Math History** The slide rule used two number lines that slid against each other. The scale on each was logarithmic, so the properties of logarithms could be applied to multiply and divide numbers.

a. Explain how the product of 2 and 3 is shown on the slide rule.

b. How does this show the product property of logarithms?

Find the domain of each function.

70. $f(x) = \log(x^2 - 4)$

71. $f(x) = \log x - \log(x - 1)$

72. $f(x) = \log\left(\dfrac{x}{x^2 - 1}\right)$

73. $f(x) = \log\left(\dfrac{1}{x}\right)^2$

74. $f(x) = -\sqrt{\log(x + 1)}$

75. $f(x) = \sqrt{-2\log(-x)}$

76. Prove: $\log_b a^p = p\log_b a$.

77. Simplify $\log_9 3^{2x}$.

Solve.

78. $\log_x 25 = 2$

79. $\log_x(-8) = 3$

80. $0 = \log_x 1$

SPIRAL REVIEW

Solve. *(Lesson 2-1)*

81. $9 = 3(x - 14)$

82. $4(x + 1) = 3(2x - 6)$

83. $-20 + 8n = n + 29$

84. $8\left(n + \dfrac{3}{4}\right) = 10n - 4$

Express each number in terms of *i*. *(Lesson 5-5)*

85. $3\sqrt{-16}$

86. $-\frac{1}{2}\sqrt{-40}$

87. $4\sqrt{-8}$

88. $\sqrt{-125}$

Write each exponential equation in logarithmic form. *(Lesson 7-3)*

89. $5^3 = 125$

90. $10^{-1} = 0.1$

91. $36^{0.5} = 6$

92. $4^x = 256$

Evaluate. *(Lesson 7-3)*

93. $\log_{12} 1$

94. $\log_5 25$

95. $\log_{16} 4$

96. $\log_{625} 0.04$

CONCEPT CONNECTION

Exponential Functions and Logarithms

Charged Up There are more than 1 billion credit cards in circulation in the United States, and the average American carries a credit card debt of approximately \$8600. Given that many credit cards charge an annual percentage rate (APR) of 18.3%, it can be difficult to escape the "credit hole."

The formula shown below can be used to compute the monthly payment M that is necessary to pay off a credit card balance P in a given number of years t. In the formula, r is the annual percentage rate and n is the number of payments per year.

$$M = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}$$

1. Suppose that you have a balance of \$8600 on a credit card with an APR of 18.3%. What monthly payment should you make in order to pay off the debt in exactly five years?

2. How much money do you end up paying altogether over the five years?

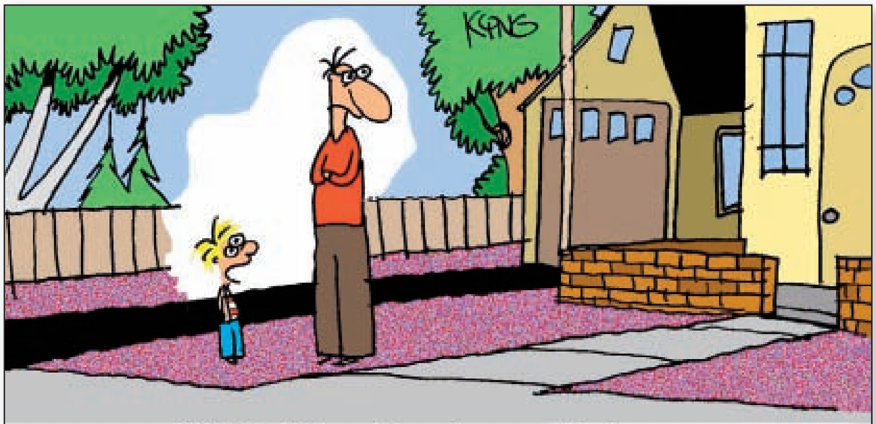
In order to calculate the number of years necessary for a given payment schedule, the formula can be written as shown.

$$t = \frac{\log\left(1 - \frac{Pr}{Mn}\right)}{-n\log\left(1 + \frac{r}{n}\right)}$$

3. If you can afford only a monthly payment of \$160, how long will it take to pay off the credit card debt?

4. Suppose you can afford a monthly payment of \$130. Will you be able to pay off the debt? If so, how long will it take? If not, why not?

5. What is the minimum monthly payment that will work toward paying off the debt?



"I didn't time to mow the lawn, so I used your credit card to have it carpeted."

READY TO GO ON?

CHAPTER 7

SECTION 7A

Quiz for Lesson 7-1 Through 7-4

7-1 Exponential Functions, Growth, and Decay

Tell whether the function shows growth or decay. Then graph.

1. $f(x) = \left(\frac{1}{4}\right)^x$ **2.** $f(x) = \frac{1}{5}(0.2)^x$ **3.** $f(x) = 14(1.4^x)$ **4.** $f(x) = 6.4\left(1\frac{3}{8}\right)^x$

5. Suppose that the number of bacteria in a culture was 1000 on Monday and the number has been increasing at a rate of 50% per day since then.

a. Write a function representing the growth of the culture per day.

b. Graph the function, and use the graph to predict the number of bacteria in the culture the following Monday.

7-2 Inverses of Relations and Functions

Graph each relation. Then graph its inverse.

6.

x	–1	0	1	2	3
y	0	4	8	12	16

7.

x	0	1	2	3	4
y	–1	$-\frac{1}{3}$	$\frac{1}{3}$	1	$1\frac{2}{3}$

Graph each function. Then write and graph the inverse.

8. $f(x) = x + 2.1$ **9.** $f(x) = \frac{3}{4} - x$ **10.** $f(x) = 5x + 4$ **11.** $f(x) = 0.4\left(\frac{x}{2} + 1.5\right)$

12. Rebekah's computer repair bill includes $210 for parts and $55 per hour for labor. Her bill can be expressed as a function of hours x by $f(x) = 210 + 55x$. Find the inverse function. Use it to find the number of hours of labor if her bill was $402.50.

7-3 Logarithmic Functions

Write the exponential equation in logarithmic form.

13. $3^2 = 9$ **14.** $17.6^0 = 1$ **15.** $2^{-2} = 0.25$ **16.** $0.5^x = 0.0625$

Write the logarithmic equation in exponential form.

17. $\log_4 64 = 3$ **18.** $\log_{\frac{1}{5}} 25 = -2$ **19.** $\log_{0.99} 1 = 0$ **20.** $\log_e x = 5$

21. Use the given x-values to graph $f(x) = \left(\frac{5}{6}\right)^x$; $x = -1, 0, 1, 2, 3$. Then graph the inverse function.

7-4 Properties of Logarithms

Express as a single logarithm. Simplify, if possible.

22. $\log_3 81 + \log_3 9$ **23.** $\log_{\frac{1}{5}} 25 + \log_{\frac{1}{5}} 5$ **24.** $\log_{1.2} 2.16 - \log_{1.2} 1.5$

Simplify each expression.

25. $\log_4 256^2$ **26.** $\log_7 343$ **27.** $17^{\log_{17} 0.73}$

Evaluate.

28. $\log_{27} 243$ **29.** $\log_{10} 0.01$ **30.** $\log_5 625$

7-5 Exponential and Logarithmic Equations and Inequalities

Objectives

Solve exponential and logarithmic equations and inequalities.

Solve problems involving exponential and logarithmic equations.

Vocabulary

exponential equation

logarithmic equation

Who uses this?

Exponential scales are used to measure light in photography. (See Exercise 40.)

An **exponential equation** is an equation containing one or more expressions that have a variable as an exponent. To solve exponential equations:

- Try writing them so that the bases are all the same. If $b^x = b^y$, then $x = y$ $(b \neq 0, b \neq 1)$.
- Take the logarithm of both sides. If $a = b$, then $\log a = \log b$ $(a > 0, b > 0)$.

California Standards

11.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Also covered: **14.0**

EXAMPLE 1 Solving Exponential Equations

Solve and check.

A $8^x = 2^{x+6}$

$(2^3)^x = 2^{x+6}$ — *Rewrite each side with the same base; 8 is a power of 2.*

$2^{3x} = 2^{x+6}$ — *To raise a power to a power, multiply exponents.*

$3x = x + 6$ — *Bases are the same, so the exponents must be equal.*

$x = 3$ — *Solve for x.*

Check

8^x	2^{x+6}
8^3	2^{3+6}
8^3	2^9
512	512 ✔

The solution is $x = 3$.

B $5^{x-2} = 200$

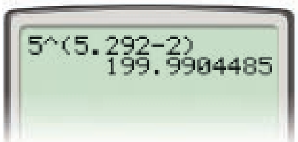
$\log 5^{x-2} = \log 200$ — *200 is not a power of 5, so take the log of both sides.*

$(x-2)\log 5 = \log 200$ — *Apply the Power Property of Logarithms.*

$x - 2 = \dfrac{\log 200}{\log 5}$ — *Divide both sides by log 5.*

$x = 2 + \dfrac{\log 200}{\log 5} \approx 5.292$

Check Use a calculator.



The solution is $x \approx 5.292$.

Helpful Hint

When you use a rounded number in a check, the result will not be exact, but it should be reasonable.

CHECK IT OUT! **Solve and check.**

1a. $3^{2x} = 27$ **1b.** $7^{-x} = 21$ **1c.** $2^{3x} = 15$

EXAMPLE 2 Money Application

You can choose a prize of either a $20,000 car or one penny on the first day, double that (2 cents) on the second day, and so on for a month. On what day would you receive more than the value of the car?

$20,000 is 2,000,000 cents. On day 1, you would receive 1 cent, or 2^0 cents. On day 2, you would receive 2 cents, or 2^1 cents, and so on. So, on day n you would receive 2^{n-1} cents.

Solve $2^{n-1} > 2 \times 10^6$.	*Write 2,000,000 in scientific notation.*
$\log 2^{n-1} > \log(2 \times 10^6)$	*Take the log of both sides.*
$(n-1)\log 2 > \log 2 + \log 10^6$	*Use the Power Property and Product Property.*
$(n-1)\log 2 > \log 2 + 6$	*$\log 10^6$ is 6.*
$n - 1 > \dfrac{\log 2 + 6}{\log 2}$	*Divide both sides by log 2.*
$n > \approx \dfrac{0.301 + 6}{0.301} + 1$	*Evaluate by using a calculator.*
$n > \approx 21.93$	*Round this up to the next whole number.*

Beginning on day 22, you would receive more than the value of the car.

Check On day 22, you would receive 2^{22-1} cents.

$$2^{22-1} = 2^{21} = 2{,}097{,}152 \text{ cents, or } \$20{,}971.52.$$

CHECK IT OUT! **2.** In Example 2, suppose that you receive triple the amount each day. On what day would you receive at least a million dollars?

A **logarithmic equation** is an equation with a logarithmic expression that contains a variable. You can solve logarithmic equations by using the properties of logarithms.

$$\text{If } \log_b x = \log_b y \text{ then } x = y$$

EXAMPLE 3 Solving Logarithmic Equations

Solve.

A $\log_3(x-5) = 2$

$3^{\log_3(x-5)} = 3^2$	*Use 3 as the base for both sides.*
$x - 5 = 9$	*Use inverse properties to remove 3 to the log base 3.*
$x = 14$	*Simplify.*

B $\log 45x - \log 3 = 1$

$\log\left(\dfrac{45x}{3}\right) = 1$	*Write as a quotient.*
$\log(15x) = 1$	*Divide.*
$10^{\log 15x} = 10^1$	*Use 10 as a base for both sides.*
$15x = 10$	*Use inverse properties on the left side.*
$x = \dfrac{2}{3}$	

Remember!
Review the properties of logarithms from Lesson 7-4.

Solve.

C $\log_4 x^2 = 7$

$2\log_4 x = 7$ *Power Property of Logarithms*

$\log_4 x = \frac{7}{2}$ *Divide both sides by 2 to isolate $\log_4 x$.*

$x = 4^{\frac{7}{2}}$ *Definition of a logarithm*

$x = (2^2)^{\frac{7}{2}}$ *4 is a power of 2.*

$x = 2^7$, or 128

D $\log x + \log(x+9) = 1$

$\log x(x+9) = 1$ *Product Property of Logarithms.*

$10^{\log x(x+9)} = 10^1$ *Exponential form*

$x(x+9) = 10$ *Use the inverse properties.*

$x^2 + 9x - 10 = 0$ *Multiply and collect terms.*

$(x-1)(x+10) = 0$ *Factor.*

$x - 1 = 0$ or $x + 10 = 0$ *Set each of the factors equal to zero.*

$x = 1$ or $x = -10$ *Solve.*

Caution!

Watch out for calculated solutions that are not solutions of the original equation.

Check Check both solutions in the original equation.

$\log x + \log(x+9)$	1
$\log 1 + \log(1+9)$	1
$\log 1 + \log 10$	1
$0 + 1$	1
1	1 ✔

$\log x + \log(x+9)$	1
$\log(-10) + \log(-10+9)$	1 ✘

$\log(-10)$ is undefined.

The solution is $x = 1$.

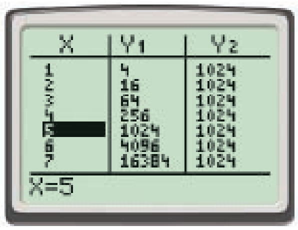
CHECK IT OUT! **3a.** Solve $3 = \log 8 + 3\log x$. **3b.** Solve $2\log x - \log 4 = 0$.

EXAMPLE 4 Using Tables and Graphs to Solve Exponential and Logarithmic Equations and Inequalities

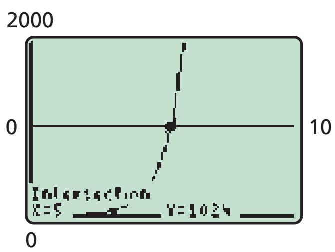
Use a table and graph to solve.

A $2^{2x} = 1024$

Use a graphing calculator. Enter **2^(2X)** as **Y1** and **1024** as **Y2**.



In the table, find the x-value where Y1 and Y2 are equal.



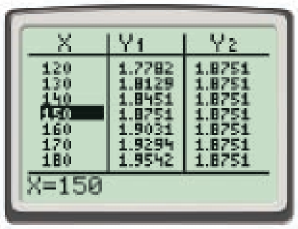
In the graph, find the x-value at the point of intersection.

The solution is $x = 5$.

Use a table and graph to solve.

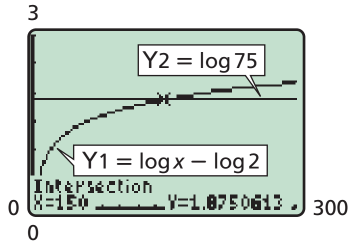
B $\log x - \log 2 \leq \log 75$

Use a graphing calculator. Enter $\log x - \log 2$ as **Y1** and $\log 75$ as **Y2**.



X	Y1	Y2
120	1.7782	1.8751
130	1.8129	1.8751
140	1.8451	1.8751
150	1.8751	1.8751
160	1.9031	1.8751
170	1.9294	1.8751
180	1.9542	1.8751

X=150



In the table, find the x-values where Y1 is less than or equal to Y2.

In the graph, find the x-value at the point of intersection.

The solution set is $\{x \mid 0 < x \leq 150\}$.

Check Use algebra.

$$\log x - \log 2 \leq \log 75$$

$$\log\left(\frac{x}{2}\right) \leq \log 75$$ *Quotient Property of Logarithms*

$$10^{\log\left(\frac{x}{2}\right)} \leq 10^{\log 75}$$ *Use 10 as a base for both sides.*

$$\frac{x}{2} \leq 75$$ *Inverse Property*

$$x \leq 150 \checkmark$$ *log x is only defined for x > 0.*

CHECK IT OUT! **Use a table and graph to solve.**

4a. $2^x = 4^{x-1}$ **4b.** $2^x > 4^{x-1}$ **4c.** $\log x^2 = 6$

THINK AND DISCUSS

1. Explain why a and b must be equal if $\log a = \log b$ $(a > 0, b > 0)$.

2. Give only the first step you would use to solve each equation.

a. $\log x^5 = 10$ **b.** $\log 2x + \log 2 = 1$

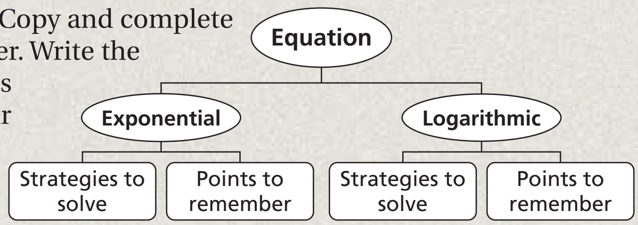
c. $x^4 = 100$ **d.** $\log(x + 1000) = 2$

e. $\log(x + 4) + \log x = 2$ **f.** $\log_6(x + 6) = 3$

3. Explain whether a logarithmic equation can have a negative number as a solution. Justify your answer. Give an example, if possible.

Know it! Note

4. GET ORGANIZED Copy and complete the graphic organizer. Write the strategies and points to remember in your own words for both exponential and logarithmic equations.



7-5 Exercises

California Standards
11.1, 🔑 14.0, 24.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 7-5
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. Vocabulary You can solve a(n) __?__ by taking the logarithm of both sides. (*exponential equation* or *logarithmic equation*)

SEE EXAMPLE 1 p. 522

Solve and check.

2. $4^{2x} = 32^{\frac{1}{2}}$ **3.** $9^x = 3^{x-2}$ **4.** $2^x = 4^{x+1}$

5. $4^x = 10$ **6.** $\left(\frac{1}{4}\right)^{2x} = \left(\frac{1}{2}\right)^x$ **7.** $2.4^{3x+1} = 9$

SEE EXAMPLE 2 p. 523

8. Population The population of a small coastal resort town, currently 3400, grows at a rate of 3% per year. This growth can be expressed by the exponential equation $P = 3400(1 + 0.03)^t$, where P is the population after t years. Find the number of years it will take for the population to exceed 10,000.

SEE EXAMPLE 3 p. 523

Solve.

9. $\log_2(7x + 1) = \log_2(2 - x)$ **10.** $\log_6(2x + 3) = 3$

11. $\log 72 - \log\left(\frac{2x}{3}\right) = 0$ **12.** $\log_3 x^9 = 12$

13. $\log_7(3 - 4x) = \log_7\left(\frac{x}{3}\right)$ **14.** $\log 50 + \log\left(\frac{x}{2}\right) = 2$

15. $\log x + \log(x + 48) = 2$ **16.** $\log\left(x + \frac{3}{10}\right) + \log x + 1 = 0$

SEE EXAMPLE 4 p. 524

Use a table and graph to solve.

17. $2^{2x+1} = 256$ **18.** $2^x 3^x \leq 7776$ **19.** $2\log x^4 = 16$ **20.** $x > 10\log x$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
21–26	1
27	2
28–33	3
34–36	4

Extra Practice
Skills Practice p. S17
Application Practice p. S38

Solve and check.

21. $2^{x-1} = \frac{1}{64}$ **22.** $\left(\frac{1}{4}\right)^x = 8^{x-1}$ **23.** $\left(\frac{1}{5}\right)^{x-2} = 125^{\frac{x}{2}}$

24. $\left(\frac{1}{2}\right)^{-x} = 1.6$ **25.** $(1.5)^{x-1} = 14.5$ **26.** $3^{\frac{x}{2}+1} = 12.2$

27. Pets A veterinarian has instructed Harrison to give his 75 lb dog one 325 mg aspirin tablet for arthritis. The amount of aspirin A remaining in the dog's body after t minutes can be expressed by $A = 325\left(\frac{1}{2}\right)^{\frac{t}{15}}$. Write and solve a logarithmic inequality to find the time it takes for the amount of aspirin to drop below 50 mg.

Solve.

28. $\log_3(7x) = \log_3(2x + 0.5)$ **29.** $\log_2\left(1 + \frac{x}{2}\right) = 4$

30. $\log 5x - \log(15.5) = 2$ **31.** $\log_5 x^4 = 2.5$

32. $\log x - \log\left(\frac{x}{100}\right) = x$ **33.** $2 - \log 3x = \log\left(\frac{x}{12}\right)$

Use a table and graph to solve.

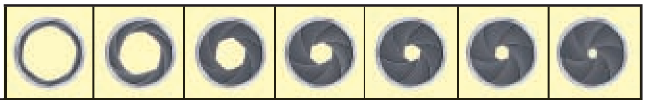
34. $2 \cdot 3^{x-1} = 162$ **35.** $4x < 2^{x+1}$ **36.** $\log(2x - 17) + \log x \geq 2$

37. Solve $\log x = \log(x^2 - 12)$. Explain your answer.

38. Solve $5^{2x} = 100$ to the nearest hundredth.

39. Solve $2^{x+2} = 64$ using more than one method.

40. **Photography** On many cameras, the amount of light admitted through the lens can be controlled by changing the size of the opening, or *aperture*. The size of the aperture is measured as an f-stop setting. The relationship between the f-stop and the amount of light admitted can be represented by the equation $n = \log_2 \frac{1}{\ell}$, where n is the change in f-stop setting from the starting value, f/5.6.



F-stop Setting	f/2	f/2.8	f/4	f/5.6	f/8	f/11	f/16
Change in F-stop Setting	−3	−2	−1	0	1	2	3

a. Solve the equation for ℓ when the f-stop setting is increased to f/16.

b. Solve the equation for n when the light admitted through the lens is twice the amount at f/5.6. What is the f-stop setting? Use a calculator to verify the solution.

LINK

Music

Pianos could be considered both percussion and stringed instruments, with hammers that hit each string. Pianos are usually located near the drums in an orchestra.

41. **Music** The frequency of a note on the piano, in Hz, is related to its position on the keyboard by the function $f(n) = 440 \cdot 2^{\frac{n}{12}}$, where n is the number of keys above or below the note concert A. (A negative value for n means that the key is to the left of, or lower on the keyboard than, concert A.) Find the position n of the key that has a frequency of 110 Hz.

42. **Finance** Suppose that \$250 is deposited into an account that pays 4.5% compounded quarterly. The equation $A = P\left(1 + \frac{r}{4}\right)^n$ gives the amount A in the account after n quarters for an initial investment P that earns interest at a rate r. Solve for n to find how long it will take for the account to contain at least \$500. (*Hint:* Divide both sides by P first.)

43. **Critical Thinking** How many real-number solutions are there for $\log x^2 < 2\log x$? Use a calculator to graph and verify the answer. Explain what the graph indicates about the answer.

44. **ERROR ANALYSIS** When a student solved $\log x + 4 = 8$, he arrived at 99,999,996. Give a possible reason for the error.

45. **Write About It** Describe two methods you can use to solve an exponential equation. Give an example of when you would use each method.

CONCEPT CONNECTION

46. This problem will prepare you for the Concept Connection on page 552.

The number of farms in Iowa (in thousands) can be modeled by $N(t) = 119(0.987)^t$, where t is the number of years since 1980.

a. Has the number of farms in Iowa been increasing or decreasing since 1980? How can you tell?

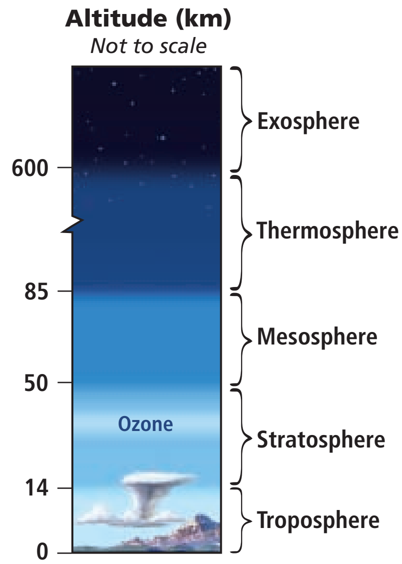
b. Find the number of farms in Iowa in 1980 and 2000.

c. According to the model, when will be the number of farms in Iowa be about 80,000?

47. **Meteorology** In one part of the atmosphere where the temperature is a constant –70°F, pressure can be expressed as a function of altitude by the equation $P(h) = 128(10)^{-0.0682h}$, where P is the atmospheric pressure in kilopascals (kPa) and h is the altitude in kilometers above sea level. The pressure ranges from 2.55 kPa to 22.9 kPa in this region.

   a. What are the lowest and highest altitudes where this model is appropriate? In what part of the atmosphere is the model useful?

   b. **What if...?** A kilopascal is 0.145 psi. Would the model predict a sea-level pressure less than or greater than the actual sea-level pressure, 14.7 psi? Explain.



STANDARDIZED TEST PREP

48. What is the solution of the equation $b^x = c$?

   (A) $x = \dfrac{\log b}{\log c}$ (B) $x = \dfrac{\log c}{\log b}$ (C) $x = \dfrac{\log b}{c}$ (D) $x = \dfrac{\log c}{b}$

49. What is the solution of $\log(x - 21) = 2 - \log x$?

   (F) $x = 4$ (G) $x = \dfrac{25}{4}$ (H) $x = \dfrac{21}{2}$ (J) $x = 25$

50. Which expression has the greatest value when $p = 5$ and $q = 2$?

   (A) $\log 2p - \log 3q$
   (B) $\log p^2 - \log q^3$
   (C) $2\log q - 3\log p$
   (D) $\log p - \log q$

CHALLENGE AND EXTEND

51. If $\log_x x = x$, can the equation be solved for x? Explain.

52. Solve $x = 0.125^{\log_2 5}$ algebraically.

53. For what domain is $\log_3 36 - \log_3 x > 1$? Use a calculator to graph and support your solution.

SPIRAL REVIEW

54. **Photography** It costs \$0.75 to develop an 8-by-10-inch photograph and \$0.35 to develop a 4-by-6-inch photograph. Eli has \$5.25. Use x as the number of 8-by-10-inch photographs and y as the number of 4-by-6-inch photographs. *(Lesson 2-5)*

   a. Write an inequality for the number of each type of photograph Eli can buy.

   b. Graph the inequality. How many 4-by-6-inch photographs can Eli buy if he buys four 8-by-10-inch photographs?

Find the determinant of each matrix. *(Lesson 4-4)*

55. $\begin{bmatrix} 4 & 2 \\ 1 & 7 \end{bmatrix}$ 56. $\begin{bmatrix} -1 & -5 \\ 9 & 10 \end{bmatrix}$ 57. $\begin{bmatrix} \frac{1}{2} & -1 \\ 0 & 6 \end{bmatrix}$ 58. $\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ 6 & 9 \end{bmatrix}$

Use inverse operations to find $f^{-1}(x)$. *(Lesson 7-2)*

59. $f(x) = 4x + 3$ 60. $f(x) = 6(x - 2)$ 61. $f(x) = \dfrac{x}{3} + 9$ 62. $f(x) = \dfrac{7x - 1}{5}$

Connecting Algebra to **Probability**

See Skills Bank page S70

Exponents in Probability

You can use exponents to determine a probability when a certain experiment is repeated.

California Standards

11.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Also covered: **14.0**

Recall that the probability P of an event is $P(\text{Event } E) = \dfrac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$.

For example, when rolling a number cube with six possible outcomes, the probability of rolling an odd prime number, 3 or 5, is $\frac{2}{6}$, or $\frac{1}{3}$. The probability of rolling an odd prime number two rolls in a row is $\frac{1}{3} \cdot \frac{1}{3}$. If the probability of an event is r and the events are independent, then the probability of getting the same result when the event is repeated n times is $P(\text{Event } E \text{ occurring } n \text{ times in succession}) = r^n$

Examples

A machine on an assembly line makes an acceptable product 90% of the time. The machine makes 10 samples of the product.

1 **What is the probability that all 10 samples are acceptable, to the nearest percent?**

$P(\text{All 10 samples are acceptable}) = 0.9^{10}$ *Substitute 0.9 for r and 10 for n in r^n.*

≈ 0.35 *Use a calculator.*

The probability that all 10 samples are acceptable is about 35%.

2 **At what number of samples does the probability fall below 10%?**

You can solve an inequality.

$0.9^n < 0.1$

$\log 0.9^n < \log 0.1$ *Take the log of both sides.*

$n \log 0.9 < \log 0.1$ *Use the Power Property of Logarithms.*

$n > \dfrac{\log 0.1}{\log 0.9}$ *← = −1* *← ≈ −0.0458*

$n > \approx 21.85$

For 22 or more samples, the probability that all are acceptable drops below 10%.

Try This

1. You toss a coin 6 times. Find the probability of getting heads every time.
2. You toss a number cube 10 times. Find the probability that no roll is a six.
3. A basketball player has a 70% chance of making each free throw. For what number of free throws does the probability of making them all drop below 10%?
4. A test contains multiple-choice questions with 4 choices for each question. For what number of questions does the probability of guessing all of them correctly drop below 0.01%?

7-6 Algebra LAB

Prove Laws of Logarithms

The table summarizes the laws of logarithms that you have used in this chapter.

Use with Lesson 7-6

California Standards

11.0 Students prove simple laws of logarithms.
Also covered: **11.2**

Laws of Logarithms
Product Property of Logarithms For any positive numbers m, n, and b $(b \neq 1)$, $\log_b mn = \log_b m + \log_b n$.
Quotient Property of Logarithms For any positive numbers m, n, and b $(b \neq 1)$, $\log_b \frac{m}{n} = \log_b m - \log_b n$.
Power Property of Logarithms For any real numbers p and any positive numbers a and b $(b \neq 1)$, $\log_b a^p = p \log_b a$.

You can use properties of real numbers and properties of exponents to prove these laws of logarithms.

Activity

Prove the Product Property of Logarithms.

1 Identify the related property of exponents.

Logarithms	⟷	Exponents
Product Property of Logarithms $\log_b mn = \log_b m + \log_b n$		Product of Powers Property $a^m a^n = a^{m+n}$

2 Write a proof that uses the definition of a logarithm and the related property of exponents.

Let $\log_b m = x$ and $\log_b n = y$. Then by the definition of a logarithm, $b^x = m$ and $b^y = n$.

Multiplying gives $b^x b^y = mn$, so $b^{x+y} = mn$ by the Product of Powers Property. By the definition of a logarithm, $\log_b mn = x + y$, and by substitution, $\log_b mn = \log_b m + \log_b n$.

Try This

Prove each law of logarithms.

1. Quotient Property of Logarithms

2. Power Property of Logarithms

3. Prove that for any positive numbers a and b $(a \neq 1, b \neq 1)$, $(\log_b a)(\log_a b) = 1$.

4. Prove that if $\log_2(x+5) - \log_2 x = 1$, then $x = 5$.

Tell whether each proof is valid. If the proof is not valid, explain the error.

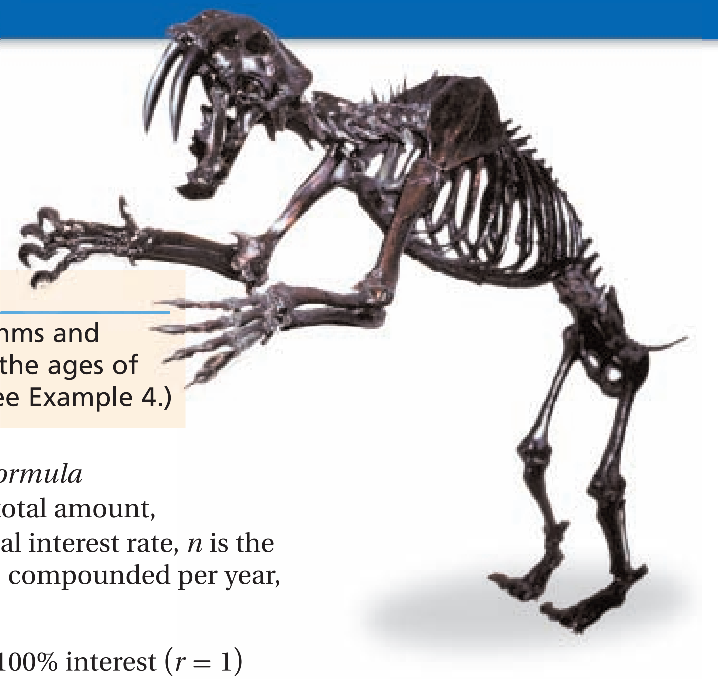
5. Prove that if $e^{\ln x + \ln x} = 4$, then $x = 2$.

It is given that $10^{\log x + \log x} = 16$. By the Product Property of Logarithms, $10^{\log(x+x)} = 16$ or $10^{\log(2x)} = 16$. Therefore, $2x = 16$ by the Inverse Property of Logarithms and Exponents. So $x = 8$ by the Division Property of Equality.

6. Prove that $\log x = \frac{\log_2 x}{\log_2 10}$.

By the Inverse Property of Logarithms and Exponents, $10^{\log x} = x$. Take the base 2 logarithm of both sides to get $\log_2 10^{\log x} = \log_2 x$. By the Power Property of Logarithms, $\log x(\log_2 10) = \log_2 x$, and by the Division Property of Equality, $\log x = \frac{\log_2 x}{\log_2 10}$.

7-6 The Natural Base, *e*



Objectives

Use the number *e* to write and graph exponential functions representing real-world situations.

Solve equations and problems involving *e* or natural logarithms.

Vocabulary

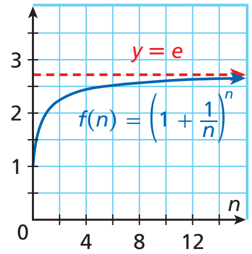
natural logarithm
natural logarithmic function

Why learn this?

Scientists use natural logarithms and carbon dating to determine the ages of ancient bones and fossils. (See Example 4.)

Recall the *compound interest formula* $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the total amount, P is the principal, r is the annual interest rate, n is the number of times the interest is compounded per year, and t is the time in years.

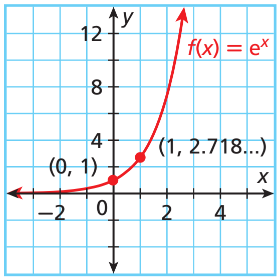
Suppose that \$1 is invested at 100% interest $(r = 1)$ compounded n times for one year as represented by the function $f(n) = \left(1 + \frac{1}{n}\right)^n$.



As n gets very large, interest is *continuously compounded*. Examine the graph of $f(n) = \left(1 + \frac{1}{n}\right)^n$. The function has a horizontal asymptote. As n becomes infinitely large, the value of the function approaches approximately 2.7182818.... This number is called ***e***. Like π, the constant e is an irrational number.

Caution!

The decimal value of e looks like it repeats: 2.718281828.... The value is actually 2.7182818284590.... There is no repeating portion.



Exponential functions with e as a base have the same properties as the functions you have studied. The graph of $f(x) = e^x$ is like other graphs of exponential functions, such as $f(x) = 3^x$.

The domain of $f(x) = e^x$ is all real numbers. The range is $\{y \mid y > 0\}$.

EXAMPLE 1 Graphing Exponential Functions

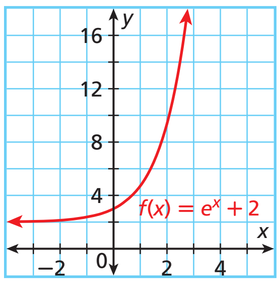
California Standards

🔑 **12.0 Students** know the laws of fractional exponents, **understand exponential functions, and use these functions in problems involving exponential growth and decay.** *Also covered:* **13.0, 11.1**

Graph $f(x) = e^x + 2$.

Make a table. Because e is irrational, the table values are rounded to the nearest tenth.

x	–3	–2	–1	0	1	2	3
$f(x) = e^x + 2$	2.0	2.1	2.4	3	4.7	9.4	22.1

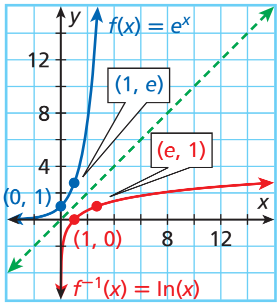


CHECK IT OUT! **1.** Graph $f(x) = e^x - 3$.

A logarithm with a base of e is called a **natural logarithm** and is abbreviated as "ln" (rather than as $\log_e$). Natural logarithms have the same properties as log base 10 and logarithms with other bases.

Know it! Note

The **natural logarithmic function** $f(x) = \ln x$ is the inverse of the natural exponential function $f(x) = e^x$.



The domain of $f(x) = \ln x$ is $\{x \mid x > 0\}$.
The range of $f(x) = \ln x$ is all real numbers.

All of the properties of logarithms from Lesson 7-4 also apply to natural logarithms.

EXAMPLE 2 Simplifying Expressions with *e* or ln

Simplify.

A $\ln e^{-2t}$

$\ln e^{-2t} = -2t$

B $e^{\ln(t-1)}$

$e^{\ln(t-1)} = t - 1$

C $e^{5\ln x}$

$e^{5\ln x} = e^{\ln x^5} = x^5$

CHECK IT OUT! **Simplify.**

2a. $\ln e^{3.2}$ **2b.** $e^{2\ln x}$ **2c.** $\ln e^{x+4y}$

The formula for continuously compounded interest is $A = Pe^{rt}$, where A is the total amount, P is the principal, r is the annual interest rate, and t is the time in years.

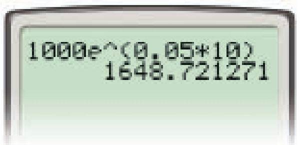
EXAMPLE 3 *Economics Application*

What is the total amount for an investment of $1000 invested at 5% for 10 years compounded continuously?

$A = Pe^{rt}$ — *Substitute 1000 for P, 0.05 for r, and 10 for t.*

$A = 1000e^{0.05(10)}$

$A \approx 1648.72$ — *Use the e^x key on a calculator.*



The total amount is $1648.72.

CHECK IT OUT! **3.** What is the total amount for an investment of $100 invested at 3.5% for 8 years and compounded continuously?

The *half-life* of a substance is the time it takes for half of the substance to breakdown or convert to another substance during the process of decay. Natural decay is modeled by the function below.

N_0 is the initial amount (at $t = 0$). k is the decay constant.

$$N(t) = N_0 e^{-kt}$$

$N(t)$ is the amount remaining. t is the time.

EXAMPLE 4 *Paleontology Application*

A paleontologist uncovers a fossil of a saber-toothed cat in California. He analyzes the fossil and concludes that the specimen contains 15% of its original carbon-14. Carbon-14 has a half-life of 5730 years. Use carbon-14 dating to determine the age of the fossil.

Step 1 Find the decay constant for carbon-14.

$N(t) = N_0 e^{-kt}$	*Use the natural decay function.*
$\frac{1}{2} = 1e^{-k(5730)}$	*Substitute 1 for N_0, 5730 for t, and $\frac{1}{2}$ for $N(t)$, because half of the initial quantity will remain.*
$\ln\frac{1}{2} = \ln e^{-5730k}$	*Simplify and take the ln of both sides.*
$\ln 2^{-1} = -5730k$	*Write $\frac{1}{2}$ as 2^{-1}, and simplify the right side.*
$-\ln 2 = -5730k$	*$\ln 2^{-1} = -1\ln 2 = -\ln 2$*
$k = \frac{\ln 2}{5730} \approx 0.00012$	

Step 2 Write the decay function and solve for t.

$N(t) = N_0 e^{-0.00012t}$	*Substitute 0.00012 for k.*
$15 = 100e^{-0.00012t}$	*Substitute 100 for N_0 and 15 for $N(t)$, since $N(t)$ is 15% of N_0.*
$0.15 = e^{-0.00012t}$	*Divide both sides by 100.*
$\ln 0.15 = \ln e^{-0.00012t}$	*Take the ln of both sides.*
$\ln 0.15 = -0.00012t$	*Simplify.*
$t = -\frac{\ln 0.15}{0.00012} \approx 15{,}809$	

The fossil is approximately 15,800 years old.

CHECK IT OUT!

4. Determine how long it will take for 650 mg of a sample of chromium-51, which has a half-life of about 28 days, to decay to 200 mg.

THINK AND DISCUSS

1. Tell how e and π are alike. Tell how they are different.

2. Explain how e and ln are related.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Fill in each box to compare and contrast the two kinds of logarithms. Give general forms and examples. Simplify, if appropriate.

	Common Logarithms	Natural Logarithms
Base		
Logarithmic Form		
Exponential Form		
$\log_b 1$		
$\log_b b$		
$\log_b b^x$		
$b^{\log_b x}$		

7-6 Exercises

California Standards
9.0, 11.1, 12.0, 13.0, 14.0

GUIDED PRACTICE

1. **Vocabulary** Write the logarithm of a number x to the natural base e as a function of x. This function is called the __?__.

SEE EXAMPLE 1 p. 531

Graph.

2. $f(x) = e^x - 4$
3. $f(x) = -e^x$
4. $f(x) = 4 - e^x$
5. $f(x) = e^{1-x}$

SEE EXAMPLE 2 p. 532

Simplify.

6. $\ln e^1$
7. $\ln e^{x-y}$
8. $\ln e^{\left(-\frac{x}{3}\right)}$
9. $e^{\ln 2x}$
10. $e^{3\ln x}$

SEE EXAMPLE 3 p. 532

11. **Economics** Emma receives \$7750 and invests it in an account that earns 4% interest compounded continuously. What is the total amount of her investment after 5 years?

SEE EXAMPLE 4 p. 533

12. **Physics** Technetium-99m, a radioisotope used to image the skeleton and the heart muscle, has a half-life of about 6 hours. Find the decay constant. Use the decay function $N(t) = N_0 e^{-kt}$ to determine the amount of a 250 mg dose that remains after 24 hours.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
13–16	1
17–20	2
21	3
22	4

Extra Practice
Skills Practice p. S17
Application Practice p. S38

Graph.

13. $f(x) = e^x + 1$
14. $f(x) = e^x - 1$
15. $f(x) = 1 - e^x$
16. $f(x) = 10 - e^x$

Simplify.

17. $\ln e^0$
18. $\ln e^{2a}$
19. $e^{\ln(c+2)}$
20. $e^{4\ln x}$

21. **Economics** Aidan has \$7565 in his checking account. He invests \$5000 of it in an account that earns 3.5% interest compounded continuously. What is the total amount of his investment after 3 years?

22. **Environment** An accident in 1986 at the Chernobyl nuclear plant in the Ukraine released a large amount of plutonium (Pu-239) into the atmosphere. The half-life of Pu-239 is about 24,110 years. Find the decay constant. Use the function $N(t) = N_0 e^{-kt}$ to find what remains of an initial 20 grams of Pu-239 after 5000 years. How long will it take for these 20 grams to decay to 1 gram?

23. **Calculator** Find the approximate values of $\ln 10$ and $\log e$.
    a. How are these numbers related?
    b. How can you use the change of base formula to support your answer?

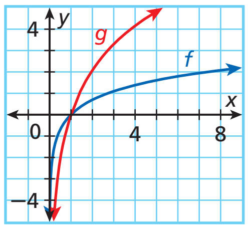
24. Show that $\ln x = \ln 10 \times \log x$.

25. **Multi-Step** Newton's law of cooling states that the temperature of an object decreases exponentially as a function of time, according to $T = T_s + \left(T_0 - T_s\right)e^{-kt}$, where T_0 is the initial temperature of the liquid, T_s is the surrounding temperature, and k is a constant. For a time in minutes, the constant for coffee is approximately 0.283. The corner coffee shop has an air temperature of 70°F and serves coffee at 206°F. Coffee experts say coffee tastes best at 140°F.
    a. How long does it take for the coffee to reach its best temperature?
    b. The air temperature on the patio is 86°F. How long does it take for coffee to reach its best temperature there?
    c. Graph the cooling functions from parts **a** and **b.** Use the graph to find the time it takes for the coffee to cool to 71°F.

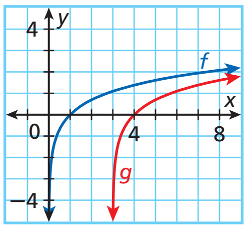
26. Graph the functions $y = \frac{\ln x}{\ln 6}$ and $y = \frac{\log x}{\log 6}$. Explain how the graphs compare with each other and with the graph of $y = \log_6 x$.

Match each transformation of $f(x) = \ln x$ with one of the following graphs.

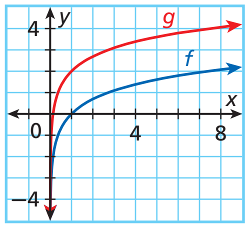
A.



B.



C.



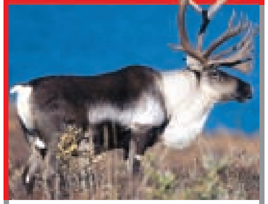
27. $g(x) = \ln(x - 3)$

28. $g(x) = 3\ln x$

29. $g(x) = \ln x + 3$

LINK

Ecology



The George River herd, the largest caribou herd in the world, reached its peak population in 1993 at about 776,000.

30. **Ecology** The George River herd of caribou in Canada was estimated to be about 4700 in 1954 and grew at an exponential rate to about 472,000 in 1984.

a. Use the exponential growth function $P(t) = P_0e^{kt}$, where P_0 is the initial population and $P(t)$ is the population at time t, to determine the growth factor k.

b. **What if...?** If the herd had continued to grow at the same rate, what would its population be in 2010?

Solve.

31. $\ln 5 + \ln x = 1$

32. $\ln 5 - \ln x = 3$

33. $\ln 10 + \ln x^2 = 10$

34. $2\ln x - 2 = 0$

35. $4\ln x - \ln x^4 = 0$

36. $e^{\ln x^3} = 8$

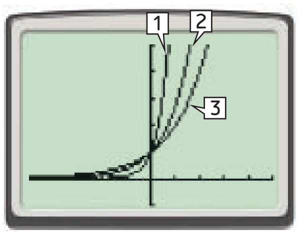
37. **Logistics** A *logistic function*, such as $f(x) = \frac{1}{(1 + e^{-x})}$, can be used to describe the spread of an epidemic in a population.

a. Graph the function.

b. How many asymptotes does the function have?

c. Describe the function in the context of the real-world situation of an epidemic.

38. **Critical Thinking** The graphs of $f(x) = 2^x$, $f(x) = 10^x$, and $f(x) = e^x$ are shown.



a. Identify the graph of each function.

b. Name the coordinates of the point that all three functions have in common.

c. Explain why this point is common to all three functions.

39. **Write About It** Compare compounding interest continuously with compounding daily. How much more is an investment worth when compounding interest continuously? Include an example.

CONCEPT CONNECTION

40. This problem will prepare you for the Concept Connection on page 552.

In 1990, there were 33,500 farms in North Dakota. In 2000, there were 30,800.

a. Find the value of k for the exponential function $N(t) = N_0e^{kt}$ to model the number of farms.

b. Use your model to predict the number of farms in North Dakota in 2010.

c. From 1990 to 2000, the average farm increased from 1209 acres to 1279 acres. Use an exponential model to predict the average farm size in 2010.

STANDARDIZED TEST PREP

41. Which group shows values in the order from least to greatest?

(A) $\log e$, $\ln 10$, $\log 10$, $\ln 1$

(B) $\ln 1$, $\log e$, $\ln 10$, $\log 10$

(C) $\ln 1$, $\log e$, $\log 10$, $\ln 10$

(D) $\ln 1$, $\log 10$, $\ln 10$, $\log e$

42. Which expression is NOT equal to x where $x \neq 0$?

(F) $e^{\ln x}$ (G) $\ln e^{x}$ (H) $x \ln e$ (J) $x + \ln e$

43. Which expression is equal to $\log 50$?

(A) $\ln 50 \div \ln 10$ (B) $\ln(50 \div 10)$ (C) $\ln 50 + \ln 10$ (D) $\ln 50(\ln 10)$

44. **Short Response** Write an expression that is equivalent to $-\ln x$ without using a negative sign.

CHALLENGE AND EXTEND

45. **Finance** For how many compounding periods in a year would the yield of an investment after 1 year at 8% interest be at least 99.9% of the yield if interest were compounded continuously? Does changing the interest rate change the answer? Explain.

46. Graph the function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{x^2}{2}\right)}$. Describe the graph, the domain, and the range.

47. Consider the graph of $f(x) = \ln x$.

a. What function represents the reflection of f across the y-axis?

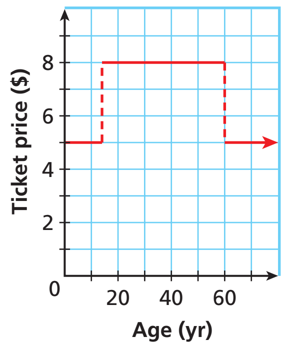
b. What function represents the reflection of f across the x-axis?

c. What function represents the reflection of f across both axes?

d. Graph the function and the three reflections. Name any asymptotes that the four graphs have in common.

SPIRAL REVIEW

48. **Entertainment** The graph shows the price of a movie ticket by age of the viewer. Sketch a graph to represent each situation below, and identify the transformation of the original graph that it represents. *(Lesson 1-8)*



a. Before 5:00 P.M., tickets are half price.

b. The maximum age of viewer for each ticket price is decreased by 3 years.

c. The price for each ticket doubles for a newly released movie.

Write a function that transforms $f(x) = -2x^2 + 3x - 4$ in each of the following ways. Support your solution by using a graphing calculator. *(Lesson 6-8)*

49. Translate up 5 units

50. Translate left 2 units

51. Reflect across the x-axis

52. Stretch horizontally by a factor of 2

Express as a single logarithm. Simplify, if possible. *(Lesson 7-4)*

53. $\log_2 8 + \log_2 \frac{1}{2}$

54. $\log_4 64 - \log_4 1$

55. $\log_3 243 - \log_3 2187$

56. $\log_5 25 + \log_5 125$

57. $\log_8 8 + \log_8 \frac{1}{8}$

58. $\log x^2 - \log x$

7-7 Transforming Exponential and Logarithmic Functions

Objectives

Transform exponential and logarithmic functions by changing parameters.

Describe the effects of changes in the coefficients of exponential and logarithmic functions.

Who uses this?

Psychologists can use transformations of exponential functions to describe knowledge retention rates over time. (See Example 5.)

The hippocampus, in blue, directs the storage of memory in the brain.

You can perform the same transformations on exponential functions that you performed on polynomial, quadratic, and linear functions.

Know it! Note

Transformations of Exponential Functions

Transformation	$f(x)$ Notation	Examples	
Vertical translation	$f(x) + k$	$y = 2^x + 3$	3 units up
		$y = 2^x - 6$	6 units down
Horizontal translation	$f(x - h)$	$y = 2^{x-2}$	2 units right
		$y = 2^{x+1}$	1 unit left
Vertical stretch or compression	$af(x)$	$y = 6(2^x)$	stretch by 6
		$y = \frac{1}{2}(2^x)$	compression by $\frac{1}{2}$
Horizontal stretch or compression	$f\left(\frac{1}{b}x\right)$	$y = 2^{\left(\frac{1}{5}x\right)}$	stretch by 5
		$y = 2^{3x}$	compression by $\frac{1}{3}$
Reflection	$-f(x)$	$y = -2^x$	across x-axis
	$f(-x)$	$y = 2^{-x}$	across y-axis

Helpful Hint

It may help you remember the direction of the shift if you think of "***h*** is for **h**orizontal."

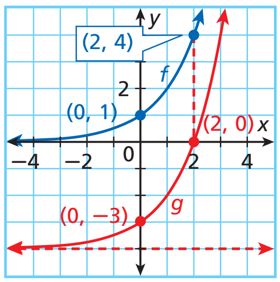
California Standards

12.0 Students know the laws of fractional exponents, **understand exponential functions, and use these functions in problems involving exponential growth and decay.**

EXAMPLE 1 Translating Exponential Functions

Make a table of values, and graph the function $g(x) = 2^x - 4$. Describe the asymptote. Tell how the graph is transformed from the graph of $f(x) = 2^x$.

x	−2	−1	0	1	2	3
$g(x)$	−3.75	−3.5	−3	−2	0	4



The asymptote is $y = -4$, and the graph approaches this line as the value of x decreases. The transformation moves the graph of $f(x) = 2^x$ down 4 units. The range changes to $\{y \mid y > -4\}$.

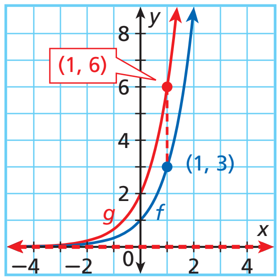
CHECK IT OUT!

1. Make a table of values, and graph $j(x) = 2^{x-2}$. Describe the asymptote. Tell how the graph is transformed from the graph of $f(x) = 2^x$.

EXAMPLE 2 Stretching, Compressing, and Reflecting Exponential Functions

Graph the exponential function. Find the y-intercept and the asymptote. Describe how the graph is transformed from the graph of its parent function.

A $g(x) = 2(3^x)$

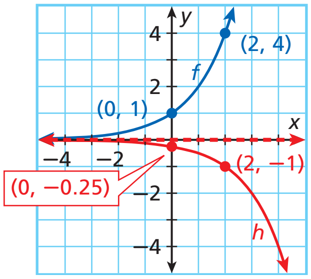


parent function: $f(x) = 3^x$

y-intercept: 2, asymptote: $y = 0$

The graph of $g(x)$ is a vertical stretch of the parent function $f(x) = 3^x$ by a factor of 2.

B $h(x) = -\frac{1}{4}(2^x)$



parent function: $f(x) = 2^x$

y-intercept: $-\frac{1}{4}$, asymptote: $y = 0$

The graph of $h(x)$ is a reflection of the parent function $f(x) = 2^x$ across the x-axis and a vertical compression by a factor of $\frac{1}{4}$. The range is $\{y \mid y < 0\}$.

CHECK IT OUT! **Graph the exponential function. Find the y-intercept and the asymptote. Describe how the graph is transformed from the graph of its parent function.**

2a. $h(x) = \frac{1}{3}(5^x)$ **2b.** $g(x) = 2(2^{-x})$

Remember!

Transformations of $\ln x$ work the same way because $\ln x$ means $\log_e x$.

Because a log is an exponent, transformations of logarithmic functions are similar to transformations of exponential functions. You can stretch, reflect, and translate the graph of the parent logarithmic function $f(x) = \log_b x$.

Examples are given in the table below for $f(x) = \log x$.

Know it! Note

Transformations of Logarithmic Functions

Transformation	$f(x)$ Notation	Examples	
Vertical translation	$f(x) + k$	$y = \log x + 3$	3 units up
		$y = \log x - 4$	4 units down
Horizontal translation	$f(x - h)$	$y = \log(x - 2)$	2 units right
		$y = \log(x + 1)$	1 unit left
Vertical stretch or compression	$af(x)$	$y = 6\log x$	stretch by 6
		$y = \frac{1}{2}\log x$	compression by $\frac{1}{2}$
Horizontal stretch or compression	$f\left(\frac{1}{b}x\right)$	$y = \log\left(\frac{1}{5}x\right)$	stretch by 5
		$y = \log(3x)$	compression by $\frac{1}{3}$
Reflection	$-f(x)$	$y = -\log x$	across x-axis
	$f(-x)$	$y = \log(-x)$	across y-axis

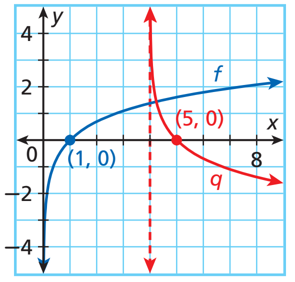
Math Builders

For more on transformations, see the Transformation Builder on page MB4.

EXAMPLE 3 Transforming Logarithmic Functions

Graph each logarithmic function. Find the asymptote. Then describe how the graph is transformed from the graph of its parent function.

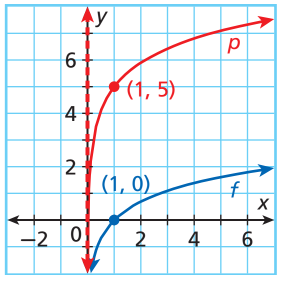
A $q(x) = -\ln(x - 4)$



asymptote: $x = 4$

The graph of $q(x)$ is a translation of the parent function $f(x) = \ln x$ 4 units right and a reflection across the x-axis. The domain is $\{x \mid x > 4\}$.

B $p(x) = 3\log x + 5$



asymptote: $x = 0$

The graph of $p(x)$ is a vertical stretch of the parent function $f(x) = \log x$ by a factor of 3 and a translation 5 units up.

CHECK IT OUT!

3. Graph the logarithmic function $p(x) = -\ln(x + 1) - 2$. Find the asymptote. Then describe how the graph is transformed from the graph of its parent function.

EXAMPLE 4 Writing Transformed Functions

Write each transformed function.

A $f(x) = 0.2^x$ is translated 2 units right, compressed vertically by a factor of $\frac{1}{3}$, and reflected across the x-axis.

$f(x) = 0.2^x$ *Begin with the parent function.*

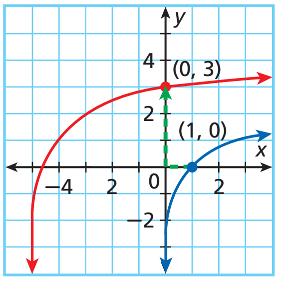
$g(x) = 0.2^{x-2}$ *To translate 2 units right, replace x with x − 2.*

$= \left(-\frac{1}{3}\right)0.2^{x-2}$ *Compress vertically by $\frac{1}{3}$ and reflect across the x-axis.*

B $f(x) = \ln x$ is translated 1 unit left and 3 units up and horizontally stretched by a factor of 5.

$f(x) = \ln\left(\frac{x}{5} + 1\right) + 3$

When you write a transformed function, you many want to graph it as a check.



CHECK IT OUT!

4. Write the transformed function when $f(x) = \log x$ is translated 3 units left and stretched vertically by a factor of 2.

EXAMPLE 5 Problem-Solving Application

PROBLEM SOLVING

A group of students retake the written portion of a driver's test after several months without reviewing the material. A model used by psychologists describes retention of the material by the function $a(t) = 85 - 15\log(t + 1)$, where a is the average score at time t (in months). Describe how the model is transformed from its parent function. Then use the model to predict the number of months when the average score falls below 70.

1 Understand the Problem

The **answers** will be the description of the transformations in $a(t) = 85 - 15\log(t + 1)$ and the number of months when the score falls below 70.

List the important information:

- The model is the function $a(t) = 85 - 15\log(t + 1)$.
- The function is a transformation of $f(t) = \log(t)$.
- The problem asks for t when $a < 70$.

2 Make a Plan

Rewrite the function in a more familiar form, and then use what you know about the effect of changing the parent function to describe the transformations. Substitute known values into $a(t) = 85 - 15\log(t + 1)$, and solve for the unknown.

3 Solve

Rewrite the function, and describe the transformations.

$$a(t) = 85 - 15\log(t + 1)$$

$$a(t) = -15\log(t + 1) + 85 \qquad \textit{Commutative Property}$$

The graph of $f(t) = \log(t)$ is reflected across the x-axis, vertically stretched by a factor of 15, and translated 85 units up and 1 unit left. The domain $\{t \mid t \geq 0\}$ makes sense in the problem.

Find the time when the average score drops below 70.

$$70 > -15\log(t + 1) + 85 \qquad \textit{Substitute 70 for a(t) and replace = with >.}$$

$$-15 > -15\log(t + 1) \qquad \textit{Subtract 85 from both sides.}$$

$$1 < \log(t + 1) \qquad \textit{Divide by −15, and reverse the inequality symbol.}$$

$$10^1 < t + 1 \qquad \textit{Change to exponential form.}$$

$$9 < t$$

The model predicts a score below 70 after 9 months.

4 Look Back

It is reasonable that scores would drop from 85 to below 70 nine months after the students take the test without reviewing the material.

CHECK IT OUT!

5. What if...? When would the average score drop to 0? Is your answer reasonable?

THINK AND DISCUSS

1. Describe the domain of $f(x) = \log_b(-x)$.

2. Explain how the process of transforming exponential and logarithmic functions is similar to transforming quadratic functions.

3. Tell which transformations of $f(x) = a^x$ change the domain or range. Tell which transformations of $f(x) = \log_b x$ change the domain or range. Are these transformations the same?

Know it! Note

4. GET ORGANIZED Copy and complete the graphic organizer. Give an example of an indicated transformation for both types of exponential and logarithmic functions. Remember, e is a constant.

Transformation	$f(x) = 5^x$ $f(x) = e^x$	$f(x) = \log_b x$ $f(x) = \ln x$
Vertical translation		
Horizontal translation		
Reflection		
Vertical stretch		
Vertical compression		

7-7 Exercises

California Standards
10.0, 11.1, 12.0, 15.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 7-7
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 537

Make a table of values, and graph each function. Describe the asymptote. Tell how the graph is transformed from the graph of $f(x) = 3^x$.

1. $g(x) = 3^x + 2$ **2.** $h(x) = 3^x - 2$ **3.** $j(x) = 3^{x+1}$

SEE EXAMPLE 2 p. 538

Graph each exponential function. Find the y-intercept and the asymptote. Describe how the graph is transformed from the graph of its parent function.

4. $g(x) = 3(4^x)$ **5.** $h(x) = \frac{1}{3}(4^x)$ **6.** $j(x) = -\frac{1}{3}(4^x)$

7. $k(x) = -2(4^x)$ **8.** $m(x) = -(4^{-x})$ **9.** $n(x) = e^{2x}$

SEE EXAMPLE 3 p. 539

Graph each logarithmic function. Find the asymptote. Then describe how the graph is transformed from the graph of its parent function.

10. $g(x) = 2.5\log x$ **11.** $h(x) = 2.5\log(x + 3)$ **12.** $j(x) = -\frac{1}{3}\ln x + 1.5$

SEE EXAMPLE 4 p. 539

Write each transformed function by using the given parent function and the indicated transformations.

13. The parent exponential function $f(x) = 0.7^x$ is horizontally stretched by a factor of 3, reflected across the x-axis, and translated 2 units left.

14. The parent logarithmic function $f(x) = \log x$ is translated 12 units right, vertically compressed by a factor of $\frac{1}{2}$, and translated 25 units up.

SEE EXAMPLE 5 p. 540

15. Forestry The height of a poplar tree in feet, at age t years can be modeled by the function $h(t) = 6 + 3\ln(t + 1)$. Describe how the model is transformed from its parent function. Then use the model to predict the number of years when the height will exceed 17 feet.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–18	1
19–24	2
25–27	3
28–30	4
31	5

Extra Practice

Skills Practice p. S17
Application Practice p. S38

Make a table of values, and graph each function. Describe the asymptote. Tell how the graph is transformed from the graph of $f(x) = 5^x$.

16. $g(x) = 5^x - 1$ **17.** $h(x) = 5^{x+2}$ **18.** $j(x) = 5^{x-1} - 1$

Graph each exponential function. Find the *y*-intercept and the asymptote. Describe how the graph is transformed from the graph of its parent function.

19. $g(x) = 4\left(\frac{1}{2}\right)^x$ **20.** $h(x) = 0.25\left(\frac{1}{2}\right)^x$ **21.** $j(x) = -0.25\left(\frac{1}{2}\right)^x$

22. $k(x) = -\left(\frac{1}{2}\right)^{\frac{x}{2}}$ **23.** $m(x) = 4\left(\frac{1}{2}\right)^{-x}$ **24.** $n(x) = -4\left(\frac{1}{2}\right)^{-x}$

Graph each logarithmic function. Find the asymptote. Describe how the graph is transformed from the graph of its parent function.

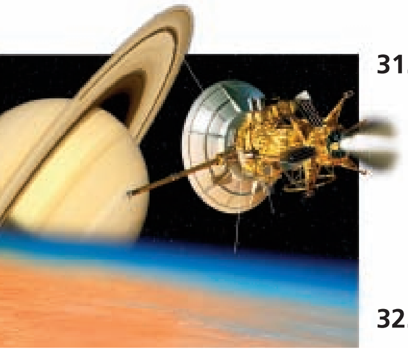
25. $g(x) = \ln(x - 5)$ **26.** $h(x) = \frac{4}{5}\log(x + 3) - 2$ **27.** $m(x) = -4\log x$

Write each transformed function.

28. The function $f(x) = \left(\frac{1}{2}\right)^x$ is translated 4 units right, reflected across the *x*-axis, and vertically stretched by a factor of 1.5.

29. The function $f(x) = \ln x$ is translated 3 units left, horizontally compressed by a factor of $\frac{1}{4}$ and translated 0.5 units down.

30. The function $f(x) = e^x$ is horizontally stretched by a factor of 3, reflected across the *y*-axis, and translated 1 unit right.



31. Space Electric power for the *Cassini* spacecraft is provided by the decay of plutonium-238 contained inside its generators. The power output of the generators in watts (W) is modeled by $P(t) = 870e^{-\frac{t}{127}}$, where t is the number of years since the manufacture date. Describe how the model is transformed from its parent function. Suppose the instruments on *Cassini* require at least 600 W to function. Use the model to predict the number of years that the instruments on *Cassini* will function properly.

32. Critical Thinking What vertical transformation of $f(x) = e^x$ is equivalent to the horizontal translation $g(x) = e^{x+2}$? Write the transformed function.

For Exercises 33–38, match each order of transformation of $f(x) = e^x$ with its transformed function.

33. stretch by a factor of 2, reflect across the *x*-axis, and translate 5 units down.

34. stretch by a factor of 2, translate 5 units down, and reflect across the *x*-axis

35. reflect across the *x*-axis, stretch by a factor of 2, and translate 5 units down.

36. reflect across the *x*-axis, translate 5 units down, and stretch by a factor of 2.

37. translate 5 units down, stretch by a factor of 2, and reflect across the *x*-axis.

38. translate 5 units down, reflect across the *x*-axis, and stretch by a factor of 2.

A. $g(x) = -2e^x - 5$

B. $g(x) = 2[-(e^x - 5)]$

C. $g(x) = 2(-e^x - 5)$

D. $g(x) = 2(-e^x) -5$

E. $g(x) = -(2e^x - 5)$

F. $g(x) = -2(e^x - 5)$

39. Which, if any, of the functions in A–F above are equivalent?

Tell whether each statement is sometimes, always, or never true.

40. A vertical translation of $f(x) = \log x$ changes its asymptote.

41. A vertical translation of $f(x) = e^x$ changes its asymptote.

42. A horizontal translation of $f(x) = \log x$ has a range of $\mathbb{R}$.

43. The graph of a transformation of $f(x) = \ln x$ intersects the graph of $f(x)$.

44. **Banking** The function $A(t) = 1000\left(1 + \frac{r}{n}\right)^{nt}$ can be used to calculate the growth of an investment of \$1000 in an account where the interest is compounded n times per year at an annual rate r. Suppose that you invest \$1000 in such an account compounded quarterly (4 times per year).

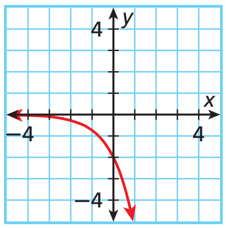
a. What annual rate would double your investment in 5 years?

b. At an annual rate of 3.5%, how long (to the nearest year) would it take for your investment to double?

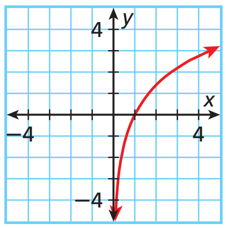
c. What does the model predict for the amount in the account after 10 years if the investment continues to grow at an annual rate of 3.5%?

Match each equation with one of the following graphs.

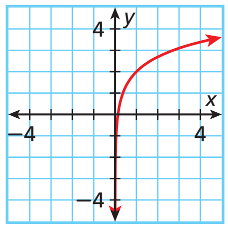
A.



B.



C.



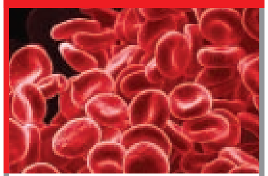
45. $f(x) = \ln x + 2$

46. $f(x) = -2e^x$

47. $f(x) = 2\ln x$

LINK

Medicine



Insulin is a hormone that allows the body's cells, including red blood cells, to absorb glucose. Synthetic insulin can help diabetics maintain a healthy level of blood sugar.

48. **Medicine** A dose of synthetic insulin breaks down in the bloodstream over time. The amount of insulin in the blood with an initial dose of A_0 mg, under some conditions, is given by $A = A_0 0.97^t$, where t is the time in minutes. The standard dose is 10 mg. Describe each transformation.

a. The initial dose is changed from 10 mg to 20 mg.

b. The breakdown of the medicine does not begin for 5 minutes.

c. The breakdown time period is increased from one-minute intervals to two-minute intervals.

d. **What if...?** The breakdown rate of 0.97 is reduced to 0.95. Is this a transformation of A?

49. **Critical Thinking** Describe how changing the value of h and changing the value of k differ in the effect on the graph of $f(x) = a\left(b^{x-h}\right) + k$.

50. **Write About It** Explain how to translate, reflect, stretch, and compress the graph of $f(x) = b^x$.

Concept Connection

51. This problem will prepare you for the Concept Connection on page 552.

The number of farms in a county is modeled by $N(t) = 1257(0.99)^t$, where t is the number of years since 1990.

a. One-third of the farms in the county always produce soybeans. Write a new function that models the number of soybean farms.

b. Write a new function that gives the number of soybean farms m months after January 1, 1990.

c. How many soybean farms were there at the end of May 1991?

STANDARDIZED TEST PREP

52. Which function is vertically stretched by a factor of 3 and translated 2 units left from its parent function?

Ⓐ $f(x) = 3(2^{x+2})$

Ⓑ $f(x) = 2^{3x} - 2$

Ⓒ $f(x) = 3\log(x - 2)$

Ⓓ $f(x) = \log(3x + 2)$

53. Which list shows the functions in order from the most compressed horizontally from $f(x) = \log x$ to the most stretched?

Ⓕ $f(x) = \log(x - 10)$, $f(x) = \log(10x)$, $f(x) = \log\left(\frac{x}{10}\right)$

Ⓖ $f(x) = \log\left(\frac{x}{10}\right)$, $f(x) = \log(x - 10)$, $f(x) = \log(10x)$

Ⓗ $f(x) = \log(10x)$, $f(x) = \log(x - 10)$, $f(x) = \log\left(\frac{x}{10}\right)$

Ⓙ $f(x) = \log\left(\frac{x}{10}\right)$, $f(x) = \log(10x)$, $f(x) = \log(x - 10)$

54. The trade-in value of Cindy's car is \$4500 and decreases by about 40% per year. Which choice represents the trade-in value as a function of time?

Ⓐ $f(t) = 4500(0.4)^t$

Ⓑ $f(t) = 4500(0.6)^t$

Ⓒ $f(t) = 0.6(4500)^t$

Ⓓ $f(t) = 0.4(4500)^t$

CHALLENGE AND EXTEND

55. **Critical Thinking** Consider the function $f(x) = \log x$.

a. Identify the transformation applied to $f(x)$ to create $g(x) = \log x + 1$.

b. Identify the transformation applied to $f(x)$ to create $h(x) = \log(10x)$.

c. Use a graphing calculator to compare the graphs and tables of both functions. What do you notice?

d. Use the properties of logarithms to explain your answer to part **c.**

56. **Graphing Calculator** Graph the function $y = -\ln(x + 2)$ in the standard window. Make a conjecture explaining why the calculator screen appears to show that the graph stops abruptly and does not extend infinitely in two directions.

57. What can you say about the value of $f(x)$ as the value of x gets closer and closer to h in the standard equation for a logarithmic function, $f(x) = a\log(x - h) + k$, given that a, h, and $k \geq 0$?

SPIRAL REVIEW

Find the minimum or maximum value. Then state the domain and range of the function. *(Lesson 5-2)*

58. $f(x) = x^2 - 2x + 5$ **59.** $f(x) = 4x^2 + x - 5$ **60.** $f(x) = -x^2 - x + 1$

61. The table shows the volumes of containers of varying heights. Write a cubic function to model the volume. *(Lesson 6-9)*

Height (in.)	5	7	10	12	15
Volume (qt)	5.5	12.5	33.5	56.8	109.5

Simplify each expression. *(Lesson 7-6)*

62. $\ln e^{x+2}$ **63.** $\ln e^{-5x}$ **64.** $e^{\ln(x-1)}$ **65.** $e^{\ln\left(\frac{x}{4}\right)}$

7-8 Curve Fitting with Exponential and Logarithmic Models

Objectives
Model data by using exponential and logarithmic functions.

Use exponential and logarithmic models to analyze and predict.

Vocabulary
exponential regression
logarithmic regression

Who uses this?
Gem cutters know that values of precious stones of similar quality are exponentially related to the gems' weights. (See Example 2.)

Analyzing data values can identify a pattern, or repeated relationship, between two quantities.

Look at this table of values for the exponential function $f(x) = 2(3^x)$.

Remember!
For linear functions (first degree), first differences are constant. For quadratic functions, second differences are constant, and so on.

x	-1	0	1	2	3
$f(x)$	$\frac{2}{3}$	2	6	18	54

$\times 3 \quad \times 3 \quad \times 3 \quad \times 3$

Notice that the *ratio* of each *y*-value and the previous one is constant. Each value is three times the one before it, so the ratio of function values is constant for equally spaced *x*-values. This data can be fit by an exponential function of the form $f(x) = ab^x$.

EXAMPLE 1 Identifying Exponential Data

Determine whether f is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.

California Standards
12.0 Students know the laws of fractional exponents, **understand exponential functions, and use these functions in problems involving exponential growth and decay.**

A

x	-1	0	1	2	3
$f(x)$	-3	-1	1	3	5

$+2 \quad +2 \quad +2 \quad +2$ *First differences*

y is a linear function of x.

B

x	-1	0	1	2	3
$f(x)$	$\frac{1}{2}$	1	2	4	8

$+\frac{1}{2} \quad +1 \quad +2 \quad +4$

Ratios $\frac{1}{\frac{1}{2}} = \frac{2}{1} = \frac{4}{2} = \frac{8}{4} = 2$

This data set is exponential, with a constant ratio of 2.

CHECK IT OUT! **Determine whether y is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.**

1a.

x	-1	0	1	2	3
$f(x)$	$2.\overline{6}$	4	6	9	13.5

1b.

x	-1	0	1	2	3
$f(x)$	-3	2	7	12	17

In Chapters 2 and 5, you used a graphing calculator to perform *linear regressions* and *quadratic regressions* to make predictions. You can also use an *exponential model,* which is an exponential function that represents a real data set.

Once you know that data are exponential, you can use **ExpReg** (exponential regression) on your calculator to find a function that fits. This method of using data to find an exponential model is called an **exponential regression**. The calculator fits exponential functions to ab^x, so translations cannot be modeled.

EXAMPLE 2 *Gemology Application*

The table gives the approximate values of diamonds of the same quality. Find an exponential model for the data. Use the model to estimate the weight of a diamond worth $2325.

Diamond Values

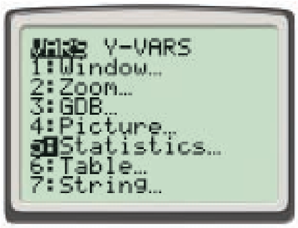
Weight (carats)	Value ($)
0.5	920
1.0	1160
2.0	1580
3.0	2150
4.0	2900

Step 1 Enter the data into two lists in a graphing calculator. Use the exponential regression feature.

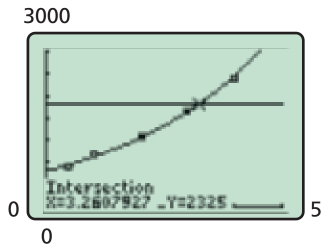
An exponential model is $V(w) \approx 814.96(1.38)^w$, where V is the diamond value and w is the weight in carats.

Step 2 Graph the data and the function model to verify that it fits the data.

To enter the regression equation as **Y1** from the **Y=** screen, press **VARS**, choose **5:Statistics**, press **ENTER**, scroll to the **EQ** menu and select **1:RegEQ.**



Enter 2325 as **Y2**. Use the intersection feature. You may need to adjust the window dimensions to find the intersection.



A diamond weighing about 3.26 carats will have a value of $2325.

> **Remember!**
> If you do not see r^2 and **r** when you calculate regression, use **2nd** **0** (CATALOG) and turn these features on by selecting **DiagnosticOn**.

CHECK IT OUT!

2. Use exponential regression to find a function that models this data. When will the number of bacteria reach 2000?

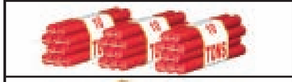
Time (min)	0	1	2	3	4	5
Bacteria	200	248	312	390	489	610

Many natural phenomena can be modeled by natural log functions. You can use a **logarithmic regression** to find a function.

EXAMPLE 3 ***Physics Application***

The table gives the Richter scale equivalent for an explosion involving a quantity of TNT. Find a natural log model for the data. Use the model to estimate the number of tons of TNT that would be the equivalent of an earthquake measuring 6.5 on the Richter scale.

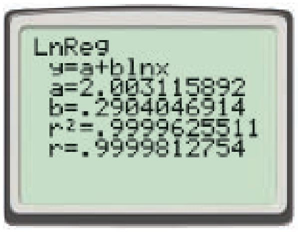
Richter Scale TNT Equivalence

TNT (tons)	Magnitude
	2.0
	3.0
	4.0
	5.0

Helpful Hint

Most calculators that perform logarithmic regression use ln rather than log.

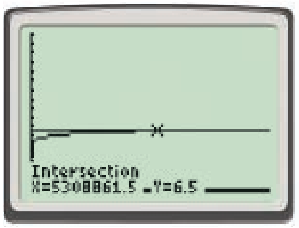
Enter the data into two lists in a graphing calculator. Then use the logarithmic regression feature. Press **STAT** **CALC 9:LnReg.** A logarithmic model is $R(t) \approx 2 + 0.29 \ln t$, where R is the Richter scale reading and t is the equivalent number of tons of TNT.



The calculated value of r^2 shows that the function fits the data.

Graph the data and function model to verify that it fits the data.

Use the intersection feature to find x when y is 6.5. The TNT equivalent of an earthquake measuring 6.5 on the Richter scale is about 5.3 million tons.



CHECK IT OUT!

3. Use logarithmic regression to find a function that models this data. When will the speed reach 8.0 m/s?

Time (min)	1	2	3	4	5	6	7
Speed (m/s)	0.5	2.5	3.5	4.3	4.9	5.3	5.6

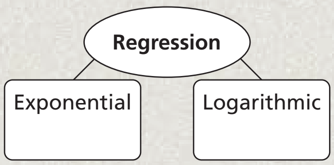
THINK AND DISCUSS

1. Explain how you can determine whether or not a data set can be fit by an exponential function of the form $f(x) = ab^x$.

2. Explain why having only two data points is not enough to tell you whether the data set is exponential or logarithmic.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Show the procedures and tools for finding an exponential or logarithmic model.



7-8 Exercises

California Standards
1.0, 8.0, 12.0, 14.0

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Homework Help Online
KEYWORD: MB7 7-8
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** __?__ is useful when data can be modeled by a function of the form $f(x) = ab^x$. (*Exponential regression* or *Logarithmic regression*)

SEE EXAMPLE 1 p. 545

Determine whether f is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.

2.

x	–1	0	1	2	3
$f(x)$	$-2\frac{5}{7}$	–1	11	95	683

3.

x	–1	0	1	2	3
$f(x)$	27	18	12	8	$5\frac{1}{3}$

4.

x	–1	0	1	2	3
$f(x)$	5	1	–3	–7	–11

5.

x	–1	0	1	2	3
$f(x)$	$2\frac{1}{4}$	3	4	$5\frac{1}{3}$	$7\frac{1}{9}$

SEE EXAMPLE 2 p. 546

6. **Physics** The table gives the approximate number of degrees Fahrenheit above room temperature of a cup of tea as it cools. Find an exponential model for the data. Use the model to estimate how long it will take the tea to reach a temperature that is less than 40 degrees above room temperature.

Cooling Tea

Time (min)	0	1	2	3	4
Degrees above room temperature (°F)	132	120	110	101	93

SEE EXAMPLE 3 p. 547

7. **Community** The table shows the population milestones for a small town following its incorporation. Find a natural log model for the data. Use the model to predict how long it will take for the population to reach 8000.

Town Population Milestones

Time (mo)	6	18	42	90	150
Population	3000	4000	5000	6000	7000

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
8–11	1
12	2
13	3

Extra Practice
Skills Practice p. S17
Application Practice p. S38

Determine whether f is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.

8.

x	–1	0	1	2	3
$f(x)$	1.25	1	0.75	0.5	0.25

9.

x	–5	–3	1	3	5
$f(x)$	20	6	2	12	30

10.

x	–1	0	1	2	3
$f(x)$	0.667	1	1.5	2.25	3.375

11.

x	–1	0	1	2	3
$f(x)$	–16	–8	–4	–2	–1

12. **Social Studies** The table gives the United States Hispanic population from 1980 to 2000. Find an exponential model for the data. Use the model to predict when the Hispanic population will exceed 120 million.

United States Hispanic Population

Years After 1970	10	20	30
Population (millions)	14.6	22.5	35.3

Source: Census 2000

13. **Telecommunication** The table gives the number of telecommuters in the United States from 1990 to 2000. Find an exponential model for the data. Use the model to estimate when the number of telecommuters will exceed 100 million.

U.S. Telecommuters											
Years After 1990	0	1	2	3	4	5	6	7	8	9	10
Telecommuters (millions)	4.4	5.5	6.6	7.3	9.1	8.5	8.7	11.1	15.7	19.6	23.6

Source: Federal Highway Administration

14. **Ecology** Data on an endangered crane species indicate that their numbers are increasing. The table shows the population size over the last 55 years. Find a logarithmic model for the data. Predict the year when the population will reach 500.

Crane Population					
Population Size	18	40	85	120	185
Years Since 1940	5	22	40	47	57

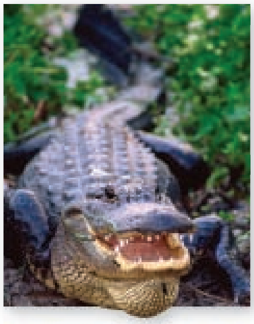
Decide whether the data set is exponential, and if it is, use exponential regression to find a function that models the data.

15.

x	1	2	3	4
$f(x)$	11	95	683	4799

16.

x	−1	0	2	3
$f(x)$	4	2	0.5	0.25

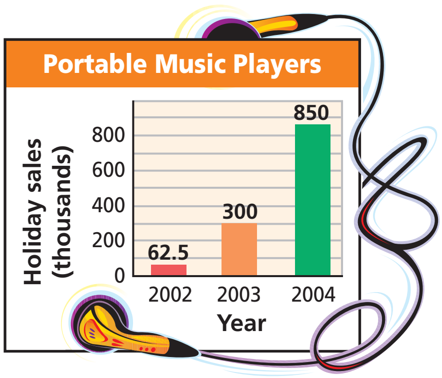


American alligator, Everglades National Park

17. **Critical Thinking** According to one source, the population of nesting wading birds in the wetlands of the Florida Everglades Park System has decreased from more than a half-million in the 1930s to less than 15,000 today. What do you need to know to determine whether this decrease in numbers is exponential? Explain.

18. **Ecology** One research study showed that the rate of calf survival in Yellowstone elk herds depends on spring snow depths. At snow depths of about 5000 mm, the rate of survival is about 0.9 per hundred cows; at 6700 mm it is about 0.3; and at 8250 mm, it is about 0.17. Find an exponential function to model the data. Use the model to predict the calf survival rate per hundred cows at snow depths of 4000 mm.

19. **Technology** Holiday season sales of a portable digital music player are shown in the graph. Assume that growth rate continues in the same way. Write an exponential function to model the data. Use the model to predict sales in three years.



20. **Data Collection** Use a graphing calculator and a temperature probe to measure the temperature of a refrigerated liquid from the time it is taken from refrigeration. Use the list feature to subtract the temperatures from room temperature. Find a model for the difference from room temperature over time. Describe the model and explain why you chose it.

21. **Make a Conjecture** Make a table of values for an exponential function with $x = 1, 2, 3, \ldots 8$. Find the first differences, second differences, and third differences. Make a conjecture about the nth differences, assuming that the domain of the function is all natural numbers.

Concept Connection

22. This problem will prepare you for the Concept Connection on page 552.

The table shows the total amount of farmland in Vermont since 1970.

a. Use exponential regression to find a function that models the data.

b. According to the model, by what percent does the amount of farmland decrease each year?

c. Predict the amount of farmland in 2010.

Farmland in Vermont

Year	Farmland (thousands of acres)
1970	2010
1980	1740
1990	1440
2000	1270

23. **Recreation** At the Autosport Show in Birmingham, England, in January 2001, karting champion Stuart Ziemelis demonstrated an electric race kart with a top speed of over 100 mi/h and acceleration from 0-to-60 mi/h in less than 4 s. The *difference* between the race kart's speed S and its top speed can be modeled by $(100 - S) = 100(0.795)^t$, where t is the time in seconds after the start.

a. Predict the electric race kart's speed at 1, 2, and 8 s.

b. Use your answers to part **a** to verify that the speed is an exponential function of time. Use exponential regression to find the function that models the speed.

24. **Write About It** Describe how to tell whether data is exponential rather than linear, quadratic, or cubic.

25. **Biology** The number of fronds of duckweed present during an experiment are given in the table.

Day	0	2	3	4	5	6
Fronds	18	32	43	57	76	101

a. Which fits the data better, an exponential function or a linear function?

b. Enter the day numbers in L1 and the logarithm of each number of fronds in L2 (use either log or ln). Which fits this data better, an exponential function or a linear function? Why?

Standardized Test Prep

26. Which situation can be modeled by an exponential function?

(A) A cost that increases by $100 each month

(B) The area of a square as the length increases by increments of 10 cm

(C) The radius of a spiral that gets 10% larger with each rotation

(D) A population that doubles as the time doubles

27. Which data set is exponential?

(F) (0, 0.1), (1, 0.5), (2, 2.5), (3, 12.5)

(G) (0, –1), (1, 0), (2, 7), (3, 20)

(H) (0, –1), (1, 0.5), (2, 2), (3, 3.5)

(J) (0, –1), (1, 2), (2, 11), (3, 26)

28. **Gridded Response** Find the missing value if f is an exponential function.

x	0	1	2	3
y	2	3.5	■	10.71875

CHALLENGE AND EXTEND

29. Find an exponential function that goes through the points $(2, 48)$ and $(4, 300)$. Show your work.

30. **Environment** Helena works in a chemistry laboratory. Due to equipment failure, she may have inhaled toxic fumes. Five hours after the incident, a blood sample shows a toxin concentration of 0.01006 mg/cm^3. Two hours later, another sample detects a concentration of 0.00881 mg/cm^3. Assume that concentration varies exponentially with time.

a. Write an exponential function to model the data.

b. There is a health risk if the toxin concentration was as high as 0.015 mg/cm^3. Was the initial concentration above this level?

c. Helena can return to work when the concentration drops below 0.00010 mg/cm^3. How many hours (to the nearest hour) after the incident will this be?

31. The calculator uses logarithms to fit data to exponential and logarithmic functions. Determine what domains or ranges of data cause the calculator to get an error when using the exponential regression and logarithmic regression functions.

SPIRAL REVIEW

Solve. *(Lesson 5-3)*

32. $|-5x| = 45$ **33.** $|x + 4| = 0$ **34.** $|2x - 4| = 3$ **35.** $2|2x| + 1 = 10$

Find the zeros of each function by factoring. *(Lesson 5-3)*

36. $f(x) = x^2 + 2x - 3$ **37.** $f(x) = 3x^2 + 24x$

38. $f(x) = 2x^2 + 10x + 12$ **39.** $f(x) = x^2 + 9x - 36$

Solve and check. *(Lesson 7-5)*

40. $\frac{1}{64} = 4^{x+5}$ **41.** $81^x = 3^{x+4}$ **42.** $8^{\frac{x}{3}} = \left(\frac{1}{2}\right)^{x+2}$ **43.** $216^x = 6^{2x}$

Career Path

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Colleen Murray
Real estate agent

Q: What high school math classes did you take?

A: Algebra 1, Geometry, Business Math, and Algebra 2.

Q: How did you become a real estate agent?

A: After high school, I took an online training course in real estate. Then I had to pass a state license exam.

Q: How is math used in real estate?

A: We calculate house prices, interest rates, payments, taxes, closing costs, commissions, and other fees. I use geometry to calculate areas and formulas to convert between units of measurement, such as square feet to acres.

Q: What are your future plans?

A: I may look into becoming a broker. Then I can supervise other agents and manage my own office.

CHAPTER 7

SECTION 7B

CONCEPT CONNECTION

Applying Exponential and Logarithmic Functions

Down on the Farm According to data from the U.S. Department of Agriculture, the number of farms in the United States has been decreasing over the past several decades. During this time, however, the average size of each farm has increased.

1. From 1940 to 1980, the average size A of a U.S. farm can be modeled by the function $A(t) = 174e^{0.022t}$, where t is the number of years since 1940. What was the average farm size in 1940? in 1980?

2. In what year did the average farm size reach 250 acres?

3. During the period from 1940 to 1980, how many years did it take for the average farm size to double?

4. The table shows the number of farms in the United States since 1940. Find an exponential model for the data.

Farms in the United States

Year	Number of Farms (millions)
1940	6.35
1950	5.65
1960	3.96
1970	2.95
1980	2.44
1990	2.15
2000	2.17

5. Predict the number of farms in the United States in 2010.

6. According to your model, how many years does it take for the number of farms to decrease by 50%?

7. According to your model, when will the number of farms in the United States fall below 1 million?

READY TO GO ON?

Quiz for Lessons 7-5 Through 7-8

7-5 Exponential and Logarithmic Equations and Inequalities

Solve.

1. $3^x = \frac{1}{27}$ **2.** $49^{x+4} < 7^{\frac{x}{2}}$ **3.** $13^{3x-1} = 91$ **4.** $2^{x+4} = 20$

5. $\log_4(x-1) \geq 3$ **6.** $\log_2 x^{\frac{1}{3}} = 5$ **7.** $\log 16x - \log 4 = 2$

8. $\log x + \log(x+3) = 1$

9. Suppose that you deposit $500 into an account that pays 3.5% compounded quarterly. The equation $A = P\left(1 + \frac{r}{4}\right)^n$ gives the amount A in the account after n quarters for an initial investment of P that earns interest at a rate of r. Use logarithms to solve for n to find how long it will take for the account to contain at least $2000.

7-6 The Natural Base, *e*

Graph.

10. $f(x) = e^x + 3$ **11.** $f(x) = 3 - e^x$ **12.** $f(x) = \frac{e^x}{3}$ **13.** $f(x) = 3(e^x - 1)$

Simplify.

14. $\ln e^2$ **15.** $\ln e^{\frac{x}{2}}$ **16.** $e^{\ln(1-3a)}$ **17.** $\ln e^{b+5}$

18. Carbon-14 is a useful dating tool for specimens between 500 and 25,000 years old, such as ancient manuscripts and artifacts. Carbon-14's half-life is 5730 years.

a. Use the formula $\frac{1}{2} = e^{-kt}$ to find the value of the decay constant for carbon-14.

b. Use the decay function $N_t = N_0 e^{-kt}$ to determine how much of 10 grams of carbon-14 will remain after 1000 years.

7-7 Transforming Exponential and Logarithmic Functions

Graph the function. Find the *y*-intercept and asymptote. Describe how the graph is transformed from the graph of the parent function.

19. $g(x) = 1.5(3^x)$ **20.** $k(x) = e^{\frac{x}{2}}$

Graph the function. Find the *x*-intercept and asymptote. Describe how the graph is transformed from the graph of the parent function.

21. $n(x) = 3.5\log(x+1)$ **22.** $p(x) = -\ln(x+2)$

Write the transformed function.

23. $f(x) = 0.5^x$ is horizontally compressed by a factor of $\frac{1}{2}$ and reflected across the x-axis.

7-8 Curve Fitting with Exponential and Logarithmic Models

Determine whether *y* is an exponential function of *x*. If so, find the constant ratio. Then use exponential regression to find a function that models the data.

24.

x	0	1	2	3	4	5
y	1.5	3	6	12	24	48

25.

x	0	1	2	3	4	5
y	1.5	2.4	3.3	4.2	5.1	6.0

CHAPTER 7 Study Guide: Review

asymptote 490
base 490
common logarithm 506
exponential decay 490
exponential equation 522
exponential function 490
exponential growth 490
exponential regression 546
inverse function 499
inverse relation 498
logarithm 505
logarithmic equation 523
logarithmic function 507
logarithmic regression 546
natural logarithm 531
natural logarithmic function 532

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ has a base of *e*.

2. A(n) ___?___ is a line that a graphed function approaches but does not touch.

3. To graph a(n) ___?___, reflect each point in the relation across the line $y = x$.

7-1 Exponential Functions, Growth, and Decay *(pp. 490–496)*

12.0

EXAMPLE

A quantity of a certain vitamin is eliminated from the bloodstream at about 15% per hour.

- Will the function that represents this situation show growth or decay?

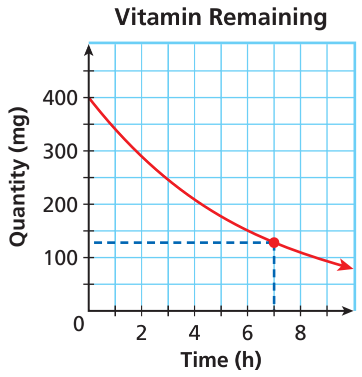
  It will show decay because the quantity decreases.

- Write a function to show the amount of the vitamin that remains *t* hours after the peak level of 400 mg.

  $f(x) = 400(0.85)^t$

- Graph the function. Use the graph to predict the amount remaining after 7 hours.

  After 7 hours, about 130 mg are left.



EXERCISES

Tell whether the function shows growth or decay. Then graph.

4. $f(x) = 0.5(1.25)^x$

5. $f(x) = 0.5\left(\frac{3}{2}\right)^x$

6. $f(x) = 2.5(0.25)^x$

7. $f(x) = 2(1 + 0.25)^x$

Use the following data to answer the questions.

The student population in a small resort town has increased by 2% per year for the last 5 years. This year's population is 765 students.

8. Will the function that represents this situation show growth or decay?

9. Suppose that the student population continues to follow the same trend. Write a function to show the number of students as a function of the year, starting with the current year.

10. Graph the function.

11. Use the graph to predict the number of students in 5 years.

12. When will the population exceed 1000 students?

7-2 Inverses of Relations and Functions *(pp. 498–504)*

24.0

EXAMPLE

- **Graph the function $f(x) = \frac{4}{5} - 3x$. Then write its inverse and graph.**

$y = -3x + \frac{4}{5}$ *Set y = f(x) and graph*

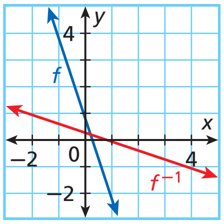
$x = -3y + \frac{4}{5}$ *Interchange x and y.*

$3y = -x + \frac{4}{5}$ *Solve for y.*

$y = -\frac{1}{3}x + \frac{4}{15}$

Write the inverse and graph.

$f^{-1}(x) = -\frac{1}{3}x + \frac{4}{15}$



EXERCISES

13. Graph the relation and connect the points. Then graph and write the inverse.

x	−1	0	1	2	3
y	1	0.2	0.04	0.008	0.001

This year the population of a species decreased by 3% from last year.

14. Write an expression for the size of the population this year P_T as a function of last year's population P_L.

15. Write an expression for the year as a function of the size of the population.

16. The formula $M = \frac{5}{8}K$ gives the approximate distance in miles as a function of kilometers. Write and use the inverse of this function to express 25 miles in kilometers.

7-3 Logarithmic Functions *(pp. 505–511)*

11.0, 11.1, 14.0

EXAMPLES

- **Write the exponential equation $9^{1.5} = 27$ in logarithmic form.**

$9^{1.5} = 27$

$\log_9 27 = 1.5$ *A logarithm is an exponent.*

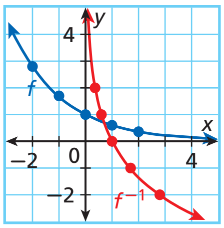
- **Evaluate $\log_4 64$.**

Because $4^3 = 64$, $\log_4 64 = 3$.

- **Graph $f(x) = 0.6^x$. Then graph its inverse. Describe the domain and range of the inverse function.**

x	−2	−1	0	1	2
f(x)	2.8	1.7	1	0.6	0.4

To graph the inverse, reverse each ordered pair.



For the inverse function, the domain is $\{x \mid x > 0\}$, and the range is $\mathbb{R}$.

EXERCISES

Write each exponential equation in logarithmic form.

17. $3^5 = 243$ **18.** $1 = 9^0$ **19.** $\left(\frac{1}{3}\right)^{-3} = 27$

Write each logarithmic equation in exponential form.

20. $\log_2 16 = 4$ **21.** $\log 10 = 1$ **22.** $2 = \log_{0.6} 0.36$

Evaluate by using mental math.

23. $\log_7 49$ **24.** $\log_{0.5} 0.25$

25. $\log_{12}\left(\frac{1}{12}\right)$ **26.** $\log 0.01$ **27.** $\log_2 1$

28. Make a table of ordered pairs for $f(x) = \left(\frac{1}{2}\right)^x$. Graph the function and its inverse. Describe the domain and range of the inverse function.

7-4 Properties of Logarithms *(pp. 512–519)*

11.0, 11.1, 11.2, 13.0, 14.0, 15.0

EXAMPLES

Express as a single logarithm and simplify.

- $\log 25 + \log 40$
  $= \log(25 \cdot 40) = \log 1000 = 3$
- $\log_5 125 - \log_5 25$
  $= \log_5\left(\frac{125}{25}\right) = \log_5 5 = 1$
- $\log_3 8^2$
  $= 2\log_3 8 = 2 \cdot 2 = 4$
- **Evaluate $\log_5 16$.**
  $= \frac{\log 16}{\log 5}$ *Use the change of base formula.*
  $\approx \frac{1.2}{0.7} \approx 1.72$ *Use a calculator to evaluate.*

EXERCISES

Express as a single logarithm and simplify.

29. $\log_2 8 + \log_2 16$ **30.** $\log 100 + \log 10{,}000$

31. $\log_2 128 - \log_2 2$ **32.** $\log 10 - \log 0.1$

33. $\log_5 25^2$ **34.** $\log 10^5 + \log 10^4$

35. The apparent loudness of the music today at Sam's Café was 10 decibels greater than the loudness yesterday. Apparent loudness L is given by $L = 10\log\frac{I}{I_0}$, where I is the intensity of sound, in W/m^2 and I_0 is the lowest intensity that the ear can detect. How many times more intense was the sound today than yesterday?

7-5 Exponential and Logarithmic Equations and Inequalities *(pp. 522–528)*

EXAMPLES

Solve.

- $5^x = 50$
  $\log 5^x = \log 50$
  $x\log 5 = \log 50$
  $x = \frac{\log 50}{\log 5} \approx 2.43$
- $\log_9 x^2 = 5$
  $2\log_9 x = 5$
  $\log_9 x = \frac{5}{2}$
  $x = 9^{\frac{5}{2}}$
  $x = (3^2)^{\frac{5}{2}} = 3^5 = 243$

EXERCISES

11.1, 14.0

Solve and check.

36. $3^{x-1} = \frac{1}{9}$ **37.** $\left(\frac{1}{2}\right)^x \le 64$ **38.** $\log x^{\frac{5}{2}} > 2.5$

39. $A = P(1 + r)^n$ gives amount A in an account after n years for an initial investment P that earns interest at an annual rate r. How long will it take for \$250 to increase to \$500 at 4% annual interest?

7-6 The Natural Base, *e* *(pp. 531–536)*

11.1, 12.0, 13.0

EXAMPLE

- **Simplify $e^{\ln(2s+1)}$.**
  $e^{\ln(2s+1)} = 2s + 1$ *e to the ln of a number is just the number.*
- **What is the total value of an investment of \$5000 that earned 6% interest compounded continuously for 5 years?**
  $A = 5000e^{0.06(5)}$ *Substitute in $A = Pe^{rt}$.*
  $A \approx 6749.29$ *Use a calculator.*
  The value is \$6749.29.

EXERCISES

40. The population of whooping cranes was about 22 in 1940 and grew at an exponential rate to about 194 in 2003.

a. Use the exponential growth function $P(t) = P_0e^{kt}$, where P_0 is the initial population and $P(t)$ is the population at time t, to determine the growth factor k.

b. If the flock continues to grow at the same rate, how large will it be in 2020?

7-7 Transforming Exponential and Logarithmic Functions *(pp. 537–544)*

EXAMPLES

Write each transformed function.

- $f(x) = \left(\frac{1}{3}\right)^x$ is shifted 1 unit left, stretched vertically by a factor of 2, and reflected across the y-axis.

$f(x) = \left(\frac{1}{3}\right)^x$ — *Begin with the parent function. To shift 1 unit left, replace x with x + 1.*

$f(x) = \left(\frac{1}{3}\right)^{x+1}$

$f(x) = 2\left(\frac{1}{3}\right)^{x+1}$ — *Stretch vertically by 2.*

$f(x) = 2\left(\frac{1}{3}\right)^{-(x+1)}$ — *Reflect across the y-axis.*

- $f(x) = \log x$ is shifted 2 units right and 1 unit down and is compressed vertically by a factor of 0.3.

$$f(x) = \log\left(\frac{x}{0.3} - 2\right) - 1$$

EXERCISES

12.0

Write the transformed function.

41. $f(x) = e^x$ is reflected across the x-axis, stretched vertically by a factor of 3, and shifted 2 units down.

Graph each function. Find the intercept and asymptote. Describe how the graph is transformed from the graph of the parent function.

42. $k(x) = \frac{3}{5}(1.5)^{6x}$

43. $m(x) = 2\log\left(x + \frac{1}{2}\right)$

The trade-in value of Marc's truck is $5300. A truck dealer tells him that the trade-in value of a truck decreases by about 35% each year.

44. Write an equation for the trade-in value as a function of time.

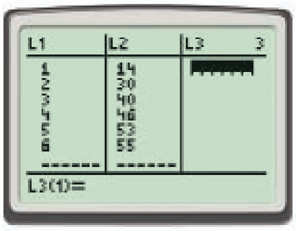
45. Describe how the graph of this function is transformed from the graph of the parent function.

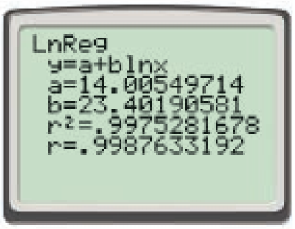
7-8 Curve Fitting with Exponential and Logarithmic Models *(pp. 545–551)*

EXAMPLES

- **Use logarithmic regression to find a function that models the increase in the number of pepper trees in a wilderness preserve over six years. Predict the year when the number of trees will reach 70.**

Year	1	2	3	4	5	6
Trees	14	30	40	46	53	55





$y \approx 14 + 23.4 \ln x$ — *Write the model. Substitute 70.*

$\ln x \approx \frac{70 - 14}{23.4} \approx 2.39$ — *There will be 70 trees in about 11 years.*

$e^{2.39} \approx 10.9$

EXERCISES

12.0

The table gives the population size of a flock of birds in one habitat over the last 55 years.

Years Since Data Was First Collected	Population Size
5	18
22	22
40	85
57	185

46. Use exponential regression, **ExpReg**, to find an exponential function that models the data.

47. Use logarithmic regression, **LnReg**, to find a logarithmic function that models the data.

48. Compare r^2-values of the two functions. Tell which function best models the data and why.

CHAPTER 7

CHAPTER TEST

Tell whether the function shows growth or decay. Then graph.

1. $f(x) = 0.4^x$

2. $f(x) = 1.3\left(\frac{2}{5}\right)^x$

3. $f(x) = \frac{7}{8}(1.1)^x$

4. $f(x) = 50(1 + 0.04)^x$

5. Gina buys a car for $13,500. Assume that its value will decrease by about 15% per year. Write an exponential function to model the value of the car. Graph the function. When will the value fall below $3000?

Graph each function. Then write its inverse and graph.

6. $f(x) = x - 1.06$

7. $f(x) = \frac{5}{6}x - 1.06$

8. $f(x) = 1.06 - \frac{5}{6}x$

9. $f(x) = \frac{1}{4}\left(1.06 - \frac{5}{6}x\right)$

Write in the alternative form (exponential or logarithmic).

10. $16^{\frac{1}{4}} = 2$

11. $16^{-0.5} = \frac{1}{4}$

12. $\log_{\frac{1}{4}} 64 = -3$

13. $\log_{81} \frac{1}{3} = -\frac{1}{4}$

Use the given *x*-values to graph each function. Then write and graph its inverse. Describe the domain and range of the inverse function.

14. $f(x) = \left(\frac{1}{4}\right)^x$; $x = -1, 0, 2, 4$

15. $f(x) = 2.5^x$; $x = -1, 0, 1, 2, 3$

16. $f(x) = 5^{-x}$; $x = -1, 0, 1, 2, 3$

Simplify.

17. $\log_4 128 - \log_4 8$

18. $\log_2 12.8 + \log_2 5$

19. $\log_3 243^2$

20. $5^{\log_5 x}$

Solve.

21. $3^{x-1} = 729^{\frac{x}{2}}$

22. $5^{1.5-x} \le 25$

23. $\log_4(x + 48) = 3$

24. $\log(6x^2) - \log 2x = 1$

25. The rate at which a liquid vitamin breaks down in the average human body can be modeled by $y = D(0.95)^x$, where y ml of the original dose D remains after x minutes. How long will it take for an original dose of 15 ml to be reduced to less than 5 ml?

26. Plutonium Pu-239 has a half-life of about 24,000 years. The formula $\frac{1}{2} = e^{-kt}$ relates the half-life t to the decay constant k for a given substance. How much of a 100-gram quantity of plutonium will remain after 5 years?

27. $f(x) = \ln x$ is shifted 2 units left and 1 unit up and is vertically stretched by a factor of 3. Write the transformed function.

28. Use logarithmic regression to find the function that models the population data in the table. In what year will the population exceed 100?

Population	50	62	78
Year	1	2	3

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 7

FOCUS ON SAT SUBJECT TESTS

The SAT Mathematics Subject Test Level 2 test is meant to be taken by students who have completed two years of algebra and one year of geometry and have studied elementary functions, trigonometry, and some precalculus topics, such as limits.

HOT TIP! The questions are placed in an order of increasing difficulty. Because each question is worth the same amount of points, answer as many of the less difficult questions as you can before tackling the more difficult ones.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. If $f^{-1}(x) = \frac{4}{3}x + 8$, what is $f(x)$?

(A) $f(x) = \frac{3}{4}(x - 8)$

(B) $f(x) = \frac{3}{4}x - 8$

(C) $f(x) = \frac{3}{4}(x + 6)$

(D) $f(x) = \frac{4}{3}(x - 8)$

(E) $f(x) = \frac{4}{3}x - 6$

2. If $f(x) = e^x$, then which of the following is $f^{-1}(7)$?

(A) e^7

(B) 7

(C) log 7

(D) ln 7

(E) $\ln\left(e^7\right)$

3. If $e^x e^{2.5} = e^{2.5x}$, what is the vaue of x?

(A) 0

(B) 1

(C) $\frac{5}{3}$

(D) $\mathbb{R}$

(E) $\varnothing$

4. What is $\log_{27} 9$?

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 2

(E) 3

5. What is the y-coordinate of the point where the graphs of $y = \log_2\left(\frac{3}{4}x - \frac{23}{4}\right)$ and $y = \log_2\left(-2x + \frac{65}{4}\right)$ intersect?

(A) -2

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 2

(E) 8

6. If $\log_9\left\{\log_2\left[\log_4(x)\right]\right\} = \frac{1}{2}$, then what is x?

(A) 1.73

(B) 8

(C) 81

(D) 6561

(E) 65,536

CHAPTER 7

STRATEGIES FOR SUCCESS

Any Question Type: Read a Test Item for Understanding

Test items given on a standardized test may vary in type from multiple choice to gridded response to short and extended response. All test items should be read thoroughly so that you recognize important information and have a complete understanding of what is being asked.

EXAMPLE 1

Extended Response The value of a computer purchased new for $2300 goes down by 15.5% each year. Write and graph an exponential function to estimate the value of the computer after 3 years. When will the value of the computer fall below $500?

READ **the problem again.**

RESTATE **the important parts of the test item by using your own words:**

What information are you given? The cost of the computer: $2300
The annual percent decrease: 15.5%

What are you asked to do?
1. Write an exponential function.
2. Graph the exponential function.
3. Estimate the computer's value after 3 years.
4. Find when the value will fall below $500.

What should your response include?
1. An exponential function
2. A graph
3. An estimated value, in dollars
4. The time in years

NOTE: Your response should include four parts.

EXAMPLE 2

Short Response Two samples of water taken Monday from a wastewater treatment holding tank have a pH of 4.2 and 4.9. To record the pH for the day, a technician finds the average pH for the two samples. What is the difference in the average pH for Monday and the sample that is most acidic?

READ **the problem again.**

RESTATE **the important parts of the test item by using your own words:**

What information are you given? The pH of two samples: 4.2, 4.9

What are you asked to do? Subtract: $\text{pH}_{\text{average}} - \text{pH}_{\text{most acidic sample}}$

Make a plan for your response. Calculate the average pH.
Identify the most acidic sample.
Find the difference.

NOTE: The question requires an intermediate step.

HOT TIP! Break a test item into parts to help you organize your approach to the problem.

Read each test item, and answer the questions that follow.

Item A

Gridded Response What is the total amount, to the nearest whole dollar, for an investment of $800 invested at 3.5% for 15 years and compounded continuously?

1. What information are you given?
2. What are you asked to find?
3. Antonio solved this problem and got an incorrect answer of $1220 after using the formula $I = Prt$. What important word(s) did Antonio overlook that may have led him to the correct formula? Explain.
4. Cleo solved this problem by using the formula $800(1 + 0.035)^{15}$ and got an answer of $1340. Did Cleo solve the problem correctly? Explain.

Item B

Gridded Response A doctor prescribed a daily 15-milligram dose of vitamin D to a 55-year-old man. The man weighs 225 pounds. The half-life of vitamin D is about 25 days. The amount A of vitamin D left after t days can be expressed by the exponential function $A = 15\left(\frac{1}{2}\right)^{\frac{t}{25}}$. Find the number of days (to the nearest day) that it takes for the initial dose of vitamin D to drop below 9 milligrams.

5. What information are you given?
6. Identify any information not necessary for your calculations. Explain.
7. Describe a plan that you can use to solve this problem.
8. A student gridded a decimal answer for his response. What part of the problem statement did he overlook?

Item C

Short Response Martha has $6435 in her home safe. She decides to take two-thirds of this amount and invest it in an account that earns 4.25% interest, compounded continuously. What is the total amount of money that Martha has in 3 years?

9. List the information given and what you are being asked to find.
10. Are there intermediate steps that you need to perform to solve the problem? If so, describe the steps.

Item D

Extended Response A runner ran a 3000 m race in 12 minutes and 48 seconds. Write a function that gives distance as a function of time. Write and use the inverse function to find the time it would take the runner to complete a 10,000 m race at the same speed.

11. How many parts are there to this question? Make a list of what needs to be included in your response.
12. What question are you to answer? What units would be acceptable for your answer?

Item E

Short Response Which data set is best represented by using a logarithmic model? Explain your reasoning, and give the function of the logarithmic model.

A)

x	1	20	40	60	80
y	88	218	341	647	980

B)

x	5	15	25	35	45
y	26	43	52	59	61

13. To determine which data set *best* represents a logarithmic model, what intermediate step must you perform to make a comparison?
14. Make a plan for your response.

CHAPTER 7

STANDARDIZED TEST PREP

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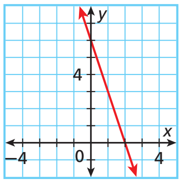
CUMULATIVE ASSESSMENT, CHAPTERS 1–7

Multiple Choice

1. Which graph is the inverse of $f(x) = -3x + 6$?

Ⓐ


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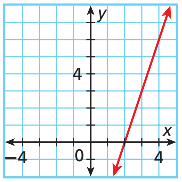

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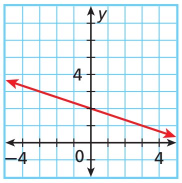

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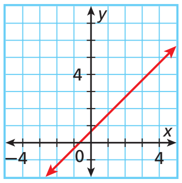


2. Which is equivalent to $\log_5 12 - \log_5 4$?

Ⓕ $\log_5 48$

Ⓖ $\log_5 8$

Ⓗ $\log_5 16$

Ⓙ $\log_5 3$

3. What is the value of x in the equation $\log_4 (x - 1)^3 = 9$?

Ⓐ $x = 27$

Ⓑ $x = 64$

Ⓒ $x = 65$

Ⓓ $x = 81$

4. The parent logarithmic function $f(x) = \ln x$ is shifted 2 units to the right and 7 units down and is horizontally stretched by a factor of 6. Which is the transformed function?

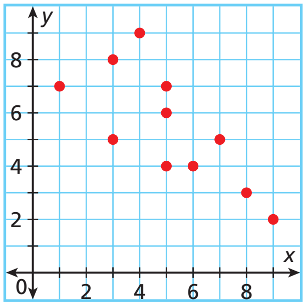
Ⓕ $f(x) = 6\ln(x - 2) - 7$

Ⓖ $f(x) = \ln\left(\frac{x}{6} - 2\right) - 7$

Ⓗ $f(x) = 6\ln(x + 2) + 7$

Ⓙ $f(x) = 6\ln\left(\frac{x}{6} + 2\right) + 7$

5. Which equation best fits the data in the scatter plot?



Ⓐ $y = -\frac{10}{11}x + 10$

Ⓑ $y = \frac{10}{11}x + 10$

Ⓒ $y = -\frac{11}{10}x + 1$

Ⓓ $y = \frac{11}{10}x + 1$

6. Which is a factor of $P(x) = 8x^3 - 26x^2 + 17x + 6$?

Ⓕ $4x - 1$

Ⓖ $x + 2$

Ⓗ $2x + 3$

Ⓙ $2x - 3$

7. Which is a function in standard form with zeros at 0 and −1?

Ⓐ $f(x) = x^2 + x - 1$

Ⓑ $f(x) = x^2 - x$

Ⓒ $f(x) = x^2 + x$

Ⓓ $f(x) = -x^2 + x$

8. The linear correlation coefficient r relating two sets of data is found to be −0.24, and the line of best fit has a y-intercept of 10. Which of the following is NOT necessarily true?

Ⓕ As the values of one set of data increase, the values of the other set decrease.

Ⓖ For positive values of x, the y-value of the line of best fit is less than 10.

Ⓗ The line of best fit is a good model for the data.

Ⓙ The line of best fit has a negative slope.

9. Which has a vertex at $(-2, -3)$?

Ⓐ $y = x^2 + 4x + 1$

Ⓑ $y = x^2 + 4x - 1$

Ⓒ $y = x^2 - 4x + 1$

Ⓓ $y = x^2 - 4x - 1$

10. What is the product $3(x + y)^4$?

Ⓕ $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Ⓖ $3x^4 + 12x^3y + 18x^2y^2 + 12xy^3 + 3y^4$

Ⓗ $81x^4 + y^4$

Ⓙ $3x^4 + 3y^4$

11. A line in $y = mx + b$ form has a positive slope and a y-intercept of 5. The slope of the line is decreased. Which of the following must be true?

Ⓐ The x-intercept of the new line is less than the x-intercept of the original line.

Ⓑ The original line and the new line intersect only at $(0, 5)$.

Ⓒ The slope of the new line is greater than 0.

Ⓓ The new line is parallel to the original line.

In Item 12, you can replace a missing number with a variable, such as *x*. Choose a different variable if there is already an *x* in the problem.

Gridded Response

12. What number is missing in the matrix?

$$\begin{bmatrix} 5 & 8 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} & 2 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} -28 & 10 \\ -2 & 8 \end{bmatrix}$$

13. Evaluate $\log_{6.25} 2.5$.

14. Find the positive zero of the equation $f(x) = x^2 + 2.6x - 7.31$ by using the quadratic formula.

15. What is the multiplicity of the root 2 in the equation $x^3 - 8x^2 + 20x - 16 = 0$?

16. Use the parent function $f(x) = x^2$. What is the horizontal compression factor of the function $f(x) = \frac{1}{2}(5x)^2 - 4$?

17. What power of 2 has a value of 268,435,456?

Short Response

18. A school is selling used computers and printers. The school sells the computers for \$500 each and the printers for \$50 each. The goal is for the school to make at least \$5200. The school expects to sell at least five computers for every two printers.

   a. Write a system of inequalities that models this situation, where x is the number of computers sold, and y is the number of printers sold.

   b. Graph the system of inequalities.

19. Radium-226, which has a half-life of 1620 years, is used in medicine for treatment of disease.

   a. Find the value of k for radium-226.

   b. How much of a 100-gram dose of radium-226 will remain after 3240 years? Round to the nearest gram.

20. Twenty equally spaced points along a 6-foot board are marked for drilling. The distance from the first and last point to the ends is equal to the space between points. What is the distance between consecutive points, to the nearest hundredth of an inch?

Extended Response

21. The chart below shows how many hours students in different grades study each night.

Grade (x)	4	6	8	10	12
Hours (y)	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

   a. Determine if the data set is exponential or logarithmic.

   b. Graph the points.

   c. Find a function to model the data. Round to the nearest ten thousandths.

   d. In which grade do students study 45 minutes each night? Round to the nearest grade.

   e. How long do third graders study each night? Round to the nearest half minute.