CHAPTER 13

Trigonometric Functions

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Chapter Project Online
KEYWORD: MB7 ChProj

Trigonometry, the study of measuring triangles, helps architects plan and design interesting buildings.

Transamerica Pyramid
San Francisco, CA

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. acute angle
2. function
3. domain
4. reciprocal

A. the set of all possible input values of a relation or function

B. an angle whose measure is greater than 90°

C. a relation with at most one y-value for each x-value

D. an angle whose measure is greater than 0° and less than 90°

E. the multiplicative inverse of a number

Ratios

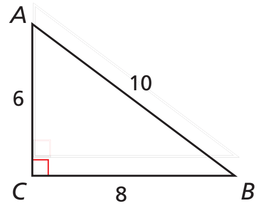
Use $\triangle ABC$ to write each ratio.

5. BC to AB

6. AC to BC

7. the length of the longest side to the length of the shortest side

8. the length of the shorter leg to the length of the hypotenuse



Classify Triangles

Classify each triangle as acute, right, or obtuse.

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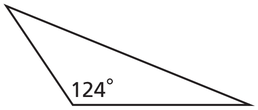

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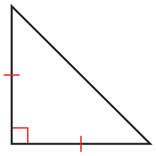

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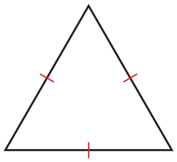
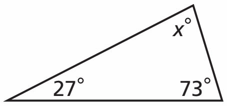


Triangle Sum Theorem

Find the value of x in each triangle.

12.


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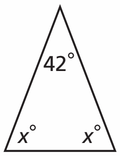

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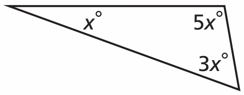


Pythagorean Theorem

Find the missing length for each right triangle with legs a and b and hypotenuse c. Round to the nearest tenth.

15. $a = 16$, $b = \square$, $c = 20$

16. $a = 3$, $b = 5$, $c = \square$

17. $a = 9$, $b = \square$, $c = 18$

18. $a = 7$, $b = 14$, $c = \square$

CHAPTER 13

Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
Preview of Trig 1.0 Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians. (Lessons 13-2, 13-3)	**notion** an idea or concept **convert** to change from one form to another	You draw angles on a coordinate plane and measure angles using degrees and radians.
Preview of Trig 2.0 Students know the definition of sine and cosine as *y*- and *x*-coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions. (Lesson 13-3) (Lab 13-3)	**unit circle** a circle with a radius of 1 unit centered at the origin on the coordinate plane	You find the values of the sine and cosine functions on the unit circle.
Preview of Trig 9.0 Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points. (Lessons 13-2, 13-3)	**compute** to calculate	You find the exact value of the sine, cosine, or tangent for special angle measures.
Preview of Trig 13.0 Students know the law of sines and the law of cosines and apply those laws to solve problems. (Lessons 13-5, 13-6)	**law** a property	You use the law of sines and the law of cosines to find angle measures and side lengths in triangles.
Preview of Trig 19.0 Students are adept at using trigonometry in a variety of applications and word problems. (Lessons 13-3, 13-4, 13-5, 13-6)	**adept** highly skilled	You use trigonometry to solve problems about architecture, astronomy, navigation, and other topics.

Standards Trig 8.0 and Trig 14.0 are also previewed in this chapter.

Reading and Writing Math

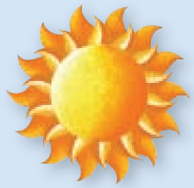
CHAPTER 13

Reading Strategy: Interpret and Read Diagrams

Diagrams are informational tools. Be sure to read and understand the information provided in these visual aids before you attempt to work a problem.

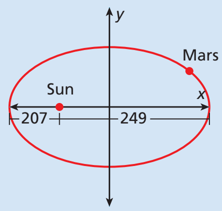
From Lesson 10-3

MULTI-STEP TEST PREP



36. This problem will prepare you for the Multi-Step Test Prep on page 758.

The figure shows the elliptical orbit of Mars, where each unit of the coordinate plane represents 1 million kilometers. As shown, the planet's maximum distance from the Sun is 249 million kilometers and its minimum distance from the Sun is 207 million kilometers.



a. The Sun is at one focus of the ellipse. What are the coordinates of the Sun?

b. What is the length of the minor axis of the ellipse?

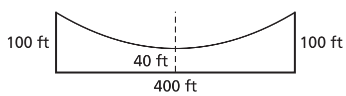
c. Write an equation that models the orbit of Mars.

1. **Examine the diagram.** A point labeled *Sun* lies on the major axis of what appears to be an ellipse. The distances from this point to the vertices are labeled 207 and 249.
2. **Reread the problem, and identify key information about the diagram.** Each unit represents 1 million kilometers. The Sun is at one focus of the ellipse.
3. **Interpret this information.** The labels 207 and 249 represent 207 million kilometers and 249 million kilometers. The length of the major axis can be found by adding these two measurements.
4. **Now you are ready to solve the problem.**

Try This

Read the problem from Chapter 10, and examine the diagram. Then answer the questions below.

26. Engineering The main cables of a suspension bridge are ideally parabolic. The cables over a bridge that is 400 feet long are attached to towers that are 100 feet tall. The lowest point of the cable is 40 feet above the bridge.



a. Find the coordinates of the vertex and the tops of the towers if the bridge represents the x-axis and the axis of symmetry is the y-axis.

1. What information is provided in the diagram?
2. What information regarding the diagram is provided in the problem?
3. What conclusions can you draw from the information related to the diagram?

Connecting Algebra to **Geometry**

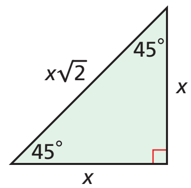
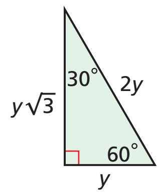
See Skills Bank page S60

Special Right Triangles

Review the relationships of the side lengths of special right triangles below. You can use these relationships to find side lengths of special right triangles.

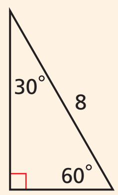
California Standards

Review of Geometry 20.0 **Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, and 90° triangles and 45°, 45°, and 90° triangles.**

Special Right Triangles		
45°-45°-90° Triangle Theorem	In any 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.	
30°-60°-90° Triangle Theorem	In any 30°-60°-90° triangle, the length of the hypotenuse is 2 times the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.	

Example

Find the unknown side lengths for the triangle shown.



The triangle is a 30°-60°-90° triangle, and the length of the hypotenuse is 8.

Step 1 Find the length of the shorter leg.

$8 = 2y$ *hypotenuse = 2 • shorter leg*

$4 = y$ *Solve for y, the length of the shorter leg.*

Step 2 Find the length of the longer leg.

$4\sqrt{3}$ *longer leg = $\sqrt{3}$ • shorter leg*

The length of the shorter leg is 4, and the length of the longer leg is $4\sqrt{3}$.

Check Use the Pythagorean Theorem.

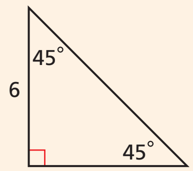
$$4^2 + \left(4\sqrt{3}\right)^2 = 8^2$$

$16 + 48$	64
64	64 ✔

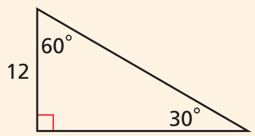
Try This

Find the unknown side lengths for each triangle.

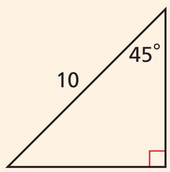
1.



2.



3.



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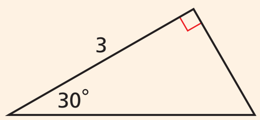


13-1 Right-Angle Trigonometry

Objectives
Understand and use trigonometric relationships of acute angles in triangles.

Determine side lengths of right triangles by using trigonometric functions.

Vocabulary
trigonometric function
sine
cosine
tangent
cosecant
secant
cotangent

California Standards

Review of Geometry 19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.

Who uses this?

Trigonometry can be used to measure the heights of objects, such as an eruption of a geyser, that cannot be measured directly. (See Example 4.)

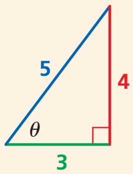
Trigonometry comes from Greek words meaning "triangle measurement." Trigonometry can be used to solve problems involving triangles.

A **trigonometric function** is a function whose rule is given by a trigonometric ratio. A *trigonometric ratio* compares the lengths of two sides of a right triangle. The Greek letter theta θ is traditionally used to represent the measure of an acute angle in a right triangle. The values of trigonometric ratios depend upon θ.

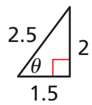
Know it! Note

Trigonometric Functions

WORDS	NUMBERS	SYMBOLS
The **sine** (sin) of angle θ is the ratio of the length of the **opposite** leg to the length of the **hypotenuse**.	$\sin\theta = \frac{4}{5}$	$\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$
The **cosine** (cos) of angle θ is the ratio of the length of the **adjacent** leg to the length of the **hypotenuse**.	$\cos\theta = \frac{3}{5}$	$\cos\theta = \frac{\text{adj.}}{\text{hyp.}}$
The **tangent** (tan) of angle θ is the ratio of the length of the **opposite** leg to the length of the **adjacent** leg.	$\tan\theta = \frac{4}{3}$	$\tan\theta = \frac{\text{opp.}}{\text{adj.}}$



The triangle shown at right is similar to the one in the table because their corresponding angles are congruent. No matter which triangle is used, the value of $\sin\theta$ is the same. The values of the sine and other trigonometric functions depend only on angle θ and not on the size of the triangle.

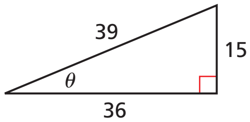


$$\sin\theta = \frac{2}{2.5} = \frac{4}{5}$$

EXAMPLE 1

Finding Trigonometric Ratios

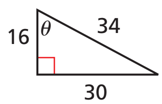
Find the value of the sine, cosine, and tangent functions for θ.



$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{15}{39} = \frac{5}{13} \qquad \cos\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{36}{39} = \frac{12}{13} \qquad \tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{15}{36} = \frac{5}{12}$$

CHECK IT OUT!

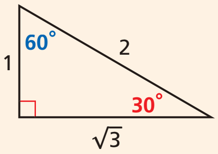
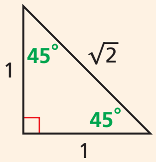
1. Find the value of the sine, cosine, and tangent functions for θ.



You will frequently need to determine the value of trigonometric ratios for 30°, 60°, and 45° angles as you solve trigonometry problems. Recall from geometry that in a 30°-60°-90° triangle, the ratio of the side lengths is $1:\sqrt{3}:2$, and that in a 45°-45°-90° triangle, the ratio of the side lengths is $1:1:\sqrt{2}$.

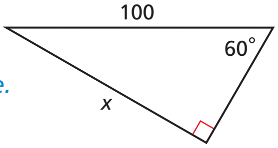
Know it! Note

Trigonometric Ratios of Special Right Triangles

Diagram	Sine	Cosine	Tangent
	$\sin 30° = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$	$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$
	$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\tan 45° = \frac{1}{1} = 1$

EXAMPLE 2 Finding Side Lengths of Special Right Triangles

Use a trigonometric function to find the value of *x*.



$\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$ *The sine function relates the opposite leg and the hypotenuse.*

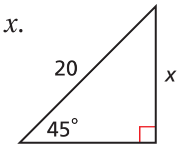
$\sin 60° = \frac{x}{100}$ *Substitute 60° for θ, x for opp., and 100 for hyp.*

$\frac{\sqrt{3}}{2} = \frac{x}{100}$ *Substitute $\frac{\sqrt{3}}{2}$ for sin 60°.*

$50\sqrt{3} = x$ *Multiply both sides by 100 to solve for x.*

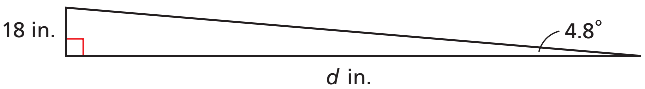
CHECK IT OUT!

2. Use a trigonometric function to find the value of *x*.



EXAMPLE 3 Construction Application

A builder is constructing a wheelchair ramp from the ground to a deck with a height of 18 in. The angle between the ground and the ramp must be 4.8°. To the nearest inch, what should be the distance *d* between the end of the ramp and the deck?



$\tan\theta = \frac{\text{opp.}}{\text{adj.}}$

$\tan 4.8° = \frac{18}{d}$ *Substitute 4.8° for θ, 18 for opp., and d for adj.*

$d(\tan 4.8°) = 18$ *Multiply both sides by d.*

$d = \frac{18}{\tan 4.8°}$ *Divide both sides by tan 4.8°.*

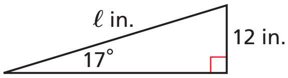
$d \approx 214$ *Use a calculator to simplify.*

The distance should be about 214 in., or 17 ft 10 in.

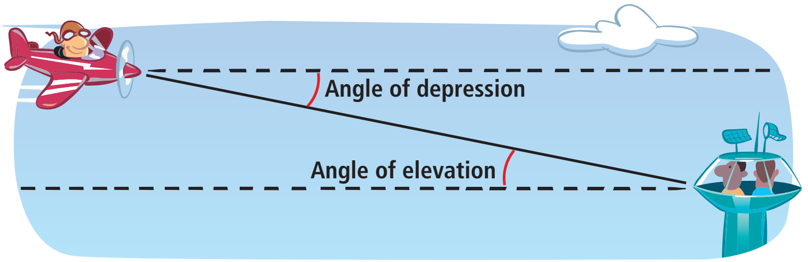
Caution!

Make sure that your graphing calculator is set to interpret angle values as degrees. Press MODE. Check that **Degree** and not **Radian** is highlighted in the third row.

3. A skateboard ramp will have a height of 12 in., and the angle between the ramp and the ground will be 17°. To the nearest inch, what will be the length ℓ of the ramp?



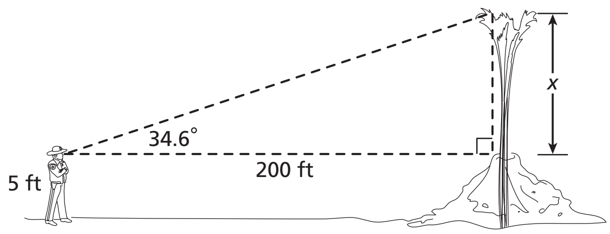
When an object is above or below another object, you can find distances indirectly by using the *angle of elevation* or the *angle of depression* between the objects.



EXAMPLE 4 Geology Application

A park ranger whose eye level is 5 ft above the ground measures the angle of elevation to the top of an eruption of Old Faithful geyser to be 34.6°. If the ranger is standing 200 ft from the geyser's base, what is the height of the eruption to the nearest foot?

Step 1 Draw and label a diagram to represent the information given in the problem.



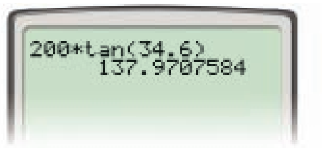
Step 2 Let x represent the height of the eruption compared with the ranger's eye level. Determine the value of x.

$\tan\theta = \frac{\text{opp.}}{\text{adj.}}$ *Use the tangent function.*

$\tan 34.6° = \frac{x}{200}$ *Substitute 34.6° for θ, x for opp., and 200 for adj.*

$200(\tan 34.6°) = x$ *Multiply both sides by 200.*

$138 \approx x$ *Use a calculator to solve for x.*



Step 3 Determine the overall height of the eruption.

$x + 5 = 138 + 5$
$= 143$

The ranger's eye level is 5 ft above the ground, so add 5 ft to x to find the overall height of the eruption.

The height of the eruption is about 143 ft.

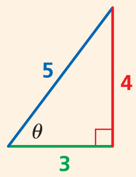
4. A surveyor whose eye level is 6 ft above the ground measures the angle of elevation to the top of the highest hill on a roller coaster to be 60.7°. If the surveyor is standing 120 ft from the hill's base, what is the height of the hill to the nearest foot?

The reciprocals of the sine, cosine, and tangent ratios are also trigonometric ratios. They are the trigonometric functions *cosecant*, *secant*, and *cotangent*.

Know it! Note

Reciprocal Trigonometric Functions

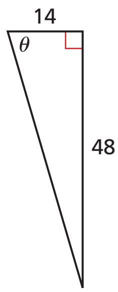
WORDS	NUMBERS	SYMBOLS
The **cosecant** (csc) of angle θ is the reciprocal of the sine function.	$\csc\theta = \frac{5}{4}$	$\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp.}}{\text{opp.}}$
The **secant** (sec) of angle θ is the reciprocal of the cosine function.	$\sec\theta = \frac{5}{3}$	$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp.}}{\text{adj.}}$
The **cotangent** (cot) of angle θ is the reciprocal of the tangent function.	$\cot\theta = \frac{3}{4}$	$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adj.}}{\text{opp.}}$



EXAMPLE 5

Finding All Trigonometric Ratios

Find the values of the six trigonometric functions for θ.



Step 1 Find the length of the hypotenuse.

$a^2 + b^2 = c^2$ *Pythagorean Theorem*

$c^2 = 14^2 + 48^2$ *Substitute 14 for a and 48 for b.*

$c^2 = 2500$ *Simplify.*

$c = 50$ *Solve for c. Eliminate the negative solution.*

Step 2 Find the function values.

$\sin\theta = \frac{48}{50} = \frac{24}{25}$ $\quad\cos\theta = \frac{14}{50} = \frac{7}{25}$ $\quad\tan\theta = \frac{48}{14} = \frac{24}{7}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{25}{24}$ $\quad\sec\theta = \frac{1}{\cos\theta} = \frac{25}{7}$ $\quad\cot\theta = \frac{1}{\tan\theta} = \frac{7}{24}$

Helpful Hint

In each reciprocal pair of trigonometric functions, there is exactly one "**co**."

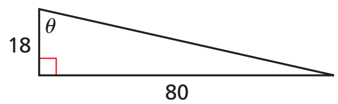
cosecant $\theta = \frac{1}{\text{sine } \theta}$

secant $\theta = \frac{1}{\textbf{co}\text{sine } \theta}$

cotangent $\theta = \frac{1}{\text{tangent } \theta}$

CHECK IT OUT!

5. Find the values of the six trigonometric functions for θ.



THINK AND DISCUSS

1. The sine of an acute angle in a right triangle is 0.6. Explain why the cosine of the other acute angle in the triangle must be 0.6.

2. If the secant of an acute angle in a right triangle is 2, which trigonometric ratio for that angle has a value of 0.5? Explain.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. For each trigonometric function, give the name, the side length ratio, and the reciprocal function.

	Sin	Cos	Tan
Function Name			
Side Length Ratio			
Reciprocal Function			

13-1 Exercises

California Standards
Review of GE19.0; 19.0, 22.0

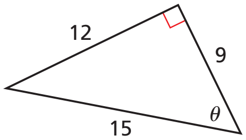
go.hrw.com
Homework Help Online
KEYWORD: MB7 13-1
Parent Resources Online
KEYWORD: MB7 Parent

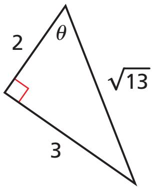
GUIDED PRACTICE

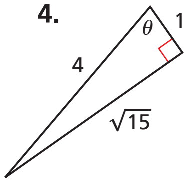
1. **Vocabulary** The ratio of the length of the opposite leg to the length of the adjacent leg of an acute angle of a right triangle is the __?__ of the angle. (*tangent* or *cotangent*)

SEE EXAMPLE 1
p. 929

Find the value of the sine, cosine, and tangent functions for θ.

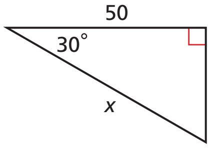
2. 

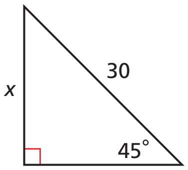
3. 

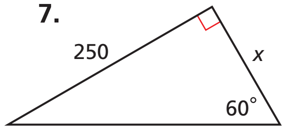
4. 

SEE EXAMPLE 2
p. 930

Use a trigonometric function to find the value of x.

5. 

6. 

7. 

SEE EXAMPLE 3
p. 930

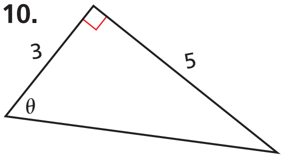
8. **Engineering** An escalator in a mall must lift customers to a height of 22 ft. If the angle between the escalator stairs and the ground floor will be 30°, what will be the length ℓ of the escalator?

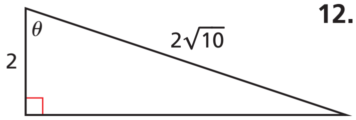
SEE EXAMPLE 4
p. 931

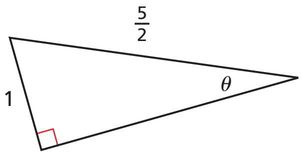
9. **Recreation** The pilot of a hot-air balloon measures the angle of depression to a landing spot to be 20.5°. If the pilot's altitude is 90 m, what is the horizontal distance between the balloon and the landing spot? Round to the nearest meter.

SEE EXAMPLE 5
p. 932

Find the values of the six trigonometric functions for θ.

10. 

11. 

12. 

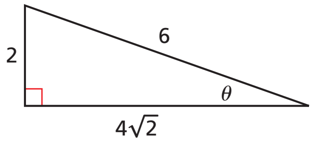
PRACTICE AND PROBLEM SOLVING

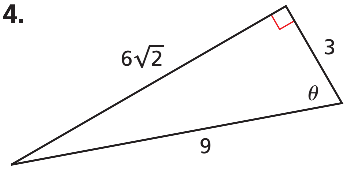
Independent Practice

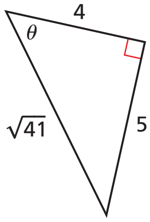
For Exercises	See Example
13–15	1
16–18	2
19	3
20	4
21–23	5

Extra Practice
Skills Practice p. S28
Application Practice p. S44

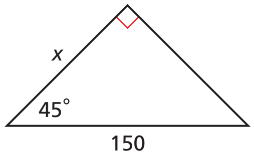
Find the value of the sine, cosine, and tangent functions for θ.

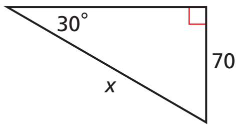
13. 

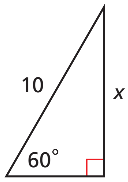
14. 

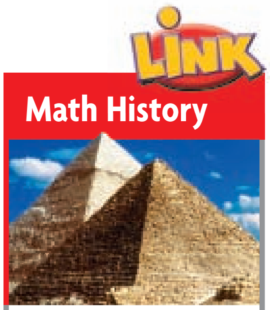
15. 

Use a trigonometric function to find the value of x.

16. 

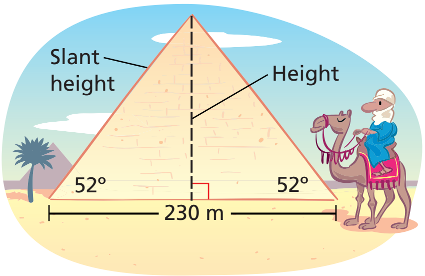
17. 

18. 



Thales of Miletus (624–547 B.C.E.) was a Greek mathematician reputed to have measured the height of the Egyptian pyramids by using the lengths of shadows and indirect measurement.

19. **History** Today, the Great Pyramid in Egypt is not as tall as when it was originally built. The square base of the pyramid has a side length of 230 m, and the sides of the pyramid meet the base at an angle of 52°.



a. What was the original height of the pyramid to the nearest meter?

b. What was the original slant height of the pyramid to the nearest meter?

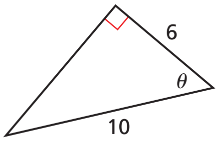
20. **Navigation** The top of the Matagorda Island Lighthouse in Texas is about 90 ft above sea level. The angle of elevation from a fishing boat to the top of the lighthouse is 10°.

a. To the nearest foot, what is the distance d between the boat and the base of the lighthouse?

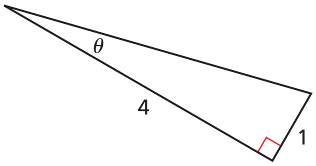
b. **What if...?** After the boat drifts for half an hour, the angle of elevation has decreased to 4.5°. To the nearest foot, how much farther has the boat moved from the lighthouse?

Find the values of the six trigonometric functions for θ.

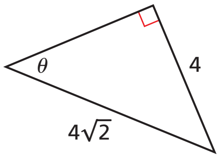
21.



22.



23.



24. **Estimation** One factor that determines a ski slope's difficulty is the slope angle. The table shows the typical slope angles for the most common difficulty categories. For each category, estimate how many meters a skier descends for every 100 m that he or she moves forward horizontally. Explain how you determined your estimates.

Slope Ratings

Symbol	Difficulty	Slope Angle
●	Beginner	5° to 10°
■	Intermediate	10° to 20°
◆	Expert	20° to 35°

25. **Multi-Step** A supply package will be dropped from an airplane to an Arctic research station. The plane's altitude is 2000 ft, and its horizontal speed is 235 ft/s. The angle of depression to the target is 14°.

a. To the nearest foot, what is the plane's horizontal distance from the target?

b. The plane needs to drop the supplies when it is a horizontal distance of 500 ft from the target. To the nearest second, how long should the pilot wait before dropping the supplies?

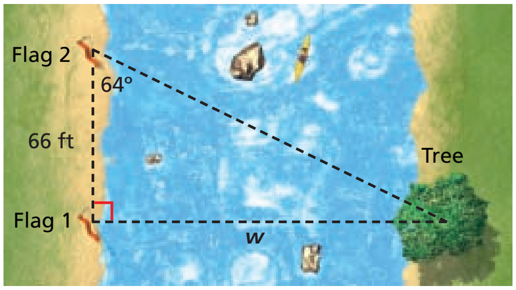
26. This problem will prepare you for the Concept Connection on page 956.

An observer on a sea cliff with a height of 12 m spots an otter through a pair of binoculars at an angle of depression of 5.7°.

a. To the nearest meter, how far is the otter from the base of the cliff?

b. Five minutes later, the observer sights the same otter at an angle of depression of 7.6°. To the nearest meter, how much closer has the otter moved to the base of the cliff?

27. **Surveying** Based on the measurements shown in the diagram, what is the width w of the river to the nearest foot?

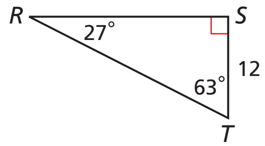


28. **Critical Thinking** Show that $\frac{\sin\theta}{\cos\theta} = \tan\theta$.

29. **Write About It** Suppose that you are given the measure of an acute angle in a right triangle and the length of the leg adjacent to this angle. Describe two different methods that you could use to find the length of the hypotenuse.

STANDARDIZED TEST PREP

Use the diagram for Exercises 30 and 31.



30. Which of the following is equal to $\cos 27°$?

Ⓐ $\csc 63°$
Ⓑ $\sec 63°$
Ⓒ $\tan 63°$
Ⓓ $\sin 63°$

31. Which expression represents the length of $\overline{RS}$?

Ⓕ $12\cot 27°$ Ⓖ $12\csc 27°$ Ⓗ $12\sin 27°$ Ⓙ $12\tan 27°$

32. If $\tan\theta = \frac{3}{4}$, what is $\cos\theta$?

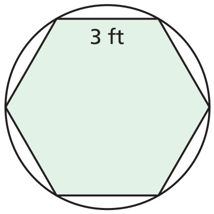
Ⓐ $\frac{3}{5}$ Ⓑ $\frac{4}{5}$ Ⓒ $\frac{5}{4}$ Ⓓ $\frac{4}{3}$

CHALLENGE AND EXTEND

33. **Geometry** Two right triangles each have an acute angle with a sine ratio of 0.6. Prove that the triangles are similar.

34. For an acute angle of a right triangle, which trigonometric ratios are always greater than 1? Which are always less than 1? Explain.

35. **Geometry** A regular hexagon with sides of length 3 ft is inscribed in a circle.



   a. Use a trigonometric ratio to find the radius of the circle.
   b. Determine the area of the hexagon.

36. Explain why the sine of an acute angle is equal to the cosine of its complement.

SPIRAL REVIEW

Solve each proportion. *(Lesson 2-2)*

37. $\frac{4}{17} = \frac{x}{136}$ 38. $\frac{60.3}{x} = \frac{6.7}{3}$ 39. $\frac{196}{x} = \frac{0.05}{9.8}$

40. The students at a high school are randomly assigned a computer password. Each password consists of 4 characters, each of which can be a letter from A to Z or a digit from 0 to 9. What is the probability that a randomly chosen password will consist only of digits? *(Lesson 11-2)*

Find the sum of each infinite series, if it exists. *(Lesson 12-5)*

41. $\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$ 42. $\sum_{n=1}^{\infty} 3n - 5$ 43. $10 + 4 + 1.6 + 0.64 + \cdots$

13-2 Angles of Rotation

Objectives
Draw angles in standard position.

Determine the values of the trigonometric functions for an angle in standard position.

Vocabulary
standard position
initial side
terminal side
angle of rotation
coterminal angle
reference angle

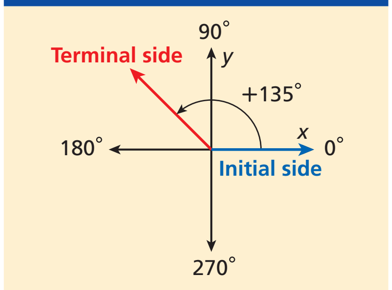
Why learn this?
You can use angles of rotation to determine the rate at which a skater must spin to complete a jump. (See Exercise 51.)

In Lesson 13-1, you investigated trigonometric functions by using acute angles in right triangles. The trigonometric functions can also be evaluated for other types of angles.

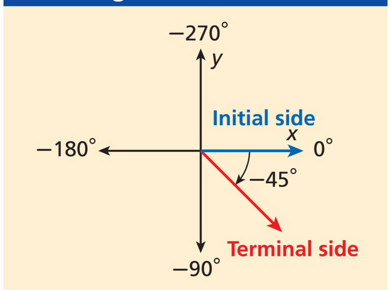
An angle is in **standard position** when its vertex is at the origin and one ray is on the positive x-axis. The **initial side** of the angle is the ray on the x-axis. The other ray is called the **terminal side** of the angle.

Know it! Note

Positive Rotation

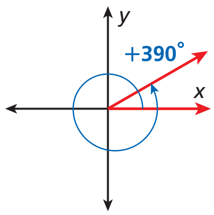


Negative Rotation



California Standards
Preview of Trigonometry 9.0 Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions **at various standard points.**
Also covered: **Preview of Trig 1.0**

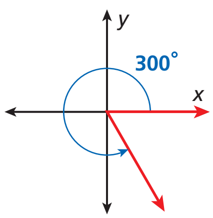
An **angle of rotation** is formed by rotating the terminal side and keeping the initial side in place. If the terminal side is rotated counterclockwise, the angle of rotation is positive. If the terminal side is rotated clockwise, the angle of rotation is negative. The terminal side can be rotated more than 360°.



EXAMPLE 1 Drawing Angles in Standard Position

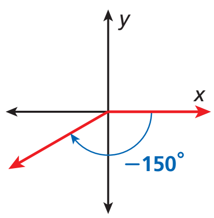
Draw an angle with the given measure in standard position.

A 300°



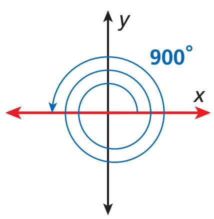
Rotate the terminal side 300° counterclockwise.

B −150°



Rotate the terminal side 150° clockwise.

C 900°



Rotate the terminal side 900° counterclockwise.

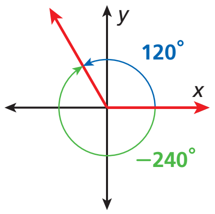
900° = 360° + 360° + 180°

Remember!
A 360° rotation is a complete rotation. A 180° rotation is one-half of a complete rotation.

CHECK IT OUT! **Draw an angle with the given measure in standard position.**

1a. 210° **1b.** 1020° **1c.** −300°

Coterminal angles are angles in standard position with the same terminal side. For example, angles measuring 120° and −240° are coterminal.



There are infinitely many coterminal angles. One way to find the measure of an angle that is coterminal with an angle θ is to add or subtract integer multiples of 360°.

EXAMPLE 2 Finding Coterminal Angles

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

A $\theta = 40°$

$40° + 360° = 400°$ *Add 360° to find a positive coterminal angle.*

$40° - 360° = -320°$ *Subtract 360° to find a negative coterminal angle.*

Angles that measure 400° and −320° are coterminal with a 40° angle.

B $\theta = 380°$

$380° - 360° = 20°$ *Subtract 360° to find a positive coterminal angle.*

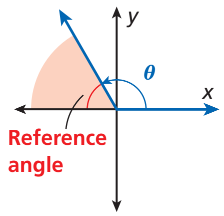
$380° - 2(360°) = -340°$ *Subtract a multiple of 360° to find a negative coterminal angle.*

Angles that measure 20° and −340° are coterminal with a 380° angle.

CHECK IT OUT! **Find the measures of a positive angle and a negative angle that are coterminal with each given angle.**

2a. $\theta = 88°$ **2b.** $\theta = 500°$ **2c.** $\theta = -120°$

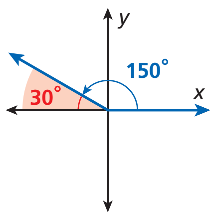
For an angle θ in standard position, the **reference angle** is the positive acute angle formed by the terminal side of θ and the x-axis. In Lesson 13-3, you will learn how to use reference angles to find trigonometric values of angles measuring greater than 90° or less than 0°.



EXAMPLE 3 Finding Reference Angles

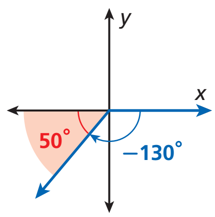
Find the measure of the reference angle for each given angle.

A $\theta = 150°$



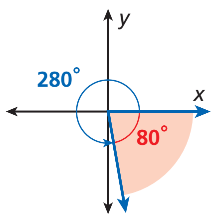
The measure of the reference angle is 30°.

B $\theta = -130°$



The measure of the reference angle is 50°.

C $\theta = 280°$

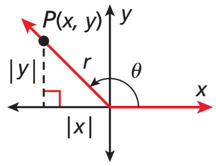


The measure of the reference angle is 80°.

CHECK IT OUT! **Find the measure of the reference angle for each given angle.**

3a. $\theta = 105°$ **3b.** $\theta = -115°$ **3c.** $\theta = 310°$

To determine the value of the trigonometric functions for an angle θ in standard position, begin by selecting a point P with coordinates (x, y) on the terminal side of the angle. The distance r from point P to the origin is given by $\sqrt{x^2 + y^2}$.



Know it! Note

Trigonometric Functions

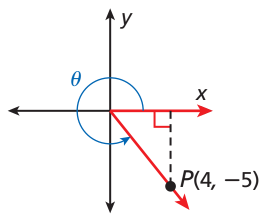
For a point $P(x, y)$ on the terminal side of θ in standard position and $r = \sqrt{x^2 + y^2}$,

SINE	COSINE	TANGENT
$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x},\ x \neq 0$

EXAMPLE 4

Finding Values of Trigonometric Functions

$P(4, -5)$ is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ.



Step 1 Plot point P, and use it to sketch a right triangle and angle θ in standard position. Find r.

$$r = \sqrt{4^2 + (-5)^2} = \sqrt{16 + 25} = \sqrt{41}$$

Helpful Hint

Because r is a distance, its value is always positive, regardless of the sign of x and y.

Step 2 Find $\sin\theta$, $\cos\theta$, and $\tan\theta$.

$$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{41}} = -\frac{5\sqrt{41}}{41}$$

$$\cos\theta = \frac{x}{r} = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$$

$$\tan\theta = \frac{y}{x} = \frac{-5}{4} = -\frac{5}{4}$$

Step 3 Use reciprocals to find $\csc\theta$, $\sec\theta$, and $\cot\theta$.

$$\csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{41}}{5} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{41}}{4} \qquad \cot\theta = \frac{1}{\tan\theta} = -\frac{4}{5}$$

CHECK IT OUT!

4. $P(-3, 6)$ is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ.

THINK AND DISCUSS

1. Describe how to determine the reference angle of an angle whose terminal side is in Quadrant III.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe how to determine the given angle or position for an angle θ.

Standard position	Reference angle
Angle θ	
Positive coterminal angle	Negative coterminal angle

13-2 Exercises

California Standards
Preview of Trig **1.0** and **9.0**; **24.0**

go.hrw.com
Homework Help Online
KEYWORD: MB7 13-2
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** If a 45° angle is in standard position, its __?__ side lies above the x-axis. (*initial* or *terminal*)

SEE EXAMPLE 1 p. 936

Draw an angle with the given measure in standard position.

2. 60° 3. −135° 4. 450° 5. −1125°

SEE EXAMPLE 2 p. 937

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

6. $\theta = 75°$ 7. $\theta = 720°$ 8. $\theta = -25°$ 9. $\theta = -390°$

SEE EXAMPLE 3 p. 937

Find the measure of the reference angle for each given angle.

10. $\theta = 95°$ 11. $\theta = -250°$ 12. $\theta = 230°$ 13. $\theta = -160°$
14. $\theta = 345°$ 15. $\theta = -130°$ 16. $\theta = -15°$ 17. $\theta = 220°$

SEE EXAMPLE 4 p. 938

***P* is a point on the terminal side of *θ* in standard position. Find the exact value of the six trigonometric functions for *θ*.**

18. $P(-3, 2)$ 19. $P(4, -2)$ 20. $P(0, -6)$ 21. $P(-3, -4)$
22. $P(5, -3)$ 23. $P(1, 6)$ 24. $P(-6, -5)$ 25. $P(-3, 6)$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
26–29	1
30–33	2
34–41	3
42–49	4

Extra Practice
Skills Practice p. S28
Application Practice p. S44

Draw an angle with the given measure in standard position.

26. −120° 27. 225° 28. −570° 29. 750°

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

30. $\theta = 254°$ 31. $\theta = 1020°$ 32. $\theta = -165°$ 33. $\theta = -610°$

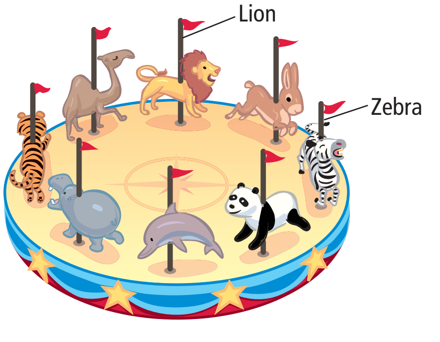
Find the measure of the reference angle for each given angle.

34. $\theta = -25°$ 35. $\theta = 50°$ 36. $\theta = -185°$ 37. $\theta = 200°$
38. $\theta = 390°$ 39. $\theta = -95°$ 40. $\theta = 160°$ 41. $\theta = 325°$

***P* is a point on the terminal side of *θ* in standard position. Find the exact value of the six trigonometric functions for *θ*.**

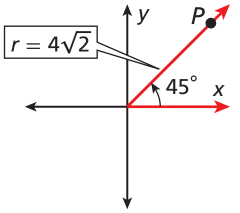
42. $P(2, -5)$ 43. $P(5, -2)$ 44. $P(-4, 5)$ 45. $P(4, 3)$
46. $P(-6, 2)$ 47. $P(3, -6)$ 48. $P(2, -4)$ 49. $P(5, 4)$

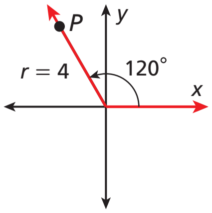
50. **Recreation** A carousel has eight evenly spaced seats shaped like animals. During each ride, the carousel makes between 8 and 9 clockwise revolutions. At the end of one ride, the carousel stops so that the lion is in the position where the zebra was when the ride started. Through how many degrees did the carousel rotate on this ride?

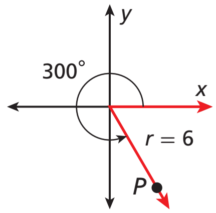


51. **Multi-Step** A double axel is a figure-skating jump in which the skater makes 2.5 revolutions in the air. If a skater is in the air for 0.66 s during a double axel, what is her average angular speed to the nearest degree per second?

Determine the exact coordinates of point *P*.

52. 

53. 

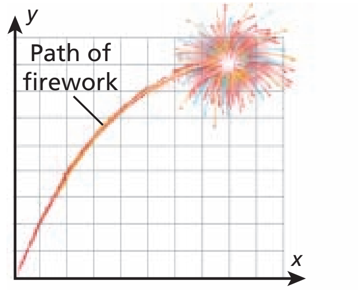
54. 

LINK

Fireworks

In professional fireworks displays, the chemicals that produce colored bursts of light are encased in spherical shells. In general, a firework rises about 100 ft for each inch of shell diameter.

55. **Fireworks** The horizontal distance x and vertical distance y in feet traveled by a firework can be modeled by the functions $x(t) = v(\cos\theta)t$ and $y(t) = -16t^2 + v(\sin\theta)t$. In these functions, v is the initial velocity of the firework, θ is the angle at which the firework is launched, and t is the time in seconds. A firework is launched with an initial velocity of 166 ft/s at an angle of 75°.



a. To the nearest foot, what is the maximum height that the firework will reach?

b. To achieve the greatest effect, the firework should explode when it reaches its maximum height. To the nearest second, how long after the launch should the firework explode?

c. To the nearest foot, what is the horizontal distance that the firework will have traveled when the maximum height is reached?

d. **What if...?** To the nearest foot, how much higher would the firework travel if it were fired at an angle of 90°?

56. **ERROR ANALYSIS** $P(2, -2)$ is a point on the terminal side of an angle θ in standard position. Two attempts at finding $\csc\theta$ are shown below. Which is incorrect? Explain the error.

A

$r = \sqrt{2^2 + (-2)^2} = \sqrt{8}$

$\csc\theta = \frac{\sqrt{8}}{2}$

$\csc\theta = \frac{2\sqrt{2}}{2} = \sqrt{2}$

B

$r = \sqrt{2^2 + (-2)^2} = \sqrt{8}$

$\csc\theta = \frac{\sqrt{8}}{-2}$

$\csc\theta = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$

CONCEPT CONNECTION

57. This problem will prepare you for the Concept Connection on page 956.

An aquarium has a cylindrical tank that rotates at a constant speed about an axis through the center of the cylinder's bases. In 1 minute, the tank rotates through an angle of 48°.

a. How long does it take the tank to make a complete rotation?

b. The tank rotates only during the aquarium's operating hours. If the aquarium is open from 9:30 A.M. to 6:00 P.M., how many rotations does the tank make in one day?

Use your calculator to find the value of each trigonometric function. Round to the nearest thousandth.

58. $\sin 260°$ **59.** $\cos(-130°)$ **60.** $\csc 200°$

Find all values of θ that have a reference angle with the given measure for $0° \le \theta < 360°$.

61. $30°$ **62.** $55°$ **63.** $82°$

64. Critical Thinking Explain how the tangent of an angle in standard position is related to the slope of the terminal side of the angle.

65. Write About It Explain how to determine whether sin 225° is positive or negative without using a calculator.

Standardized Test Prep

66. Which of the following angles have a reference angle with a measure of 30°?

I. $\theta = 120°$ **II.** $\theta = -150°$ **III.** $\theta = 330°$

Ⓐ III only Ⓑ I and II only Ⓒ II and III only Ⓓ I, II, and III

67. In standard position, the terminal side of $\angle P$ passes through point $(-3, 4)$, and the terminal side of $\angle Q$ passes through point $(3, 4)$. Which trigonometric function has the same value for both angles?

Ⓕ sine Ⓖ cosine Ⓗ tangent Ⓙ secant

68. Which angle in standard position is coterminal with an angle that measures $-120°$?

Ⓐ $\theta = 60°$ Ⓑ $\theta = 120°$ Ⓒ $\theta = 240°$ Ⓓ $\theta = 300°$

CHALLENGE AND EXTEND

***P* is a point on the terminal side of θ in standard position. Find the value of the sine, cosine, and tangent of θ in terms of *a* and *b*. Assume that *a* and *b* are positive.**

69. $P(a, b)$ **70.** $P\left(\frac{1}{a}, a\right)$ **71.** $P(a^2, ab)$

72. Write an expression that can be used to determine all of the coterminal angles of an angle that measures 50°.

73. For what values of θ, if any, are the six trigonometric functions undefined?

SPIRAL REVIEW

Use finite differences to determine the degree of the polynomial that best describes the data. *(Lesson 6-9)*

74.

x	0	1	2	3	4	5
y	−3	−1	3	9	17	27

75.

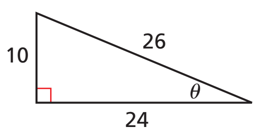
x	0	1	2	3	4	5
y	2	−2	0	14	46	102

Given $f(x) = 2x - 2$ and $g(x) = x^2 + 1$, find each value. *(Lesson 9-4)*

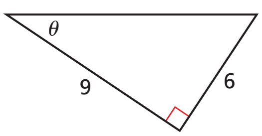
76. $f(g(3))$ **77.** $g(f(4))$ **78.** $f(g(-1))$

Find the value of the sine, cosine, and tangent functions for θ. *(Lesson 13-1)*

79.



80.



13-3 Technology LAB

Use with Lesson 13-3

Explore the Unit Circle

A *unit circle* is a circle with a radius of 1 unit centered at the origin on the coordinate plane. You can use a graphing calculator to plot a unit circle based on the cosine and sine functions. You can then use the unit circle to explore the values of these functions for various angle measures.

California Standards

Preview of Trigonometry 2.0 Students know the definition of sine and cosine as *y*- and *x*-coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.

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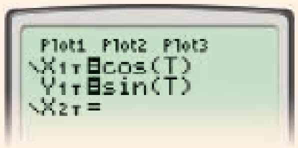
Lab Resources Online
KEYWORD: MB7 Lab13

Activity

Use the sine and cosine functions to graph a unit circle, and use it to determine sin 30° and cos 30°.

1. Press MODE and make sure that the angle mode is set to **Degree**. Set the graphing mode to **Par** (Parametric).

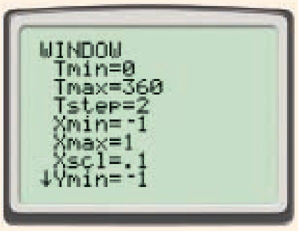


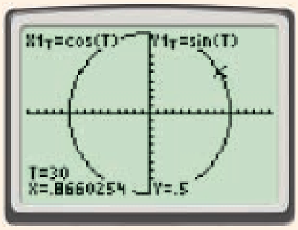
2. Press Y=, and enter **cos(T)** for $\mathbf{X_{1T}}$ and **sin(T)** for $\mathbf{Y_{1T}}$.

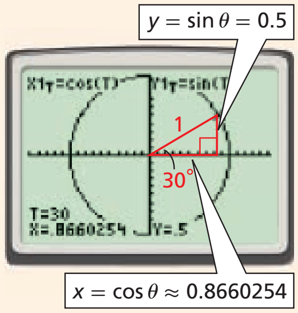
3. Press WINDOW and set **Tmin** to 0, **Tmax** to 360, and **Tstep** to 2. Set **Xmin** and **Ymin** to −1, **Xmax** and **Ymax** to 1, and **Xscl** and **Yscl** to 0.1.

4. Press ZOOM and select **5:Zsquare**. A circle with a radius of 1 unit is displayed.

5. Press TRACE. Use the arrow keys to move the cursor to the point where **T=30**.







From Lesson 13-1, you know that $\sin 30° = \frac{1}{2} = 0.5$ and that $\cos 30° = \frac{\sqrt{3}}{2} \approx 0.8660254$. These values agree with those shown on the graph.

Notice that the unit circle can be used to define the cosine and sine functions. For an angle θ in standard position whose terminal side passes through point $P(x, y)$ on the unit circle, $\sin\theta = y$ and $\cos\theta = x$.

Try This

Use the unit circle on your graphing calculator to determine the values of the sine and cosine functions of each angle.

1. $\theta = 150°$ **2.** $\theta = 244°$ **3.** $\theta = 90°$

4. Make a Conjecture How can you verify that the unit circle displayed on your graphing calculator has a radius of 1 unit?

5. Make a Conjecture Use the graph of the unit circle to explain why the sine function is negative for an angle θ in standard position if the angle's terminal side lies in Quadrants III or IV.

13-3 The Unit Circle

Objectives

Convert angle measures between degrees and radians.

Find the values of trigonometric functions on the unit circle.

Vocabulary

radian
unit circle

California Standards

Preview of Trigonometry 2.0 Students know the definition of sine and cosine as *y*- and *x*-coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.

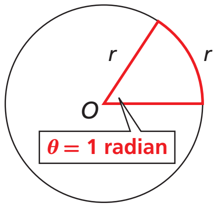
Also covered: **Preview of Trig 1.0, 9.0, 19.0**

Who uses this?

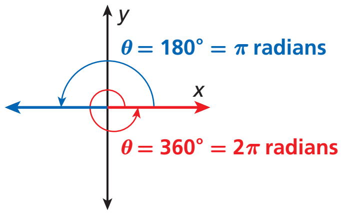
Engineers can use angles measured in radians when designing machinery used to train astronauts. (See Example 4.)

So far, you have measured angles in degrees. You can also measure angles in *radians*.

A **radian** is a unit of angle measure based on arc length. Recall from geometry that an *arc* is an unbroken part of a circle. If a central angle θ in a circle of radius r intercepts an arc of length r, then the measure of θ is defined as 1 radian.



The circumference of a circle of radius r is $2\pi r$. Therefore, an angle representing one complete clockwise rotation measures 2π radians. You can use the fact that 2π radians is equivalent to 360° to convert between radians and degrees.



Know it! Note — **Converting Angle Measures**

DEGREES TO RADIANS	RADIANS TO DEGREES
Multiply the number of degrees by $\left(\frac{\pi \text{ radians}}{180^\circ}\right)$.	Multiply the number of radians by $\left(\frac{180^\circ}{\pi \text{ radians}}\right)$.

EXAMPLE 1 **Converting Between Degrees and Radians**

Convert each measure from degrees to radians or from radians to degrees.

A -45°

$$-45^\circ\left(\frac{\pi \text{ radians}}{180^\circ}\right) = -\frac{\pi}{4} \text{ radians}$$ *Multiply by* $\left(\frac{\pi \text{ radians}}{180^\circ}\right)$.

B $\frac{5\pi}{6}$ radians

$$\left(\frac{5\pi}{6} \text{ radians}\right)\left(\frac{180^\circ}{\pi \text{ radians}}\right) = 150^\circ$$ *Multiply by* $\left(\frac{180^\circ}{\pi \text{ radians}}\right)$.

Reading Math

Angles measured in radians are often not labeled with the unit. If an angle measure does not have a degree symbol, you can usually assume that the angle is measured in radians.

CHECK IT OUT! **Convert each measure from degrees to radians or from radians to degrees.**

1a. 80° **1b.** $\frac{2\pi}{9}$ radians **1c.** −36° **1d.** 4π radians

A **unit circle** is a circle with a radius of 1 unit. For every point $P(x, y)$ on the unit circle, the value of r is 1. Therefore, for an angle θ in standard position:

$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

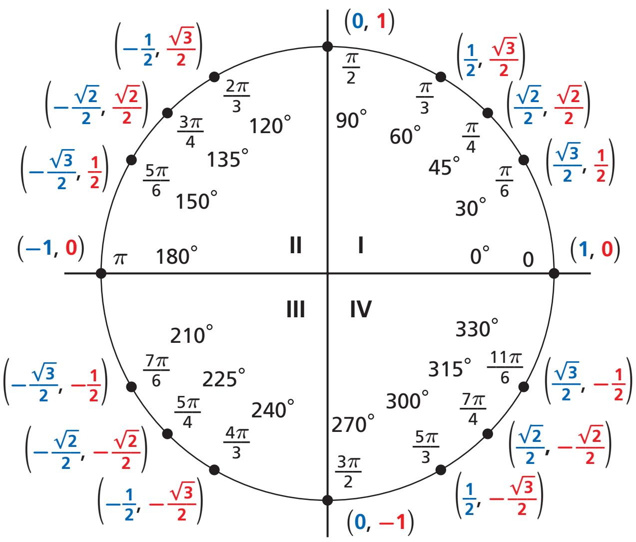
$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

$$\tan\theta = \frac{y}{x}$$

So the coordinates of P can be written as $(\cos\theta, \sin\theta)$.

The diagram shows the equivalent degree and radian measures of special angles, as well as the corresponding x- and y-coordinates of points on the unit circle.

The Unit Circle



EXAMPLE 2 Using the Unit Circle to Evaluate Trigonometric Functions

Use the unit circle to find the exact value of each trigonometric function.

A $\cos 210°$

The angle passes through the point $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ on the unit circle.

$\cos 210° = x$ *Use $\cos\theta = x$.*

$= -\frac{\sqrt{3}}{2}$

B $\tan\frac{5\pi}{3}$

The angle passes through the point $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ on the unit circle.

$\tan\frac{5\pi}{3} = \frac{y}{x}$ *Use $\tan\theta = \frac{y}{x}$.*

$= \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$

CHECK IT OUT! **Use the unit circle to find the exact value of each trigonometric function.**

2a. $\sin 315°$ **2b.** $\tan 180°$ **2c.** $\cos\frac{4\pi}{3}$

You can use reference angles and Quadrant I of the unit circle to determine the values of trigonometric functions.

Know it! Note

Trigonometric Functions and Reference Angles	
To find the sine, cosine, or tangent of θ:	
Step 1	Determine the measure of the reference angle of θ.
Step 2	Use Quadrant I of the unit circle to find the sine, cosine, or tangent of the reference angle.
Step 3	Determine the quadrant of the terminal side of θ in standard position. Adjust the sign of the sine, cosine, or tangent based upon the quadrant of the terminal side.

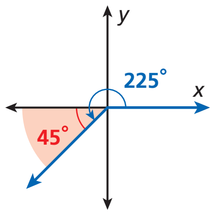
The diagram shows how the signs of the trigonometric functions depend on the quadrant containing the terminal side of θ in standard position.

QII	$\sin\theta: +$ $\cos\theta: -$ $\tan\theta: -$	$\sin\theta: +$ $\cos\theta: +$ $\tan\theta: +$	QI
QIII	$\sin\theta: -$ $\cos\theta: -$ $\tan\theta: +$	$\sin\theta: -$ $\cos\theta: +$ $\tan\theta: -$	QIV

EXAMPLE 3

Using Reference Angles to Evaluate Trigonometric Functions

Use a reference angle to find the exact value of the sine, cosine, and tangent of 225°.



Step 1 Find the measure of the reference angle.

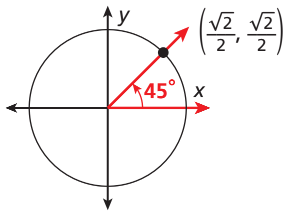
The reference angle measures 45°.

Step 2 Find the sine, cosine, and tangent of the reference angle.

$\sin 45° = \frac{\sqrt{2}}{2}$ *Use $\sin\theta = y$.*

$\cos 45° = \frac{\sqrt{2}}{2}$ *Use $\cos\theta = x$.*

$\tan 45° = 1$ *Use $\tan\theta = \frac{y}{x}$.*



Step 3 Adjust the signs, if needed.

$\sin 225° = -\frac{\sqrt{2}}{2}$ *In Quadrant III, $\sin\theta$ is negative.*

$\cos 225° = -\frac{\sqrt{2}}{2}$ *In Quadrant III, $\cos\theta$ is negative.*

$\tan 225° = 1$ *In Quadrant III, $\tan\theta$ is positive.*

CHECK IT OUT! **Use a reference angle to find the exact value of the sine, cosine, and tangent of each angle.**

3a. 270° **3b.** $\frac{11\pi}{6}$ **3c.** −30°

If you know the measure of a central angle of a circle, you can determine the length s of the arc intercepted by the angle.

$$\frac{\text{radian measure of } \theta}{\text{radian measure of circle}} \rightarrow \frac{\theta}{2\pi} = \frac{s}{2\pi r} \leftarrow \frac{\text{arc length intercepted by } \theta}{\text{arc length intercepted by circle}}$$

$\theta = \frac{s}{r}$ *Multiply each side by 2π.*

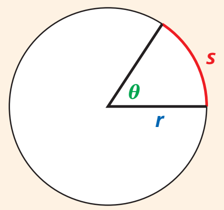
$s = r\theta$ *Solve for s.*

Know it! Note

Arc Length Formula

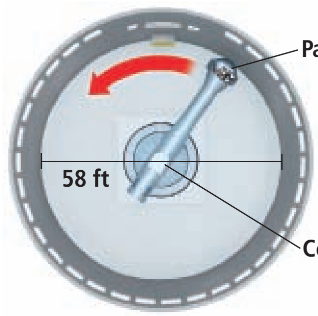
For a circle of radius r, the arc length s intercepted by a central angle θ (measured in radians) is given by the following formula.

$$s = r\theta$$



EXAMPLE 4 Engineering Application

A human centrifuge is a device used in training astronauts. The passenger cab of the centrifuge shown makes 32 complete revolutions about the central hub in 1 minute. To the nearest foot, how far does an astronaut in the cab travel in 1 second?



Step 1 Find the radius of the centrifuge.

$r = \frac{58}{2} = 29$ ft — *The radius is $\frac{1}{2}$ of the diameter.*

Step 2 Find the angle θ through which the cab rotates in 1 second.

$\frac{\text{radians rotated in 1 s}}{1 \text{ s}} = \frac{\text{radians rotated in 60 s}}{60 \text{ s}}$ — *Write a proportion.*

$\frac{\theta \text{ radians}}{1 \text{ s}} = \frac{32(2\pi) \text{ radians}}{60 \text{ s}}$ — *The cab rotates θ radians in 1 s and $32(2\pi)$ radians in 60 s.*

$60 \cdot \theta = 32(2\pi)$ — *Cross multiply.*

$\theta = \frac{32(2\pi)}{60}$ — *Divide both sides by 60.*

$\theta = \frac{16\pi}{15}$ — *Simplify.*

Step 3 Find the length of the arc intercepted by $\frac{16\pi}{15}$ radians.

$s = r\theta$ — *Use the arc length formula.*

$s = 29\left(\frac{16\pi}{15}\right)$ — *Substitute 29 for r and $\frac{16\pi}{15}$ for θ.*

$s \approx 97$ — *Simplify by using a calculator.*

The astronaut travels about 97 feet in 1 second.

CHECK IT OUT! **4.** An hour hand on Big Ben's Clock Tower in London is 14 ft long. To the nearest tenth of a foot, how far does the tip of the hour hand travel in 1 minute?

THINK AND DISCUSS

1. Explain why the tangent of a 90° angle is undefined.

2. Describe how to use a reference angle to determine the sine of an angle whose terminal side in standard position is in Quadrant IV.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, give an expression that can be used to determine the value of the trigonometric function.

Know it! Note

	Acute Angle of Right Triangle	Angle of Rotation with $P(x, y)$	Angle with $P(x, y)$ on Unit Circle
$\sin\theta$			
$\cos\theta$			
$\tan\theta$			

13-3 Exercises

California Standards
Preview of Trig **1.0**, **2.0**, **9.0**, and **19.0**; **22.0**

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KEYWORD: MB7 13-3
Parent Resources Online
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GUIDED PRACTICE

1. **Vocabulary** What is the radius of a *unit circle*? the circumference?

SEE EXAMPLE 1 p. 943

Convert each measure from degrees to radians or from radians to degrees.

2. $30°$
3. $-75°$
4. $-150°$
5. $135°$
6. $\frac{3\pi}{5}$
7. $-\frac{5\pi}{8}$
8. $-\frac{\pi}{3}$
9. $\frac{4\pi}{9}$

SEE EXAMPLE 2 p. 944

Use the unit circle to find the exact value of each trigonometric function.

10. $\sin 150°$
11. $\tan 315°$
12. $\cot \frac{11\pi}{6}$
13. $\cos \frac{2\pi}{3}$

SEE EXAMPLE 3 p. 945

Use a reference angle to find the exact value of the sine, cosine, and tangent of each angle.

14. $240°$
15. $120°$
16. $\frac{7\pi}{4}$
17. $\frac{\pi}{3}$

SEE EXAMPLE 4 p. 946

18. **Engineering** An engineer is designing a curve on a highway. The curve will be an arc of a circle with a radius of 1260 ft. The central angle that intercepts the curve will measure $\frac{\pi}{6}$ radians. To the nearest foot, what will be the length of the curve?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
19–26	1
27–30	2
31–34	3
35	4

Extra Practice

Skills Practice p. S28
Application Practice p. S44

Convert each measure from degrees to radians or from radians to degrees.

19. $240°$
20. $115°$
21. $-25°$
22. $-315°$
23. $-\frac{\pi}{9}$
24. $\frac{2\pi}{5}$
25. $\frac{7\pi}{2}$
26. $-\frac{4\pi}{3}$

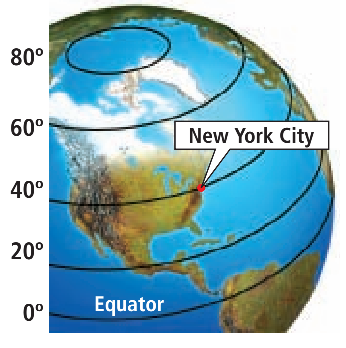
Use the unit circle to find the exact value of each trigonometric function.

27. $\tan 300°$
28. $\sin 120°$
29. $\cos \frac{5\pi}{6}$
30. $\sec \frac{\pi}{3}$

Use a reference angle to find the exact value of the sine, cosine, and tangent of each angle.

31. $225°$
32. $135°$
33. $\frac{11\pi}{6}$
34. $-\frac{5\pi}{6}$

35. **Geography** New York City is located about 40° north of the equator. If Earth's radius is about 4000 miles, approximately how many miles south would a plane need to fly from New York City to reach the equator?



Draw an angle with the given measure in standard position. Then determine the measure of its reference angle.

36. $\frac{\pi}{3}$
37. $\frac{7\pi}{4}$
38. $\frac{5\pi}{6}$

39. **Electronics** A DVD rotates through an angle of 20π radians in 1 second. At this speed, how many revolutions does the DVD make in 1 minute?

40. **Work** A cashier is unsure whether a group of customers ordered and ate a large or a medium pizza. All that remains is one crust, which has an arc length of about $4\frac{1}{4}$ in. All pizzas are divided into 12 equal pieces. If medium pizzas have a diameter of 12 in. and large pizzas have a diameter of 16 in., what size did the customers order? Explain how you determined your answer.

CONCEPT CONNECTION

41. This problem will prepare you for the Concept Connection on page 956.

A railing along an observation deck at an aquarium has a length of 22 ft. The railing is shaped like an arc that represents $\frac{1}{8}$ of a circle.

a. What is the measure, to the nearest degree, of the central angle that intercepts the railing?

b. To the nearest foot, what is the radius of the circle on which the railing is based?

Find the measure of an angle that is coterminal with each given angle.

42. $\theta = \frac{\pi}{8}$ **43.** $\theta = \pi$ **44.** $\theta = \frac{3\pi}{4}$ **45.** $\theta = -\frac{4\pi}{3}$

46. Astronomy The table shows the radius of Earth and Pluto and the number of hours that each takes to rotate on its axis.

	Radius at Equator (km)	Rotational Period (h)
Earth	6378	24
Pluto	1195	153

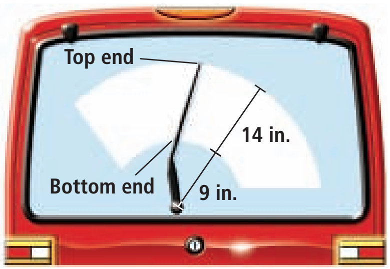
a. How many days does it take Earth to rotate through an angle of 2π radians?

b. Through what angle, in radians, does each planet rotate in 1 hour?

c. What if...? Suppose that a scientific expedition is sent to Pluto. In 1 hour, how much farther would a person at Earth's equator move than an astronaut at Pluto's equator, as a result of the planets' rotations? Round to the nearest kilometer.

47. Quadrantal angles are angles whose terminal sides lie on the x- or y-axis in standard position. Explain how to use the unit circle to determine the sine of the quadrantal angles 0, $\frac{\pi}{2}$, π, and $\frac{3\pi}{2}$ radians.

48. Multi-Step A rear windshield wiper moves through an angle of 135° on each swipe. To the nearest inch, how much greater is the length of the arc traced by the top end of the wiper blade than the length of the arc traced by the bottom end of the wiper blade?



49. Critical Thinking If P is a point on the terminal side of θ in standard position, under what conditions are the coordinates of P equal to $(\cos\theta, \sin\theta)$?

50. Write About It Explain how to determine the value of $\sin(-\theta)$ if you know the value of $\sin\theta$.

STANDARDIZED TEST PREP

51. Which angle measure is closest to 2.5 radians?

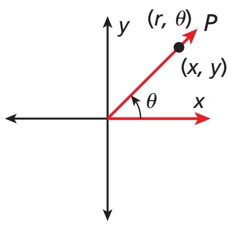
Ⓐ 90° Ⓑ 120° Ⓒ 150° Ⓓ 225°

52. What is the value of $\cot\left(\frac{5\pi}{6}\right)$?

Ⓕ $-\sqrt{3}$ Ⓖ $-\frac{\sqrt{3}}{3}$ Ⓗ $\frac{\sqrt{3}}{3}$ Ⓙ $\sqrt{3}$

53. Short Response If the tangent of an angle θ is $-\sqrt{3}$ and the cosine of θ is $\frac{1}{2}$, what is the value of the other four trigonometric functions of θ? Explain how you determined your answer.

CHALLENGE AND EXTEND



Polar Coordinates In the rectangular coordinate system, the coordinates of point P are (x, y). In the polar coordinate system, the coordinates of point P are (r, θ). Convert each point from polar coordinates to rectangular coordinates.

54. $\left(6\sqrt{2}, \frac{\pi}{4}\right)$ **55.** $\left(10, \frac{7\pi}{6}\right)$ **56.** $\left(5, \frac{5\pi}{3}\right)$

57. **Photography** A photographer taking nighttime photos of wildlife is using a searchlight attached to the roof of a truck. The light has a range of 250 m and can be rotated horizontally through an angle of 150°. Estimate the area of ground that can be lit by the searchlight without moving the truck. Explain how you determined your estimate.

58. What is the range of each of the six trigonometric functions for the domain $\{\theta \mid -90° < \theta < 90°\}$?

SPIRAL REVIEW

Graph each function, and identify its domain and range. *(Lesson 8-7)*

59. $f(x) = \sqrt{x + 4}$ **60.** $f(x) = \sqrt[3]{x} - 3$ **61.** $f(x) = -3\sqrt{x}$

Find the 12th term of each geometric sequence. *(Lesson 12-4)*

62. 900, 180, 36, 7.2, … **63.** −6, 24, −96, 384, … **64.** $\frac{1}{8}, \frac{3}{8}, 1\frac{1}{8}, 3\frac{3}{8}, \ldots$

Find the measure of the reference angle for each given angle. *(Lesson 13-2)*

65. $\theta = 135°$ **66.** $\theta = -295°$ **67.** $\theta = 175°$ **68.** $\theta = -155°$

Career Path

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Career Resources Online
KEYWORD: MB7 Career

Darryl Wright
Surveying Assistant

Q: What high school math classes did you take?

A: Algebra 1, Geometry, Algebra 2, and Trigonometry

Q: What do you like about surveying?

A: I like being outside and being in the construction industry. It's cool to see a bridge or building built where there wasn't anything before. I also like the high-tech instruments we get to use.

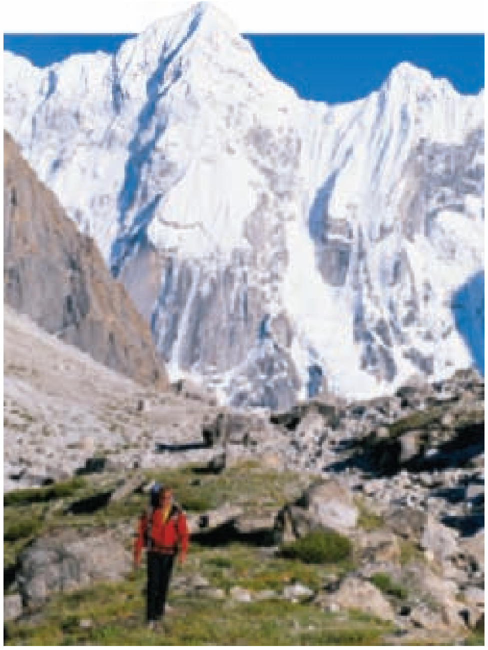
Q: How is math used in surveying?

A: Surveying is basically measuring a lot of distances and angles, so we use trigonometry and geometry all the time. I also do unit conversions to make sure the measurements are in the correct form.

Q: What are your future plans?

A: I'm taking classes at a community college and working toward an associate's degree in surveying. At the same time, I'm gaining work experience that will eventually help me become a licensed surveyor.

13-4 Inverses of Trigonometric Functions



Objectives

Evaluate inverse trigonometric functions.

Use trigonometric equations and inverse trigonometric functions to solve problems.

Vocabulary

inverse sine function
inverse cosine function
inverse tangent function

Who uses this?

Hikers can use inverse trigonometric functions to navigate in the wilderness. (See Example 3.)

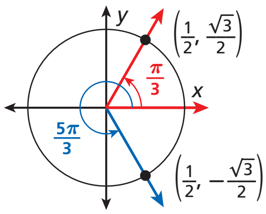
You have evaluated trigonometric functions for a given angle. You can also find the measure of angles given the value of a trigonometric function by using an *inverse trigonometric* relation.

Function	Inverse Relation
$\sin\theta = a$	$\sin^{-1} a = \theta$
$\cos\theta = a$	$\cos^{-1} a = \theta$
$\tan\theta = a$	$\tan^{-1} a = \theta$

Reading Math

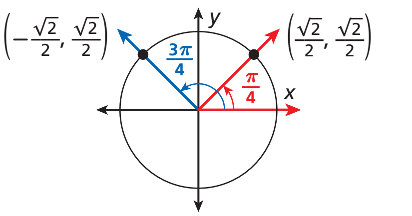
The expression $\sin^{-1}$ is read as "the inverse sine." In this notation, $^{-1}$ indicates the *inverse* of the sine function, NOT the *reciprocal* of the sine function.

The inverses of the trigonometric functions are not functions themselves because there are many values of θ for a particular value of a. For example, suppose that you want to find $\cos^{-1}\frac{1}{2}$. Based on the unit circle, angles that measure $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ radians have a cosine of $\frac{1}{2}$. So do all angles that are coterminal with these angles.



EXAMPLE 1 **Finding Trigonometric Inverses**

Find all possible values of $\sin^{-1}\frac{\sqrt{2}}{2}$.



Step 1 Find the values between 0 and 2π radians for which $\sin\theta$ is equal to $\frac{\sqrt{2}}{2}$.

$$\frac{\sqrt{2}}{2} = \sin\frac{\pi}{4}, \qquad \frac{\sqrt{2}}{2} = \sin\frac{3\pi}{4}$$

Use y-coordinates of points on the unit circle.

Step 2 Find the angles that are coterminal with angles measuring $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ radians.

$$\frac{\pi}{4} + (2\pi)n, \qquad \frac{3\pi}{4} + (2\pi)n$$

Add integer multiples of 2π radians, where n is an integer.

California Standards

Preview of Trigonometry 8.0 Students know the definitions of the inverse trigonometric functions and can graph the functions.

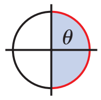
Also covered: **Preview of Trig 19.0**

CHECK IT OUT! **1.** Find all possible values of $\tan^{-1}1$.

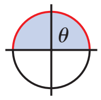
Because more than one value of θ produces the same output value for a given trigonometric function, it is necessary to restrict the domain of each trigonometric function in order to define the inverse trigonometric functions.

Trigonometric functions with restricted domains are indicated with a capital letter. The domains of the Sine, Cosine, and Tangent functions are restricted as follows.

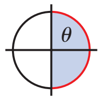
$\text{Sin}\,\theta = \sin\theta$ for $\left\{\theta \mid -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right\}$ *θ is restricted to Quadrants I and IV.*



$\text{Cos}\,\theta = \cos\theta$ for $\left\{\theta \mid 0 \le \theta \le \pi\right\}$ *θ is restricted to Quadrants I and II.*



$\text{Tan}\,\theta = \tan\theta$ for $\left\{\theta \mid -\frac{\pi}{2} < \theta < \frac{\pi}{2}\right\}$ *θ is restricted to Quadrants I and IV.*



These functions can be used to define the inverse trigonometric functions. For each value of a in the domain of the inverse trigonometric functions, there is only one value of θ. Therefore, even though $\tan^{-1} 1$ has many values, $\text{Tan}^{-1} 1$ has only one value.

Know it! Note

Inverse Trigonometric Functions

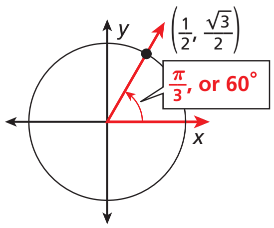
WORDS	SYMBOL	DOMAIN	RANGE
The **inverse sine function** is $\text{Sin}^{-1} a = \theta$, where $\text{Sin}\,\theta = a$.	$\text{Sin}^{-1} a$	$\{a \mid -1 \le a \le 1\}$	$\left\{\theta \mid -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right\}$ $\{\theta \mid -90^\circ \le \theta \le 90^\circ\}$
The **inverse cosine function** is $\text{Cos}^{-1} a = \theta$, where $\text{Cos}\,\theta = a$.	$\text{Cos}^{-1} a$	$\{a \mid -1 \le a \le 1\}$	$\{\theta \mid 0 \le \theta \le \pi\}$ $\{\theta \mid 0^\circ \le \theta \le 180^\circ\}$
The **inverse tangent function** is $\text{Tan}^{-1} a = \theta$, where $\text{Tan}\,\theta = a$.	$\text{Tan}^{-1} a$	$\{a \mid -\infty < a < \infty\}$	$\left\{\theta \mid -\frac{\pi}{2} < \theta < \frac{\pi}{2}\right\}$ $\{\theta \mid -90^\circ < \theta < 90^\circ\}$

Reading Math

The inverse trigonometric functions are also called the arcsine, arccosine, and arctangent functions.

EXAMPLE 2 Evaluating Inverse Trigonometric Functions

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.



A $\text{Cos}^{-1} \frac{1}{2}$

$\frac{1}{2} = \text{Cos}\,\theta$ *Find the value of θ for $0 \le \theta \le \pi$ whose Cosine is $\frac{1}{2}$.*

$\frac{1}{2} = \text{Cos}\,\frac{\pi}{3}$ *Use x-coordinates of points on the unit circle.*

$\text{Cos}^{-1}\frac{1}{2} = \frac{\pi}{3}$, or $\text{Cos}^{-1}\frac{1}{2} = 60^\circ$

B $\text{Sin}^{-1} 2$

The domain of the inverse sine function is $\{a \mid -1 \le a \le 1\}$. Because 2 is outside this domain, $\text{Sin}^{-1} 2$ is undefined.

CHECK IT OUT! **Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.**

2a. $\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ **2b.** $\text{Cos}^{-1} 0$

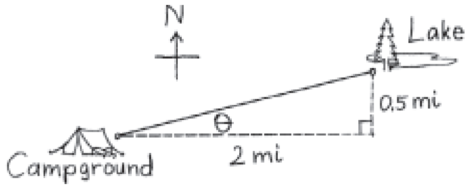
You can solve trigonometric equations by using trigonometric inverses.

EXAMPLE 3 Navigation Application

A group of hikers plans to walk from a campground to a lake. The lake is 2 miles east and 0.5 mile north of the campground. To the nearest degree, in what direction should the hikers head?

Step 1 Draw a diagram.

The hikers' direction should be based on θ, the measure of an acute angle of a right triangle.



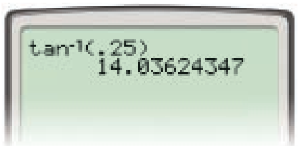
Step 2 Find the value of θ.

$\tan\theta = \dfrac{\text{opp.}}{\text{adj.}}$ *Use the tangent ratio.*

$\tan\theta = \dfrac{0.5}{2} = 0.25$ *Substitute 0.5 for opp. and 2 for adj. Then simplify.*

$\theta = \text{Tan}^{-1} 0.25$

$\theta \approx 14°$



The hikers should head 14° north of east.

Caution!

If the answer on your calculator screen is 0.2449786631 when you enter $\tan^{-1}(0.25)$, your calculator is set to radian mode instead of degree mode.

CHECK IT OUT! **Use the information given above to answer the following.**

3. An unusual rock formation is 1 mile east and 0.75 mile north of the lake. To the nearest degree, in what direction should the hikers head from the lake to reach the rock formation?

EXAMPLE 4 Solving Trigonometric Equations

Solve each equation to the nearest tenth. Use the given restrictions.

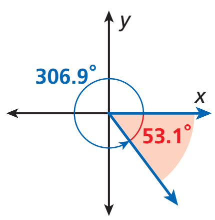
A $\cos\theta = 0.6$, **for** $0° \leq \theta \leq 180°$

The restrictions on θ are the same as those for the inverse cosine function.

$\theta = \text{Cos}^{-1}(0.6) \approx 53.1°$ *Use the inverse cosine function on your calculator.*

B $\cos\theta = 0.6$, **for** $270° < \theta < 360°$

The terminal side of θ is restricted to Quadrant IV. Find the angle in Quadrant IV that has the same cosine value as 53.1°.



$\theta \approx 360° - 53.1° \approx 306.9°$ *θ has a reference angle of 53.1°, and 270° < θ < 360°.*

CHECK IT OUT! **Solve each equation to the nearest tenth. Use the given restrictions.**

4a. $\tan\theta = -2$, for $-90° < \theta < 90°$

4b. $\tan\theta = -2$, for $90° < \theta < 180°$

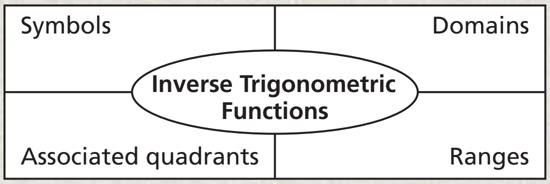
THINK AND DISCUSS

1. Given that θ is an acute angle in a right triangle, describe the measurements that you need to know to find the value of θ by using the inverse cosine function.

2. Explain the difference between $\tan^{-1} a$ and $\text{Tan}^{-1} a$.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, give the indicated property of the inverse trigonometric functions.



13-4 Exercises

California Standards
Preview of Trig **8.0** and **19.0**; **24.0**

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Homework Help Online
KEYWORD: MB7 13-4
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** Explain how the inverse tangent function differs from the reciprocal of the tangent function.

SEE EXAMPLE 1 p. 950

Find all possible values of each expression.

2. $\sin^{-1}\left(-\frac{1}{2}\right)$
3. $\tan^{-1}\frac{\sqrt{3}}{3}$
4. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

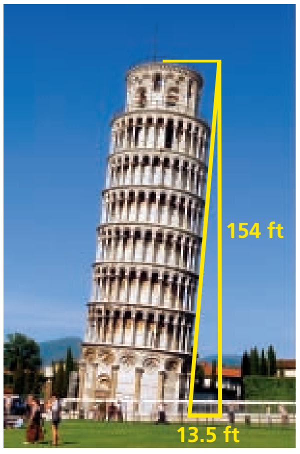
SEE EXAMPLE 2 p. 951

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.

5. $\text{Cos}^{-1}\frac{\sqrt{3}}{2}$
6. $\text{Tan}^{-1} 1$
7. $\text{Cos}^{-1} 2$
8. $\text{Tan}^{-1}(-\sqrt{3})$
9. $\text{Sin}^{-1}\frac{\sqrt{2}}{2}$
10. $\text{Sin}^{-1} 0$

SEE EXAMPLE 3 p. 952

11. **Architecture** A point on the top of the Leaning Tower of Pisa is shifted about 13.5 ft horizontally compared with the tower's base. To the nearest degree, how many degrees does the tower tilt from vertical?



SEE EXAMPLE 4 p. 952

Solve each equation to the nearest tenth. Use the given restrictions.

12. $\tan\theta = 1.4$, for $-90° < \theta < 90°$
13. $\tan\theta = 1.4$, for $180° < \theta < 270°$
14. $\cos\theta = -0.25$, for $0 \leq \theta \leq 180°$
15. $\cos\theta = -0.25$, for $180° < \theta < 270°$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–18	1
19–24	2
25	3
26–29	4

Extra Practice

Skills Practice p. S29
Application Practice p. S44

Find all possible values of each expression.

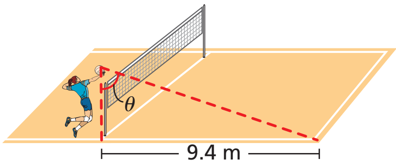
16. $\cos^{-1} 1$ **17.** $\sin^{-1} \frac{\sqrt{3}}{2}$ **18.** $\tan^{-1}(-1)$

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.

19. $\text{Sin}^{-1} \frac{\sqrt{3}}{2}$ **20.** $\text{Cos}^{-1}(-1)$ **21.** $\text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

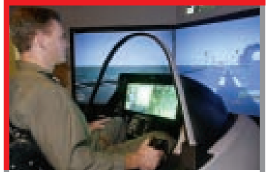
22. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **23.** $\text{Tan}^{-1}\sqrt{3}$ **24.** $\text{Sin}^{-1}\sqrt{3}$

25. Volleyball A volleyball player spikes the ball from a height of 2.44 m. Assume that the path of the ball is a straight line. To the nearest degree, what is the maximum angle θ at which the ball can be hit and land within the court?



LINK

Aviation



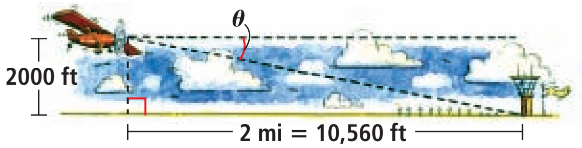
A flight simulator is a device used in training pilots that mimics flight conditions as realistically as possible. Some flight simulators involve full-size cockpits equipped with sound, visual, and motion systems.

Solve each equation to the nearest tenth. Use the given restrictions.

26. $\sin\theta = -0.75$, for $-90° \le \theta \le 90°$ **27.** $\sin\theta = -0.75$, for $180° < \theta < 270°$

28. $\cos\theta = 0.1$, for $0° \le \theta \le 180°$ **29.** $\cos\theta = 0.1$, for $270° < \theta < 360°$

30. Aviation The pilot of a small plane is flying at an altitude of 2000 ft. The pilot plans to start the final descent toward a runway when the horizontal distance between the plane and the runway is 2 mi. To the nearest degree, what will be the angle of depression θ from the plane to the runway at this point?



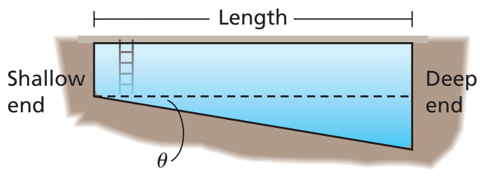
31. Multi-Step The table shows the dimensions of three pool styles offered by a construction company.

Pool Style	Length (ft)	Shallow End Depth (ft)	Deep End Depth (ft)
A	38	3	8
B	25	2	6
C	50	2.5	7

a. To the nearest tenth of a degree, what angle θ does the bottom of each pool make with the horizontal?

b. Which pool style's bottom has the steepest slope? Explain.

c. What if...? If the slope of the bottom of a pool can be no greater than $\frac{1}{6}$, what is the greatest angle θ that the bottom of the pool can make with the horizontal? Round to the nearest tenth of a degree.



32. Navigation Lines of longitude are closer together near the poles than at the equator. The formula for the length ℓ of 1° of longitude in miles is $\ell = 69.0933 \cos\theta$, where θ is the latitude in degrees.

a. At what latitude, to the nearest degree, is the length of a degree of longitude approximately 59.8 miles?

b. To the nearest mile, how much longer is the length of a degree of longitude at the equator, which has a latitude of 0°, than at the Arctic Circle, which has a latitude of about 66°N?

CONCEPT CONNECTION

33. This problem will prepare you for the Concept Connection on page 956.

Giant kelp is a seaweed that typically grows about 100 ft in height, but may reach as high as 175 ft.

a. A diver positions herself 10 ft from the base of a giant kelp so that her eye level is 5 ft above the ocean floor. If the kelp is 100 ft in height, what would be the angle of elevation from the diver to the top of the kelp? Round to the nearest tenth of a degree.

b. The angle of elevation from the diver's eye level to the top of a giant kelp whose base is 30 ft away is 75.5°. To the nearest foot, what is the height of the kelp?

Find each value.

34. $\text{Cos}^{-1}(\cos 0.4)$ **35.** $\tan\left(\text{Tan}^{-1} 0.7\right)$ **36.** $\sin\left(\text{Cos}^{-1} 0\right)$

37. Critical Thinking Explain why the domain of the Cosine function is different from the domain of the Sine function.

38. Write About It Is the statement $\text{Sin}^{-1}(\sin\theta) = \theta$ true for all values of θ? Explain.

STANDARDIZED TEST PREP

39. For which equation is the value of θ in radians a positive value?

Ⓐ $\text{Cos}\,\theta = -\frac{1}{2}$ Ⓑ $\text{Tan}\,\theta = -\frac{\sqrt{3}}{3}$ Ⓒ $\text{Sin}\,\theta = -\frac{\sqrt{3}}{2}$ Ⓓ $\text{Sin}\,\theta = -1$

40. A caution sign next to a roadway states that an upcoming hill has an 8% slope. An 8% slope means that there is an 8 ft rise for 100 ft of horizontal distance. At approximately what angle does the roadway rise from the horizontal?

Ⓕ 2.2° Ⓖ 4.6° Ⓗ 8.5° Ⓙ 12.5°

41. What value of θ makes the equation $2\sqrt{2}(\text{Cos}\,\theta) = -2$ true?

Ⓐ 45° Ⓑ 60° Ⓒ 135° Ⓓ 150°

CHALLENGE AND EXTEND

42. If $\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$, what is the value of $\text{Csc}^{-1}\left(-\sqrt{2}\right)$?

Solve each inequality for $\{\theta \mid 0 \le \theta \le 2\pi\}$.

43. $\cos\theta \le \frac{1}{2}$ **44.** $2\sin\theta - \sqrt{3} > 0$ **45.** $\tan 2\theta \ge 1$

SPIRAL REVIEW

Graph each function. Identify the parent function that best describes the set of points, and describe the transformation from the parent function. *(Lesson 1-9)*

46. $\{(-2, -4), (-1, -0.5), (0, 0), (1, 0.5), (2, 4)\}$ **47.** $\{(-4, 1), (-2, 3), (0, 5), (2, 7), (4, 9)\}$

Find the inverse of each function. Determine whether the inverse is a function, and state its domain and range. *(Lesson 9-5)*

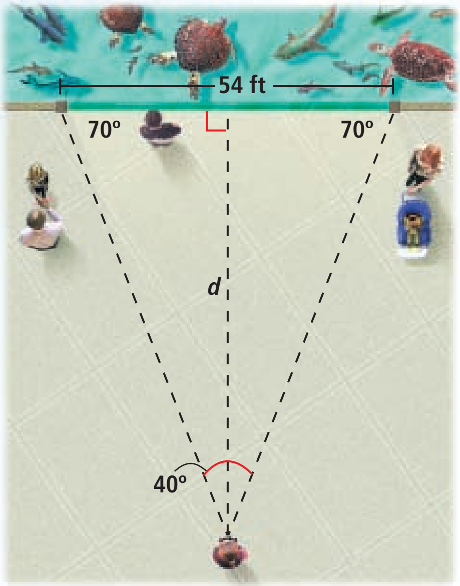
48. $f(x) = 3(x + 2)^2$ **49.** $f(x) = \frac{x}{4} + 1$ **50.** $f(x) = -2x^2 + 5$

Convert each measure from degrees to radians or from radians to degrees. *(Lesson 13-3)*

51. 240° **52.** $-\frac{5\pi}{4}$ **53.** 420°

CONCEPT CONNECTION

Trigonometry and Angles



By the Sea The Monterey Bay Aquarium in California is visited by almost 2 million people each year. The aquarium is home to more than 550 species of plants and animals.

1. The window in front of the aquarium's Outer Bay exhibit is 54 ft long. Maria's camera has a viewing angle of 40°, as shown. To the nearest foot, how far would Maria need to stand from the window in order to include the entire window in a photo? This distance is labeled d.
2. Warty sea cucumbers may be found in Monterey Bay to a depth of about 64 m. A research vessel is anchored to the seafloor by a 70 m chain that makes an angle of 56° with the ocean's surface. Is the research vessel located over water that is too deep for warty sea cucumbers? Justify your answer.
3. A crystal jellyfish in a cylindrical aquarium tank is carried by a current in a circular path with a diameter of 2.5 m. In 1 min, the jellyfish is carried 26 cm by the current. At this rate, how long will it take the current to move the jellyfish in a complete circle? Round to the nearest minute.
4. A sea otter is released from the aquarium's rehabilitation program with a radio transmitter implanted in its abdomen. The transmitter indicates that the otter is 400 m west and 80 m south of an observation deck. How many degrees south of west should an aquarium worker standing on the deck aim his binoculars in order to see the otter? Round to the nearest degree.

READY TO GO ON?

CHAPTER 13

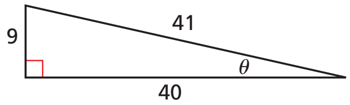
SECTION 13A

Quiz for Lessons 13-1 Through 13-4

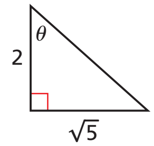
13-1 Right-Angle Trigonometry

Find the values of the six trigonometric functions for θ.

1.

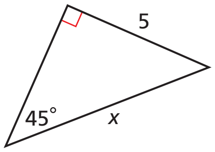


2.

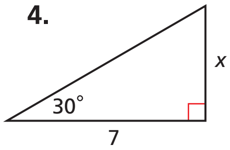


Use a trigonometric function to find the value of x.

3.



4.



5. A biologist's eye level is 5.5 ft above the ground. She measures the angle of elevation to an eagle's nest on a cliff to be 66° when she stands 50 ft from the cliff's base. To the nearest foot, what is the height of the eagle's nest?

13-2 Angles of Rotation

Draw an angle with the given measure in standard position.

6. $-270°$

7. $405°$

Point P is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ.

8. $P(12, -5)$

9. $P(-2, 7)$

13-3 The Unit Circle

Convert each measure from degrees to radians or from radians to degrees.

10. $-120°$

11. $63°$

12. $\frac{3\pi}{8}$

13. $-\frac{10\pi}{3}$

Use the unit circle to find the exact value of each trigonometric function.

14. $\cos 210°$

15. $\tan 120°$

16. $\cos \frac{\pi}{2}$

17. $\tan \frac{5\pi}{4}$

18. A bicycle tire rotates through an angle of 3.4π radians in 1 second. If the radius of the tire is 0.34 m, what is the bicycle's speed in meters per second? Round to the nearest tenth.

13-4 Inverses of Trigonometric Functions

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.

19. $\text{Sin}^{-1} \frac{\sqrt{3}}{2}$

20. $\text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

21. A driver uses a ramp when unloading supplies from his delivery truck. The ramp is 10 feet long, and the bed of the truck is 4 feet off the ground. To the nearest degree, what angle does the ramp make with the ground?

13-5 The Law of Sines

Objectives

Determine the area of a triangle given side-angle-side information.

Use the Law of Sines to find the side lengths and angle measures of a triangle.

Who uses this?

Sailmakers can use sine ratios to determine the amount of fabric needed to make a sail. (See Example 1.)

A sailmaker is designing a sail that will have the dimensions shown in the diagram. Based on these dimensions, the sailmaker can determine the amount of fabric needed.

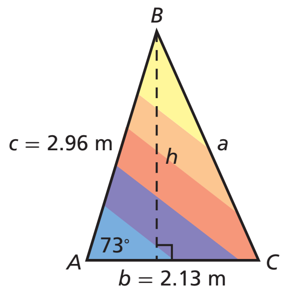
Helpful Hint

An angle and the side opposite that angle are labeled with the same letter. Capital letters are used for angles, and lowercase letters are used for sides.

The area of the triangle representing the sail is $\frac{1}{2}bh$. Although you do not know the value of h, you can calculate it by using the fact that $\sin A = \frac{h}{c}$, or $h = c \sin A$.

$\text{Area} = \frac{1}{2}bh$ *Write the area formula.*

$\text{Area} = \frac{1}{2}bc\sin A$ *Substitute c sin A for h.*



This formula allows you to determine the area of a triangle if you know the lengths of two of its sides and the measure of the angle between them.

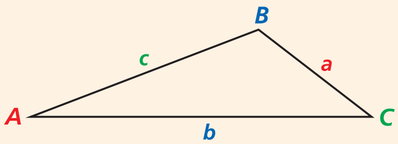
Know it! Note — **Area of a Triangle**

For $\triangle ABC$,

$\text{Area} = \frac{1}{2}bc \sin A$

$\text{Area} = \frac{1}{2}ac \sin B$

$\text{Area} = \frac{1}{2}ab \sin C$



EXAMPLE 1 Determining the Area of a Triangle

California Standards

Preview of Trigonometry
13.0 Students know the law of sines and the law of cosines **and apply those laws to solve problems.**
Also covered: **Preview of Trig 14.0 and 19.0**

Find the area of the sail shown at the top of the page. Round to the nearest tenth.

$\text{area} = \frac{1}{2}bc \sin A$ *Write the area formula.*

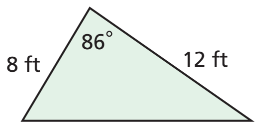
$= \frac{1}{2}(2.13)(2.96) \sin 73°$ *Substitute 2.13 for b, 2.96 for c, and 73° for A.*

≈ 3.014655113 *Use a calculator to evaluate the expression.*

The area of the sail is about 3.0 m^2.

CHECK IT OUT!

1. Find the area of the triangle. Round to the nearest tenth.



The area of $\triangle ABC$ is equal to $\frac{1}{2}bc\sin A$ or $\frac{1}{2}ac\sin B$ or $\frac{1}{2}ab\sin C$. By setting these expressions equal to each other, you can derive the Law of Sines.

$$\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$

$$bc\sin A = ac\sin B = ab\sin C \qquad \textit{Multiply each expression by 2.}$$

$$\frac{\cancel{b}\cancel{c}\sin A}{a\cancel{b}\cancel{c}} = \frac{\cancel{a}\cancel{c}\sin B}{\cancel{a}b\cancel{c}} = \frac{\cancel{a}\cancel{b}\sin C}{\cancel{a}\cancel{b}c} \qquad \textit{Divide each expression by abc.}$$

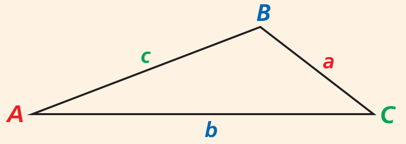
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \textit{Divide out common factors.}$$

Know it! Note

Law of Sines

For $\triangle ABC$, the Law of Sines states that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



The Law of Sines allows you to solve a triangle as long as you know either of the following:

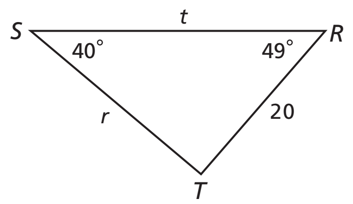
1. Two angle measures and any side length—angle-angle-side (AAS) or angle-side-angle (ASA) information
2. Two side lengths and the measure of an angle that is not between them—side-side-angle (SSA) information

EXAMPLE 2

Using the Law of Sines for AAS and ASA

Solve the triangle. Round to the nearest tenth.

A



Reading Math

The expression "solve a triangle" means to find the measures of all unknown angles and sides.

Step 1 Find the third angle measure.

$m\angle R + m\angle S + m\angle T = 180°$ — *Triangle Sum Theorem*

$49° + 40° + m\angle T = 180°$ — *Substitute 49° for m∠R and 40° for m∠S.*

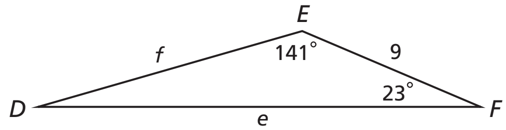
$m\angle T = 91°$ — *Solve for m∠T.*

Step 2 Find the unknown side lengths.

$\frac{\sin R}{r} = \frac{\sin S}{s}$	*Law of Sines*	$\frac{\sin S}{s} = \frac{\sin T}{t}$
$\frac{\sin 49°}{r} = \frac{\sin 40°}{20}$	*Substitute.*	$\frac{\sin 40°}{20} = \frac{\sin 91°}{t}$
$r\sin 40° = 20\sin 49°$	*Cross multiply.*	$t\sin 40° = 20\sin 91°$
$r = \frac{20\sin 49°}{\sin 40°}$	*Solve for the unknown side.*	$t = \frac{20\sin 91°}{\sin 40°}$
$r \approx 23.5$		$t \approx 31.1$

Solve the triangle. Round to the nearest tenth.

B



Step 1 Find the third angle measure.

$m\angle D = 180° - 141° - 23° = 16°$ *Triangle Sum Theorem*

Step 2 Find the unknown side lengths.

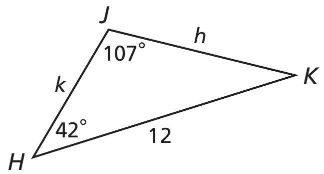
$\frac{\sin D}{d} = \frac{\sin E}{e}$ *Law of Sines* $\frac{\sin D}{d} = \frac{\sin F}{f}$

$\frac{\sin 16°}{9} = \frac{\sin 141°}{e}$ *Substitute.* $\frac{\sin 16°}{9} = \frac{\sin 23°}{f}$

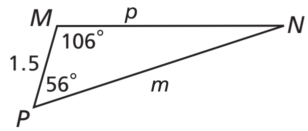
$e = \frac{9 \sin 141°}{\sin 16°} \approx 20.5$ $f = \frac{9 \sin 23°}{\sin 16°} \approx 12.8$

CHECK IT OUT! **Solve each triangle. Round to the nearest tenth.**

2a.



2b.

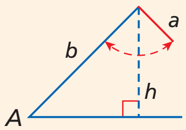


When you use the Law of Sines to solve a triangle for which you know side-side-angle (SSA) information, zero, one, or two triangles may be possible. For this reason, SSA is called the *ambiguous case*.

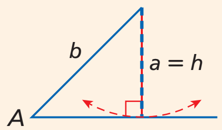
Ambiguous Case Possible Triangles

Given *a*, *b*, and m∠*A*,

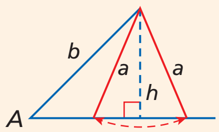
∠*A* IS ACUTE.



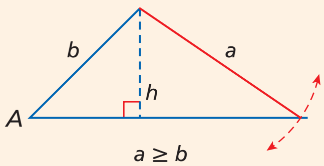
$a < h$
No triangle



$a = h$
One triangle

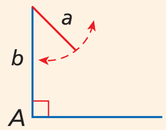


$h < a < b$
Two triangles

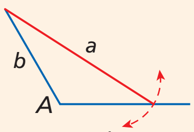


$a \geq b$
One triangle

∠*A* IS RIGHT OR OBTUSE.



$a \leq b$
No triangle



$a > b$
One triangle

Remember!

When one angle in a triangle is obtuse, the measures of the other two angles must be acute.

Know it! Note

Solving a Triangle Given *a*, *b*, and m∠*A*
1. Use the values of *a*, *b*, and m∠*A* to determine the number of possible triangles.
2. If there is one triangle, use the Law of Sines to solve for the unknowns.
3. If there are two triangles, use the Law of Sines to find m∠B_1 and m∠B_2. Then use these values to find the other measurements of the two triangles.

EXAMPLE 3 Art Application

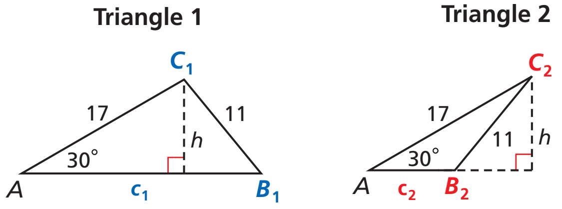
Maggie is designing a mosaic by using triangular tiles of different shapes. Determine the number of triangles that Maggie can form using the measurements $a = 11$ cm, $b = 17$ cm, and $m\angle A = 30°$. Then solve the triangles. Round to the nearest tenth.

Step 1 Determine the number of possible triangles. In this case, $\angle A$ is acute. Find h.

$$\sin 30° = \frac{h}{17} \qquad \textit{$\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$}$$

$$h = 17\sin 30° \approx 8.5 \text{ cm} \qquad \textit{Solve for h.}$$

Because $h < a < b$, two triangles are possible.



Step 2 Determine $m\angle B_1$ and $m\angle B_2$.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \textit{Law of Sines}$$

$$\frac{\sin 30°}{11} = \frac{\sin B}{17} \qquad \textit{Substitute.}$$

$$\sin B = \frac{17\sin 30°}{11} \qquad \textit{Solve for sin B.}$$

$$\sin B \approx 0.773$$

Let $\angle B_1$ represent the acute angle with a sine of 0.773. Use the inverse sine function on your calculator to determine $m\angle B_1$.

$$m\angle B_1 = \text{Sin}^{-1}\left(\frac{17\sin 30°}{11}\right) \approx 50.6°$$

Let $\angle B_2$ represent the obtuse angle with a sine of 0.773.

$$m\angle B_2 = 180° - 50.6° = 129.4° \qquad \textit{The reference angle of $\angle B_2$ is 50.6°.}$$

Helpful Hint

Because $\angle B_1$ and $\angle B_2$ have the same sine value, they also have the same reference angle.

Step 3 Find the other unknown measures of the two triangles.

Solve for $m\angle C_1$.

$$30° + 50.6° + m\angle C_1 = 180°$$
$$m\angle C_1 = 99.4°$$

Solve for $m\angle C_2$.

$$30° + 129.4° + m\angle C_2 = 180°$$
$$m\angle C_2 = 20.6°$$

Solve for c_1.

$$\frac{\sin A}{a} = \frac{\sin C_1}{c_1} \qquad \textit{Law of Sines}$$

$$\frac{\sin 30°}{11} = \frac{\sin 99.4°}{c_1} \qquad \textit{Substitute.}$$

$$c_1 = \frac{11\sin 99.4°}{\sin 30°} \qquad \textit{Solve for the unknown side.}$$

$$c_1 \approx 21.7 \text{ cm}$$

Solve for c_2.

$$\frac{\sin A}{a} = \frac{\sin C_2}{c_2}$$

$$\frac{\sin 30°}{11} = \frac{\sin 20.6°}{c_2}$$

$$c_2 = \frac{11\sin 20.6°}{\sin 30°}$$

$$c_2 \approx 7.7 \text{ cm}$$

CHECK IT OUT!

3. Determine the number of triangles Maggie can form using the measurements $a = 10$ cm, $b = 6$ cm, and $m\angle A = 105°$. Then solve the triangles. Round to the nearest tenth.

THINK AND DISCUSS

1. Explain how right triangle trigonometry can be used to determine the area of an obtuse triangle.

2. Explain why using the Law of Sines when given AAS or ASA is different than when given SSA.

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, give the conditions for which the ambiguous case results in zero, one, or two triangles.

SSA: Given *a*, *b*, and m∠*A*			
Angle *A*	**0 triangles**	**1 triangle**	**2 triangles**
Obtuse			
Acute			

13-5 Exercises

California Standards
Preview of Trig 13.0, 14.0, and 19.0

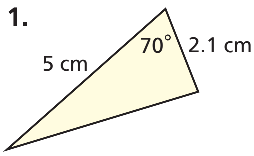
go.hrw.com
Homework Help Online
KEYWORD: MB7 13-5
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

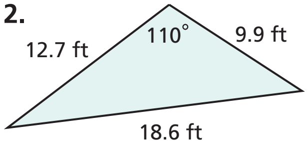
SEE EXAMPLE 1
p. 958

Find the area of each triangle. Round to the nearest tenth.

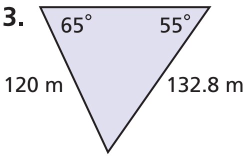
1.



2.



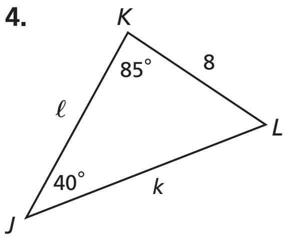
3.



SEE EXAMPLE 2
p. 959

Solve each triangle. Round to the nearest tenth.

4.



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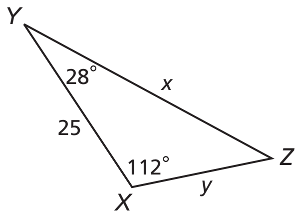

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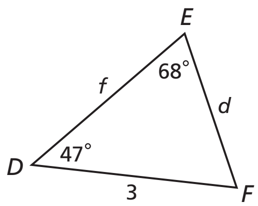

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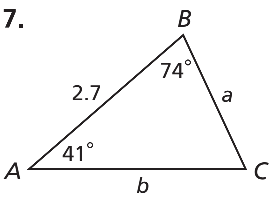

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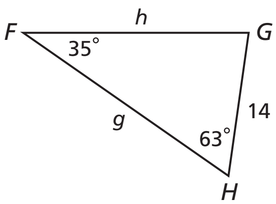
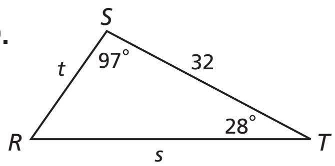


9.



SEE EXAMPLE 3
p. 961

Gardening A landscape architect is designing triangular flower beds. Determine the number of different triangles that he can form using the given measurements. Then solve the triangles. Round to the nearest tenth.

10. $a = 6$ m, $b = 9$ m, m∠$A = 55°$

11. $a = 10$ m, $b = 4$ m, m∠$A = 120°$

12. $a = 8$ m, $b = 9$ m, m∠$A = 35°$

13. $a = 7$ m, $b = 6$ m, m∠$A = 45°$

PRACTICE AND PROBLEM SOLVING

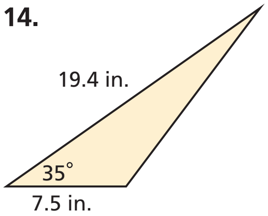
Independent Practice

For Exercises	See Example
14–16	1
17–19	2
20–23	3

Extra Practice

Skills Practice p. S29
Application Practice p. S44

Find the area of each triangle. Round to the nearest tenth.

14.


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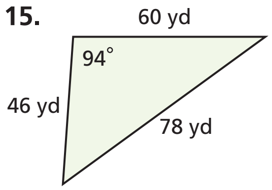
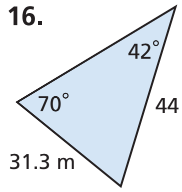
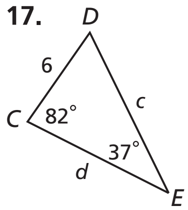
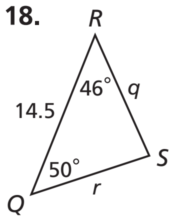
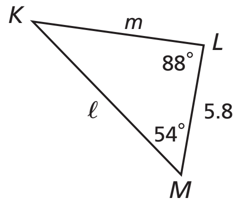


16.


Solve each triangle. Round to the nearest tenth.

17.


18.


19.


Art An artist is designing triangular mirrors. Determine the number of different triangles that she can form using the given measurements. Then solve the triangles. Round to the nearest tenth.

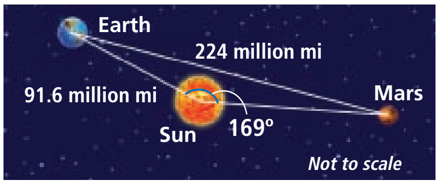
20. $a = 6$ cm, $b = 4$ cm, $m\angle A = 72°$

21. $a = 3.0$ in., $b = 3.5$ in., $m\angle A = 118°$

22. $a = 4.2$ cm, $b = 5.7$ cm, $m\angle A = 39°$

23. $a = 7$ in., $b = 3.5$ in., $m\angle A = 130°$

24. **Astronomy** The diagram shows the relative positions of Earth, Mars, and the Sun on a particular date. What is the distance between Mars and the Sun on this date? Round to the nearest million miles.



Use the given measurements to solve △*ABC*. Round to the nearest tenth.

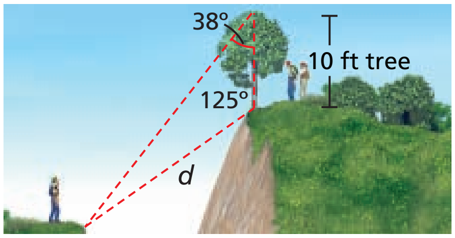
25. $m\angle A = 54°$, $m\angle B = 62°$, $a = 14$

26. $m\angle A = 126°$, $m\angle C = 18°$, $c = 3$

27. $m\angle B = 80°$, $m\angle C = 41°$, $b = 25$

28. $m\angle A = 24°$, $m\angle B = 104°$, $c = 10$

29. **Rock Climbing** A group of climbers needs to determine the distance from one side of a ravine to another. They make the measurements shown. To the nearest foot, what is the distance *d* across the ravine?



Determine the number of different triangles that can be formed using the given measurements. Then solve the triangles. Round to the nearest tenth.

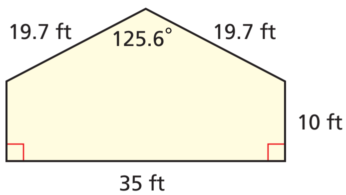
30. $m\angle C = 45°$, $b = 10$, $c = 5$

31. $m\angle B = 135°$, $b = 12$, $c = 8$

32. $m\angle A = 60°$, $a = 9$, $b = 10$

33. $m\angle B = 30°$, $a = 6$, $b = 3$

34. **Painting** Trey needs to paint a side of a house that has the measurements shown. What is the area of this side of the house to the nearest square foot?



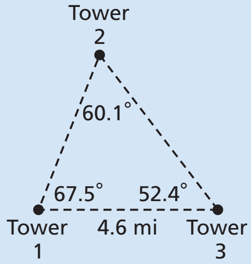
CONCEPT CONNECTION

35. This problem will prepare you for the Concept Connection on page 974.

An emergency dispatcher must determine the position of a caller reporting a fire. Based on the caller's cell phone records, she is located in the area shown.

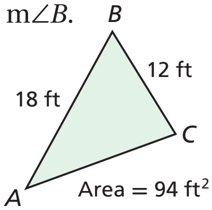
a. To the nearest tenth of a mile, what are the unknown side lengths of the triangle?

b. What is the area in square miles of the triangle in which the caller is located? Round to the nearest tenth.

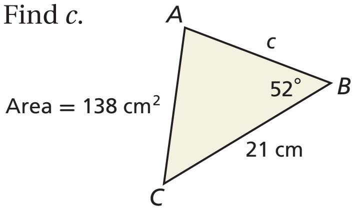


Find the indicated measurement. Round to the nearest tenth.

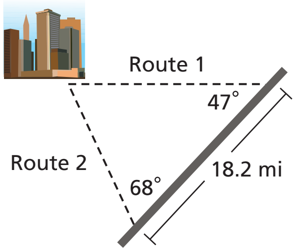
36. Find m∠*B*.



37. Find *c*.



38. Multi-Step A new road will be built from a town to a nearby highway. So far, two routes have been proposed. To the nearest tenth of a mile, how much shorter is route 2 than route 1?



39. ///ERROR ANALYSIS/// Below are two attempts at solving △*FGH* for *g*. Which is incorrect? Explain the error.

A

$$\frac{\sin 41°}{4.8} = \frac{\sin 74°}{g}$$

$$g = \frac{4.8 \sin 74°}{\sin 41°}$$

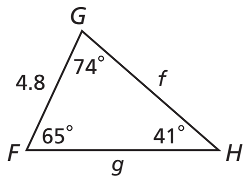
$$g \approx 7.0$$

B

$$\frac{\sin 74°}{4.8} = \frac{\sin 41°}{g}$$

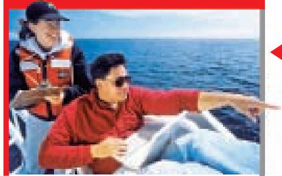
$$g = \frac{4.8 \sin 41°}{\sin 74°}$$

$$g \approx 3.3$$



LINK

Navigation



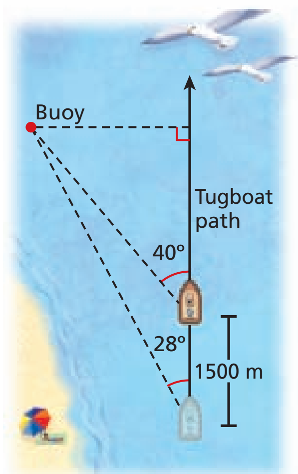
Ship captains rely heavily on GPS technology. GPS stands for "Global Positioning System," a navigation system based on trigonometry and the use of satellites. The GPS can be used to determine information such as latitude and longitude.

40. Navigation As a tugboat travels along a channel, the captain sights a buoy at an angle of 28° to the boat's path. The captain continues on the same course for a distance of 1500 m and then sights the same buoy at an angle of 40°.

a. To the nearest meter, how far is the tugboat from the buoy at the second sighting?

b. To the nearest meter, how far was the tugboat from the buoy when the captain first sighted the bouy?

c. What if...? If the tugboat continues on the same course, what is the closest that it will come to the buoy? Round to the nearest meter.



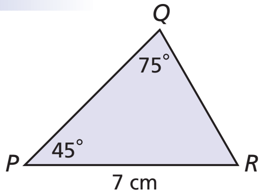
41. Critical Thinking How can you tell, without using the Law of Sines, that a triangle cannot be formed by using the measurements m∠*A* = 92°, m∠*B* = 104°, and *a* = 18?

42. Write About It Explain how to solve a triangle when angle-angle-side (AAS) information is known.

STANDARDIZED TEST PREP

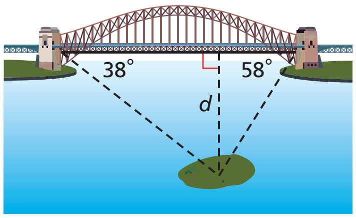
43. What is the area of $\triangle PQR$ to the nearest tenth of a square centimeter?

Ⓐ 2.4 cm^2 Ⓑ 15.5 cm^2 Ⓒ 23.5 cm^2 Ⓓ 40.1 cm^2



44. A bridge is 325 m long. From the west end, a surveyor measures the angle between the bridge and an island to be 38°. From the east end, the surveyor measures the angle between the bridge and the island to be 58°. To the nearest meter, what is the distance *d* between the bridge and the island?

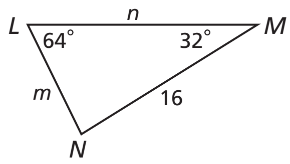
Ⓕ 171 m Ⓖ 201 m Ⓗ 217 m Ⓙ 277 m



45. **Short Response** Examine $\triangle LMN$ at right.

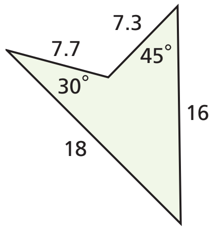
a. Write an expression that can be used to determine the value of *m.*

b. Is there more than one possible triangle that can be constructed from the given measurements? Explain your answer.



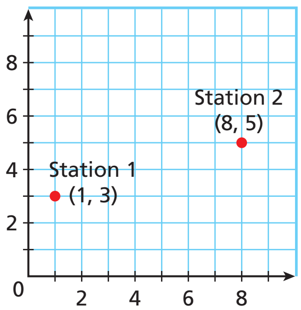
CHALLENGE AND EXTEND

46. What is the area of the quadrilateral at right to the nearest square unit?



47. **Critical Thinking** The lengths of two sides of a triangle are $a = 3$ and $b = 2\sqrt{3}$. For what values of $m\angle A$ do two solutions exist when you solve the triangle by using the Law of Sines?

48. **Multi-Step** The map shows the location of two ranger stations. Each unit on the map represents 1 mile. A ranger at station 1 saw a meteor that appeared to land about 72° north of east. A ranger at station 2 saw the meteor appear to land about 45° north of west. Based on this information, about how many miles from station 1 did the meteor land? Explain how you determined your answer.



SPIRAL REVIEW

Find the intercepts of each line, and graph the line. *(Lesson 2-3)*

49. $x + y = 5$ **50.** $3x - y = 9$ **51.** $2x + 6y = 12$

Solve each equation. *(Lesson 8-5)*

52. $\dfrac{4x}{x-1} = \dfrac{2x+9}{x-1}$ **53.** $\dfrac{7x-9}{x^2-4} = \dfrac{4}{x+2}$ **54.** $\dfrac{x}{3} - \dfrac{4x}{7} = \dfrac{x-2}{3}$

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees. *(Lesson 13-4)*

55. $\text{Cos}^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$ **56.** $\text{Tan}^{-1}\sqrt{3}$ **57.** $\text{Sin}^{-1}\dfrac{1}{2}$

13-6 The Law of Cosines

Objectives

Use the Law of Cosines to find the side lengths and angle measures of a triangle.

Use Heron's Formula to find the area of a triangle.

California Standards

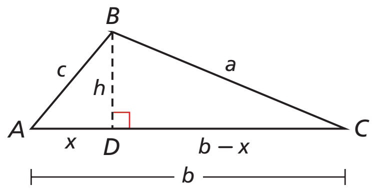
Preview of Trigonometry
13.0 Students know the law of sines and the law of cosines and apply those laws to solve problems.
Also covered: **Preview of Trig 19.0**

Who uses this?

Trapeze artists can use the Law of Cosines to determine whether they can perform stunts safely. (See Exercise 27.)

In the previous lesson, you learned to solve triangles by using the Law of Sines. However, the Law of Sines cannot be used to solve triangles for which side-angle-side (SAS) or side-side-side (SSS) information is given. Instead, you must use the Law of Cosines.

To derive the Law of Cosines, draw $\triangle ABC$ with altitude $\overline{BD}$. If x represents the length of $\overline{AD}$, the length of $\overline{DC}$ is $b - x$.



Write an equation that relates the side lengths of $\triangle DBC$.

$a^2 = (b - x)^2 + h^2$ — *Pythagorean Theorem*

$a^2 = b^2 - 2bx + x^2 + h^2$ — *Expand $(b - x)^2$.*

$a^2 = b^2 - 2bx + c^2$ — *In $\triangle ABD$, $c^2 = x^2 + h^2$. Substitute c^2 for $x^2 + h^2$.*

$a^2 = b^2 - 2b(c\cos A) + c^2$ — *In $\triangle ABD$, $\cos A = \frac{x}{c}$, or $x = c\cos A$. Substitute $c\cos A$ for x.*

$a^2 = b^2 + c^2 - 2bc\cos A$

The previous equation is one of the formulas for the Law of Cosines.

Know it! Note

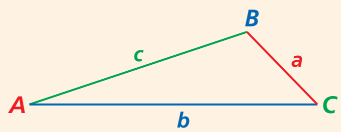
Law of Cosines

For $\triangle ABC$, the Law of Cosines states that

$a^2 = b^2 + c^2 - 2bc\cos A.$

$b^2 = a^2 + c^2 - 2ac\cos B.$

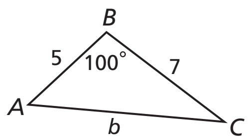
$c^2 = a^2 + b^2 - 2ab\cos C.$



EXAMPLE 1 Using the Law of Cosines

Use the given measurements to solve $\triangle ABC$. Round to the nearest tenth.

A



Step 1 Find the length of the third side.

$b^2 = a^2 + c^2 - 2ac\cos B$ — *Law of Cosines*

$b^2 = 7^2 + 5^2 - 2(7)(5)\cos 100°$ — *Substitute.*

$b^2 \approx 86.2$ — *Use a calculator to simplify.*

$b \approx 9.3$ — *Solve for the positive value of b.*

Step 2 Find an angle measure.

$\frac{\sin A}{a} = \frac{\sin B}{b}$ *Law of Sines*

$\frac{\sin A}{7} = \frac{\sin 100°}{9.3}$ *Substitute.*

$\sin A = \frac{7 \sin 100°}{9.3}$ *Solve for sin A.*

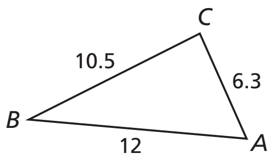
$m\angle A = \text{Sin}^{-1}\left(\frac{7 \sin 100°}{9.3}\right) \approx 47.8°$ *Solve for m∠A.*

Step 3 Find the third angle measure.

$47.8° + 100° + m\angle C \approx 180°$ *Triangle Sum Theorem*

$m\angle C \approx 32.2°$ *Solve for m∠C.*

B



Step 1 Find the measure of the largest angle, ∠C.

$c^2 = a^2 + b^2 - 2ab \cos C$ *Law of Cosines*

$12^2 = 10.5^2 + 6.3^2 - 2(10.5)(6.3) \cos C$ *Substitute.*

$\cos C \approx 0.0449$ *Solve for cos C.*

$m\angle C \approx \text{Cos}^{-1}(0.0449) \approx 87.4°$ *Solve for m∠C.*

Step 2 Find another angle measure.

$b^2 = a^2 + c^2 - 2ac \cos B$ *Law of Cosines*

$6.3^2 = 10.5^2 + 12^2 - 2(10.5)(12) \cos B$ *Substitute.*

$\cos B \approx 0.8514$ *Solve for cos B.*

$m\angle B \approx \text{Cos}^{-1}(0.8514) \approx 31.6°$ *Solve for m∠B.*

Step 3 Find the third angle measure.

$m\angle A + 31.6° + 87.4° \approx 180°$ *Triangle Sum Theorem*

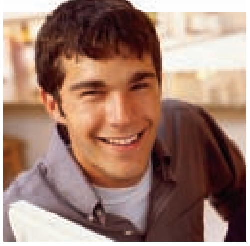
$m\angle A \approx 61.0°$ *Solve for m∠A.*

CHECK IT OUT! **Use the given measurements to solve △ABC. Round to the nearest tenth.**

1a. $b = 23$, $c = 18$, $m\angle A = 173°$ **1b.** $a = 35$, $b = 42$, $c = 50.3$

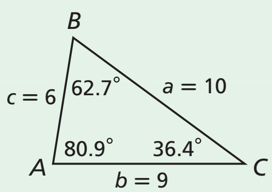
Student to Student

Solving Triangles



Stefan Maric
Wylie High School

If I solve a triangle using the Law of Sines, I like to use the Law of Cosines to check my work. I used the Law of Sines to solve the triangle below.



I can check that the length of side b really is 9 by using the Law of Cosines.

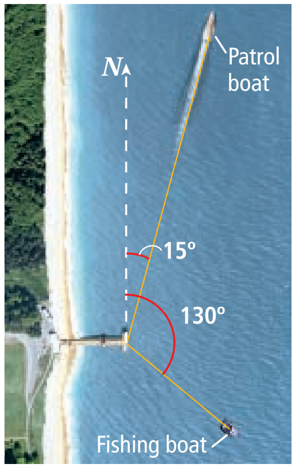
$b^2 = a^2 + c^2 - 2ac \cos B$

9^2	$10^2 + 6^2 - 2(10)(6) \cos 62.7°$
81	81.0 ✔

The Law of Cosines shows that I was right.

EXAMPLE 2 Problem-Solving Application

A coast guard patrol boat and a fishing boat leave a dock at the same time on the courses shown. The patrol boat travels at a speed of 12 nautical miles per hour (12 knots), and the fishing boat travels at a speed of 5 knots. After 3 hours, the fishing boat sends a distress signal picked up by the patrol boat. If the fishing boat does not drift, how long will it take the patrol boat to reach it at a speed of 12 knots?



1 Understand the Problem

The **answer** will be the number of hours that the patrol boat needs to reach the fishing boat.

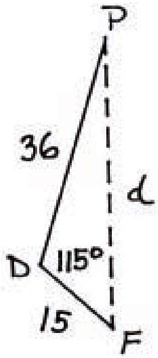
List the important information:

- The patrol boat's speed is 12 knots. Its direction is 15° east of north.
- The fishing boat's speed is 5 knots. Its direction is 130° east of north.
- The boats travel 3 hours before the distress call is given.

2 Make a Plan

Determine the angle between the boats' courses and the distance that each boat travels in 3 hours. Use this information to draw and label a diagram. Then use the Law of Cosines to find the distance d between the boats at the time of the distress call. Finally, determine how long it will take the patrol boat to travel this distance.

3 Solve



Step 1 Draw and label a diagram.

The angle between the boats' courses is $130° - 15° = 115°$. In 3 hours, the patrol boat travels $3(12) = 36$ nautical miles and the fishing boat travels $3(5) = 15$ nautical miles.

Step 2 Find the distance d between the boats.

$d^2 = p^2 + f^2 - 2pf\cos D$ — *Law of Cosines*

$d^2 = 15^2 + 36^2 - 2(15)(36)\cos 115°$ — *Substitute 15 for p, 36 for f, and 115° for D.*

$d^2 \approx 1977.4$ — *Use a calculator to simplify.*

$d \approx 44.5$ — *Solve for the positive value of d.*

Step 3 Determine the number of hours.

The patrol boat must travel about 44.5 nautical miles to reach the fishing boat. At a speed of 12 nautical miles per hour, it will take the patrol boat $\frac{44.5}{12} \approx 3.7$ hours to reach the fishing boat.

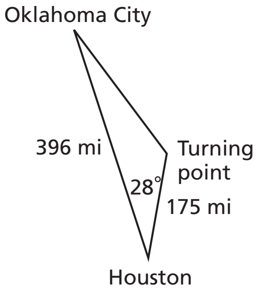
4 Look Back

To reach the fishing boat, the patrol boat will have to travel a greater distance than it did during the first 3 hours of its trip. Therefore, it makes sense that it will take the patrol boat longer than 3 hours to reach the fishing boat. An answer of 3.7 hours seems reasonable.

Helpful Hint

There are two solutions to $d^2 = 1977.4$. One is positive, and one is negative. Because d represents a distance, the negative solution can be disregarded.

2. A pilot is flying from Houston to Oklahoma City. To avoid a thunderstorm, the pilot flies 28° off of the direct route for a distance of 175 miles. He then makes a turn and flies straight on to Oklahoma City. To the nearest mile, how much farther than the direct route was the route taken by the pilot?

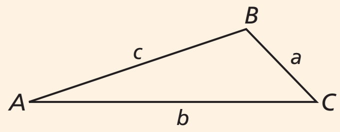


The Law of Cosines can be used to derive a formula for the area of a triangle based on its side lengths. This formula is called Heron's Formula.

Heron's Formula

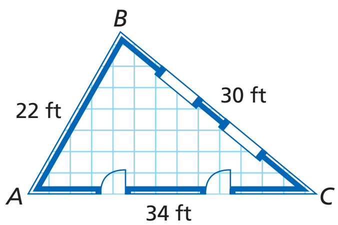
For $\triangle ABC$, where s is half of the perimeter of the triangle, or $\frac{1}{2}(a + b + c)$,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$



EXAMPLE 3 Architecture Application

A blueprint shows a reception area that has a triangular floor with sides measuring 22 ft, 30 ft, and 34 ft. What is the area of the floor to the nearest square foot?



Step 1 Find the value of s.

$s = \frac{1}{2}(a + b + c)$ — *Use the formula for half of the perimeter.*

$s = \frac{1}{2}(30 + 34 + 22) = 43$ — *Substitute 30 for a, 34 for b, and 22 for c.*

Step 2 Find the area of the triangle.

$A = \sqrt{s(s-a)(s-b)(s-c)}$ — *Heron's Formula*

$A = \sqrt{43(43-30)(43-34)(43-22)}$ — *Substitute 43 for s.*

$A \approx 325$ — *Use a calculator to simplify.*

The area of the floor is 325 ft^2.

Check Find the measure of the largest angle, $\angle B$.

$b^2 = a^2 + c^2 - 2ac\cos B$ — *Law of Cosines*

$34^2 = 30^2 + 22^2 - 2(30)(22)\cos B$ — *Substitute.*

$\cos B \approx 0.1727$ — *Solve for cos B.*

$m\angle B \approx 80.1°$ — *Solve for m∠B.*

Find the area of the triangle by using the formula area $= \frac{1}{2}ac\sin B$.

area $= \frac{1}{2}(30)(22)\sin 80.1° \approx 325\ \text{ft}^2$ ✔

3. The surface of a hotel swimming pool is shaped like a triangle with sides measuring 50 m, 28 m, and 30 m. What is the area of the pool's surface to the nearest square meter?

THINK AND DISCUSS

1. Explain why you cannot solve a triangle if you are given only angle-angle-angle (AAA) information.
2. Describe the steps that you could use to find the area of a triangle by using Heron's Formula when you are given side-angle-side (SAS) information.
3. **GET ORGANIZED** Copy and complete the graphic organizer. List the types of triangles that can be solved by using each law. Consider the following types of triangles: ASA, AAS, SAS, SSA, and SSS.

Know it! Note

13-6 Exercises

California Standards
Preview of Trig 13.0 and 19.0; 8.0

go.hrw.com
Homework Help Online
KEYWORD: MB7 13-6
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 966

Use the given measurements to solve each triangle. Round to the nearest tenth.

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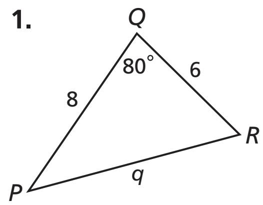

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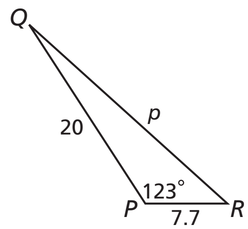

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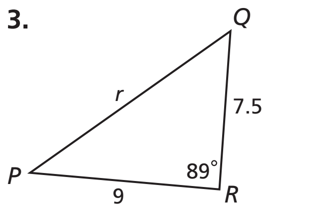

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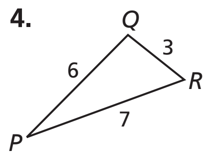

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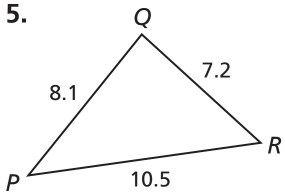

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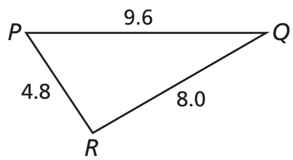
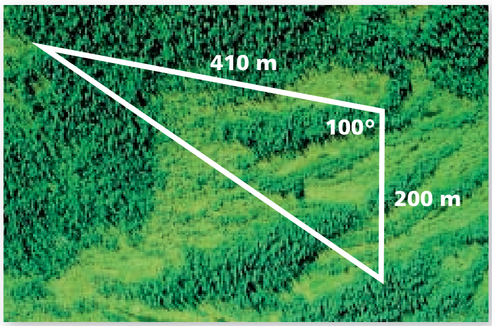


SEE EXAMPLE 2 p. 968

7. **Recreation** A triangular hiking trail is being built in the area shown. At an average walking speed of 2 m/s, how many minutes will it take a hiker to make a complete circuit around the triangular trail? Round to the nearest minute.



SEE EXAMPLE 3 p. 969

8. **Agriculture** A triangular wheat field has side lengths that measure 410 ft, 500 ft, and 420 ft. What is the area of the field to the nearest square foot?

PRACTICE AND PROBLEM SOLVING

Independent Practice

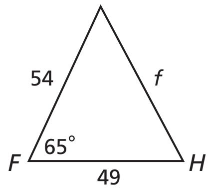
For Exercises	See Example
9–14	1
15	2
16	3

Extra Practice

Skills Practice p. S29

Application Practice p. S44

Use the given measurements to solve each triangle. Round to the nearest tenth.

9.


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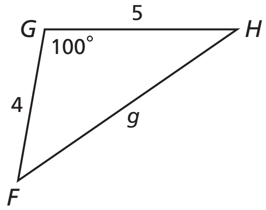

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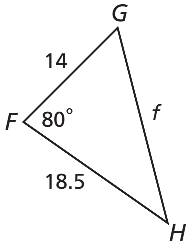

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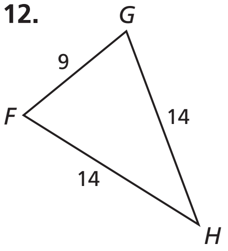

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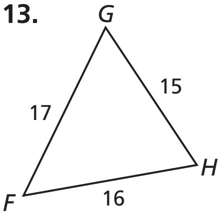
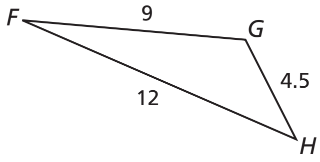
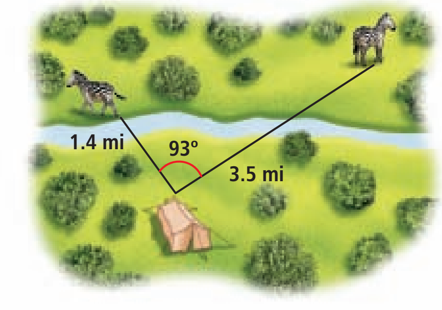


14.


15. Ecology An ecologist is studying a pair of zebras fitted with radio-transmitter collars. One zebra is 1.4 mi from the ecologist, and the other is 3.5 mi from the ecologist. To the nearest tenth of a mile, how far apart are the two zebras?



16. Art How many square meters of fabric are needed to make a triangular banner with side lengths of 2.1 m, 1.5 m, and 1.4 m? Round to the nearest tenth.

Use the given measurements to solve $\triangle ABC$. Round to the nearest tenth.

17. $m\angle A = 120°$, $b = 16$, $c = 20$

18. $m\angle B = 78°$, $a = 6$, $c = 4$

19. $m\angle C = 96°$, $a = 13$, $b = 9$

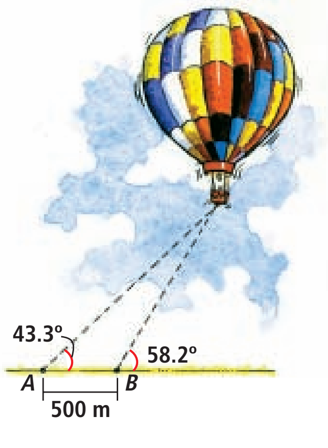
20. $a = 14$, $b = 9$, $c = 10$

21. $a = 5$, $b = 8$, $c = 6$

22. $a = 30$, $b = 26$, $c = 35$

23. Commercial Art A graphic artist is asked to draw a triangular logo with sides measuring 15 cm, 18 cm, and 20 cm. If she draws the triangle correctly, what will be the measures of its angles to the nearest degree?

24. Aviation The course of a hot-air balloon takes the balloon directly over points A and B, which are 500 m apart. Several minutes later, the angle of elevation from an observer at point A to the balloon is 43.3°, and the angle of elevation from an observer at point B to the balloon is 58.2°. To the nearest meter, what is the balloon's altitude?



25. Multi-Step A student pilot takes off from a local airstrip and flies 70° south of east for 160 miles. The pilot then changes course and flies due north for another 80 miles before turning and flying directly back to the airstrip.

a. How many miles is the third stage of the pilot's flight? Round to the nearest mile.

b. To the nearest degree, what angle does the pilot turn the plane through in order to fly the third stage?

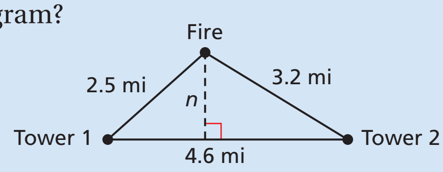
CONCEPT CONNECTION

26. This problem will help prepare you for the Concept Connection on page 974.

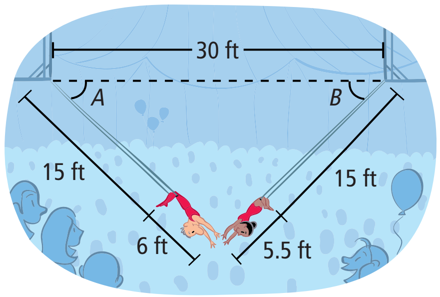
Phone records indicate that a fire is located 2.5 miles from one cell phone tower and 3.2 miles from a second cell phone tower.

a. To the nearest degree, what are the measures of the angles of the triangle shown in the diagram?

b. Tower 2 is directly east of tower 1. How many miles north of the towers is the fire? This distance is represented by n in the diagram.



27. Entertainment Two performers hang by their knees from trapezes, as shown.



a. To the nearest degree, what acute angles A and B must the cords of each trapeze make with the horizontal if the performer on the left is to grab the wrists of the performer on the right and pull her away from her trapeze?

b. What if...? Later, the performer on the left grabs the trapeze of the performer on the right and lets go of his trapeze. To the nearest degree, what angles A and B must the cords of each trapeze make with the horizontal for this trick to work?

Find the area of the triangle with the given side lengths. Round to the nearest tenth.

28. 15 in., 18 in., 24 in.

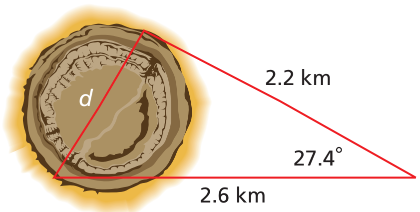
29. 30 cm, 35 cm, 47 cm

30. 28 m, 37 m, 33 m

31. 3.5 ft, 5 ft, 7.5 ft

32. Estimation The adjacent sides of a parallelogram measure 3.1 cm and 3.9 cm. The measures of the acute interior angles of the parallelogram are 58°. Estimate the lengths of the diagonals of the parallelogram without using a calculator, and explain how you determined your estimates.

33. Surveying Barrington Crater in Arizona was produced by the impact of a meteorite. Based on the measurements shown, what is the diameter d of Barrington Crater to the nearest tenth of a kilometer?



34. Travel The table shows the distances between three islands in Hawaii. To the nearest degree, what is the angle between each pair of islands in relation to the third island?

Distances Between Islands (mi)

	Kauai	Molokai	Lanai
Kauai	0	155.7	174.8
Molokai	155.7	0	26.1
Lanai	174.8	26.1	0

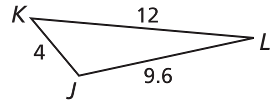
35. Critical Thinking Use the Law of Cosines to explain why $c^2 = a^2 + b^2$ for $\triangle ABC$, where $\angle C$ is a right angle.

36. Critical Thinking Can the value of s in Heron's Formula ever be less than the length of the longest side of a triangle? Explain.

37. Write About It Describe two different methods that could be used to solve a triangle when given side-side-side (SSS) information.

STANDARDIZED TEST PREP

38. What is the approximate measure of $\angle K$ in the triangle shown?



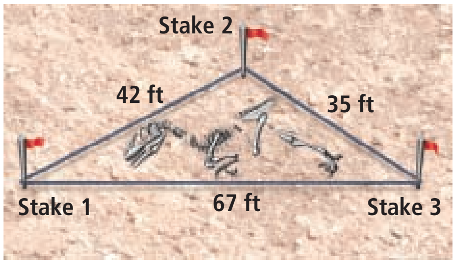
Ⓐ 30° Ⓒ 54°
Ⓑ 45° Ⓓ 60°

39. For $\triangle RST$ with side lengths r, s, and t, which equation can be used to determine r?

Ⓕ $r = \sqrt{s^2 + t^2 - 2st\sin R}$ Ⓗ $r = \sqrt{s^2 + t^2 - 2st\cos R}$

Ⓖ $r = \sqrt{s^2 - t^2 - 2st\sin R}$ Ⓙ $r = \sqrt{s^2 - t^2 - 2st\cos R}$

40. A team of archaeologists wants to dig for fossils in a triangular area marked by three stakes. The distances between the stakes are shown in the diagram. Which expression represents the dig area in square feet?



Ⓐ $\sqrt{72(30)(37)(5)}$

Ⓑ $\sqrt{48(6)(13)(19)}$

Ⓒ $\sqrt{144(42)(35)(67)}$

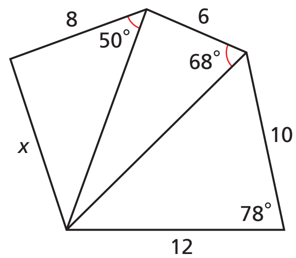
Ⓓ $\sqrt{144(102)(109)(77)}$

CHALLENGE AND EXTEND

41. Abby uses the Law of Cosines to find $m\angle A$ when $a = 2$, $b = 3$, and $c = 5$. The answer she gets is 0°. Did she make an error? Explain.

42. **Geometry** What are the angle measures of an isosceles triangle whose base is half as long as its congruent legs? Round to the nearest tenth.

43. Use the figure shown to solve for x. Round to the nearest tenth.



SPIRAL REVIEW

Solve each equation. *(Lesson 5-5)*

44. $x^2 + 25 = 0$ **45.** $3x^2 = -48$ **46.** $\frac{1}{2}x^2 + 18 = 0$

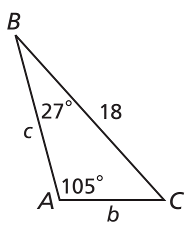
Identify the x- and y-intercepts of $f(x)$. Without graphing $g(x)$, identify its x- and y-intercepts. *(Lesson 9-3)*

47. $f(x) = 2x - 8$ and $g(x) = \frac{1}{2}f(x)$ **48.** $f(x) = x^2 - 4$ and $g(x) = -f(x)$

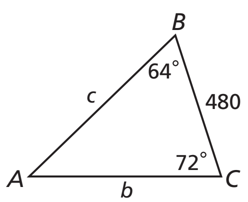
49. $f(x) = \frac{1}{2}x + 6$ and $g(x) = f(3x)$ **50.** $f(x) = x^3 + 1$ and $g(x) = -4f(x)$

Solve each triangle. Round to the nearest tenth. *(Lesson 13-5)*

51.



52.



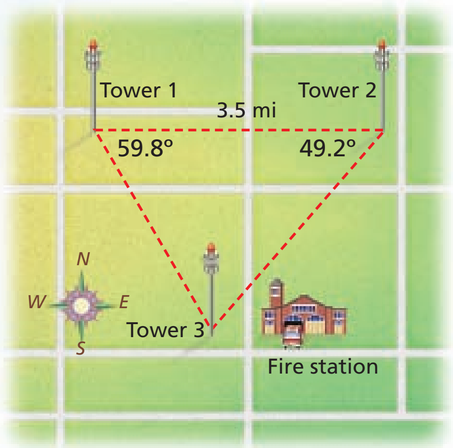
CHAPTER 13

CONCEPT CONNECTION

SECTION 13B

Applying Trigonometric Functions

Where's the Fire? A driver dials 9-1-1 on a cell phone to report smoke coming from a building. Before the driver can give the building's address, the call is cut short. The 9-1-1 dispatcher is still able to determine the driver's location based on the driver's position in relation to nearby cell phone towers.



1. When the driver makes the call, he is located in the triangular area between the three cell phone towers shown. To the nearest tenth of a mile, what is the distance between tower 3 and each of the other towers?
2. What is the area in square miles of the triangle with the three cell phone towers at its vertices? Round to the nearest tenth.
3. The driver is 1.7 miles from tower 1 and 2.4 miles from tower 2 when the call is made. Make a sketch of the triangle with tower 1, tower 2, and the driver at its vertices. To the nearest tenth of a degree, what are the measures of the angles of this triangle?
4. Tower 2 is directly east of tower 1. How many miles east of tower 1 is the driver? How many miles south of tower 1 is the driver? Round to the nearest tenth.
5. A fire station is located 1 block from tower 3. Estimate the distance between the fire station and the fire. Explain how you determined your estimate.

READY TO GO ON?

CHAPTER 13

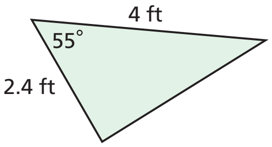
SECTION 13B

Quiz for Lessons 13-5 Through 13-6

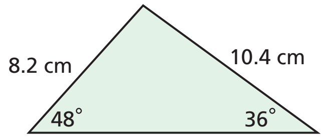
13-5 The Law of Sines

Find the area of each triangle. Round to the nearest tenth.

1.

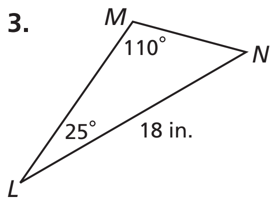


2.

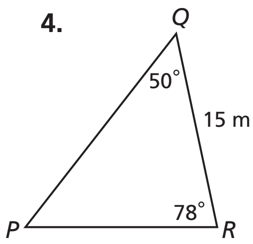


Solve each triangle. Round to the nearest tenth.

3.



4.

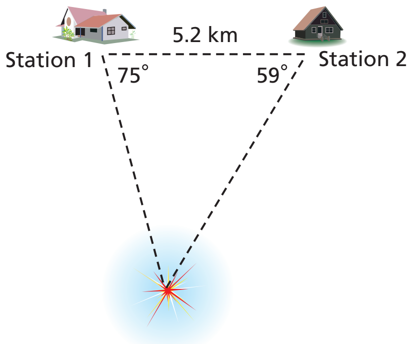


Derrick is designing triangular panes for a stained glass window. Determine the number of different triangles that he can form using the given measurements. Then solve the triangles. Round to the nearest tenth.

5. $a = 2.1$ cm, $b = 1.8$ cm, $m\angle A = 42°$

6. $a = 3$ cm, $b = 4.6$ cm, $m\angle A = 95°$

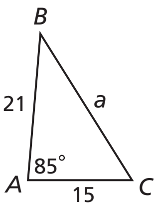
7. The rangers at two park stations spot a signal flare at the same time. Based on the measurements shown in the diagram, what is the distance between each park station and the point where the flare was set off? Round to the nearest tenth.



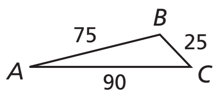
13-6 The Law of Cosines

Use the given measurements to solve each triangle. Round to the nearest tenth.

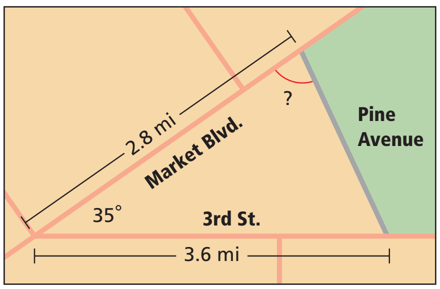
8.



9.



10. A civil engineer is working on plans for a new road called Pine Avenue. This road will intersect Market Boulevard and 3rd Street as shown. To the nearest degree, what is the measure of the angle that Pine Avenue will make with Market Boulevard?



11. A school courtyard is shaped like a triangle. Its sides measure 25 yards, 27.5 yards, and 32 yards. What is the area of the courtyard to the nearest square yard?

CHAPTER 13 Study Guide: Review

- angle of rotation 936
- cosecant 932
- cosine 929
- cotangent 932
- coterminal angle 937
- initial side. 936
- inverse cosine function. 951
- inverse sine function. 951
- inverse tangent function 951
- radian 943
- reference angle 937
- secant 932
- sine. 929
- standard position 936
- tangent 929
- terminal side 936
- trigonometric function. 929
- unit circle 944

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is a unit of angle measure based on arc length.

2. The ___?___ of an acute angle in a right triangle is the ratio of the length of the hypotenuse to the length of the opposite leg.

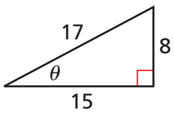
3. An angle in ___?___ has its vertex at the origin and one ray on the positive x-axis.

13-1 Right-Angle Trigonometry *(pp. 929–935)*

Review of GE 19.0

EXAMPLES

■ **Find the values of the sine, cosine, and tangent functions for θ.**

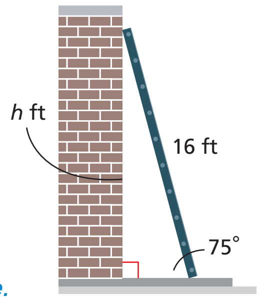


$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{8}{17}$$

$$\cos\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{15}{17}$$

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{8}{15}$$

■ **A 16 ft ladder is leaned against a building as shown. How high up the building does the ladder reach?**



$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$$

$$\sin 75° = \frac{h}{16} \quad \textit{Substitute.}$$

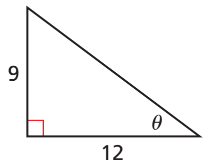
$$h = 16\sin 75° \approx 15.5 \quad \textit{Solve for h.}$$

The ladder reaches about 15.5 ft up the building.

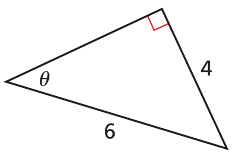
EXERCISES

Find the values of the six trigonometric functions for θ.

4.

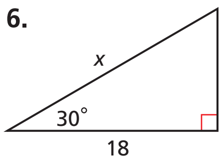


5.

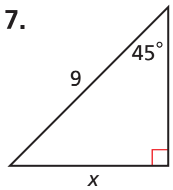


Use a trigonometric function to find the value of x.

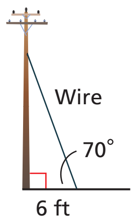
6.



7.



8. A support wire is being attached to a telephone pole as shown in the diagram. To the nearest foot, how long does the wire need to be?



9. The angle of depression from a watchtower to a forest fire is 8°. If the watchtower is 25 m high, what is the distance between the base of the tower and the fire to the nearest meter?

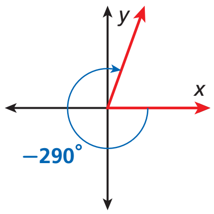
13-2 Angles of Rotation *(pp. 936–941)*

Preview of Trig **1.0** and **9.0**

EXAMPLES

■ **Draw a $-290°$ angle in standard position.**

Rotate the terminal side 290° clockwise.



■ **$P(-5, 12)$ is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions of θ.**

$$r = \sqrt{(-5)^2 + (12)^2} = 13 \qquad \textit{Find } r.$$

$$\sin\theta = \frac{y}{r} = \frac{12}{13} \qquad \cos\theta = \frac{x}{r} = \frac{-5}{13} = -\frac{5}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{12}{-5} = -\frac{12}{5} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{13}{12}$$

$$\sec\theta = \frac{1}{\cos\theta} = -\frac{13}{5} \qquad \cot\theta = \frac{1}{\tan\theta} = -\frac{5}{12}$$

EXERCISES

Draw an angle with the given measure in standard position.

10. $195°$ **11.** $-220°$ **12.** $-450°$

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

13. $\theta = 115°$ **14.** $\theta = 382°$ **15.** $\theta = -135°$

Find the measure of the reference angle for each given angle.

16. $\theta = 84°$ **17.** $\theta = 127°$ **18.** $\theta = -105°$

***P* is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ.**

19. $P(-4, 3)$ **20.** $P(5, 12)$ **21.** $P(-15, -8)$

22. $P(8, -3)$ **23.** $P(-9, -1)$ **24.** $P(-5, 10)$

13-3 The Unit Circle *(pp. 943–949)*

Preview of Trig **1.0**, **2.0**, **9.0**, and **19.0**

EXAMPLES

Convert each measure from degrees to radians or from radians to degrees.

■ $-60°$

$$-60°\left(\frac{\pi \text{ radians}}{180°}\right) = -\frac{\pi}{3} \text{ radians}$$

■ $\frac{5\pi}{3}$ radians

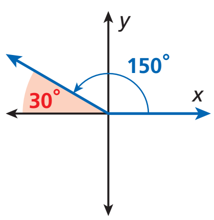
$$\left(\frac{5\pi}{3} \text{ radians}\right)\left(\frac{180°}{\pi \text{ radians}}\right) = 300°$$

■ **Use a reference angle to find the exact value of $\tan 150°$.**

Step 1 The reference angle measures 30°.

Step 2 Find the tangent of the reference angle.

$$\tan 30° = \frac{\sqrt{3}}{3}$$



Step 3 Adjust the sign, if needed.

The tangent ratio is negative if the terminal side of the angle is in Quadrant II.

$$\tan 150° = -\frac{\sqrt{3}}{3}$$

EXERCISES

Convert each measure from degrees to radians or from radians to degrees.

25. $270°$ **26.** $-120°$ **27.** $400°$

28. $\frac{\pi}{6}$ **29.** $-\frac{\pi}{9}$ **30.** $\frac{9\pi}{4}$

Use the unit circle to find the exact value of each trigonometric function.

31. $\cos 240°$ **32.** $\tan\frac{3\pi}{4}$ **33.** $\sec 300°$

Use a reference angle to find the exact value of the sine, cosine, and tangent of each angle measure.

34. $\frac{7\pi}{6}$ **35.** $300°$ **36.** $-\frac{\pi}{3}$

37. A circle has a radius of 16 in. To the nearest inch, what is the length of an arc of the circle that is intercepted by a central angle of 80°?

38. The minute hand on a clock on a town hall tower is 1.5 meters in length.

a. Find the angle in radians through which the minute hand rotates in 10 minutes.

b. To the nearest tenth of a meter, how far does the tip of the minute hand travel in 10 minutes?

13-4 Inverses of Trigonometric Functions *(pp. 950–955)*

Preview of Trig **8.0** and **19.0**

EXAMPLES

■ **Evaluate $\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. Give your answer in both radians and degrees.**

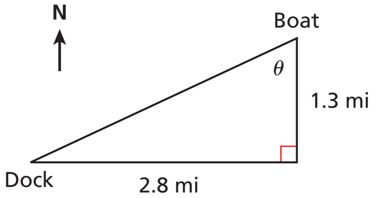
$$-\frac{\sqrt{3}}{2} = \text{Sin}\,\theta \qquad \textit{Find the value of } \theta \textit{ for } -\tfrac{\pi}{2} \le \theta \le \tfrac{\pi}{2}.$$

$$-\frac{\sqrt{3}}{2} = \text{Sin}\left(-\frac{\pi}{3}\right) \qquad \textit{Use y-coordinates of points on the unit circle.}$$

$$\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}, \text{ or } \text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -60^\circ$$

■ **A boat is 2.8 miles east and 1.3 miles north of a dock. To the nearest degree, in what direction should the boat head to reach the dock?**

Step 1 Draw a diagram.



Step 2 Find the value of θ.

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}} \qquad \textit{Use the tangent ratio.}$$

$$\tan\theta = \frac{2.8}{1.3} \qquad \textit{Substitute.}$$

$$\theta = \text{Tan}^{-1}\left(\frac{2.8}{1.3}\right) \approx 65^\circ \qquad \textit{Solve for } \theta.$$

The boat should head 65° west of south.

EXERCISES

Find all possible values of each expression.

39. $\tan^{-1}\sqrt{3}$

40. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

41. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

42. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.

43. $\text{Sin}^{-1}\left(-\frac{1}{2}\right)$

44. $\text{Tan}^{-1}\frac{\sqrt{3}}{3}$

45. $\text{Cos}^{-1}(-1)$

46. $\text{Sin}^{-1}\frac{\sqrt{2}}{2}$

47. A skateboard ramp is 39 inches long and rises to a height of 22 inches. To the nearest degree, what angle does the ramp make with the ground?

48. A parasail is a parachute that lifts a person into the air when he or she is towed by a boat. Shelley is parasailing at a height of 100 feet. If 152 feet of towline attaches her to the boat, what is the angle of depression from Shelley to the boat? Round to the nearest degree.

Solve each equation to the nearest tenth. Use the given restrictions.

49. $\sin\theta = 0.3$, for $-90^\circ \le \theta \le 90^\circ$

50. $\sin\theta = 0.3$, for $90^\circ \le \theta \le 180^\circ$

51. $\tan\theta = 2.2$, for $-90^\circ < \theta < 90^\circ$

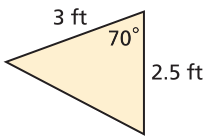
52. $\tan\theta = 2.2$, for $180^\circ \le \theta \le 270^\circ$

13-5 The Law of Sines *(pp. 958–965)*

Preview of Trig 13.0, 14.0, and **19.0**

EXAMPLES

■ **Find the area of the triangle. Round to the nearest tenth.**



$$\text{Area} = \frac{1}{2}ab\sin C \qquad \textit{Use the area formula}$$

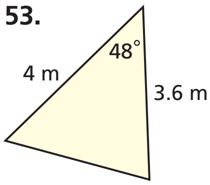
$$= \frac{1}{2}(3)(2.5)\sin 70^\circ \qquad \textit{Substitute.}$$

$$\approx 3.5 \text{ ft}^2 \qquad \textit{Evaluate.}$$

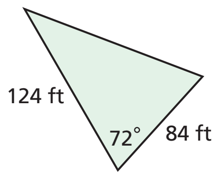
EXERCISES

Find the area of each triangle. Round to the nearest tenth.

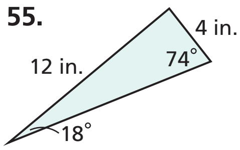
53.



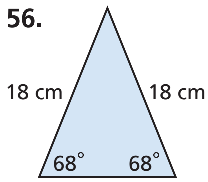
54.



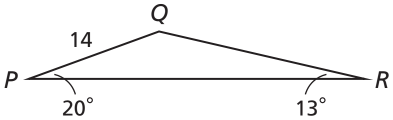
55.



56.



■ **Solve the triangle. Round to the nearest tenth.**



Step 1 Find the third angle measure.

$m\angle Q° = 180° - 20° - 13° = 147°$

Step 2 Use the Law of Sines to find the unknown side lengths.

$$\frac{\sin P}{p} = \frac{\sin R}{r} \qquad \frac{\sin Q}{q} = \frac{\sin R}{r}$$

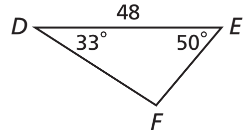
$$\frac{\sin 20°}{p} = \frac{\sin 13°}{14} \qquad \frac{\sin 147°}{q} = \frac{\sin 13°}{14}$$

$$p = \frac{14 \sin 20°}{\sin 13°} \qquad q = \frac{14 \sin 147°}{\sin 13°}$$

$$p \approx 21.3 \qquad q \approx 33.9$$

Solve each triangle. Round to the nearest tenth.

57.



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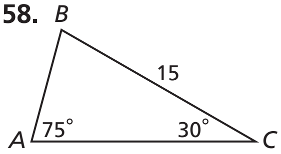
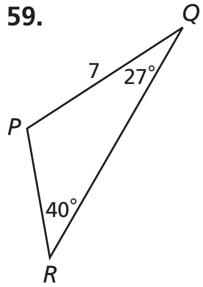
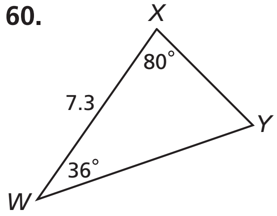


59.



60.



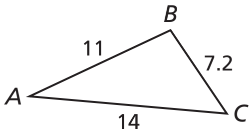
61. A graphic artist is designing a triangular logo. Determine the number of different triangles that he can form using the measurements $a = 14$ cm, $b = 16$ cm, and $m\angle A = 55°$. Then solve the triangles. Round to the nearest tenth.

13-6 The Law of Cosines *(pp. 966–973)*

Preview of Trig 13.0 and 19.0

EXAMPLES

■ **Use the given measurements to solve △ABC. Round to the nearest tenth.**



Step 1 Find the measure of the largest angle, ∠B.

$b^2 = a^2 + c^2 - 2ac \cos B$

$14^2 = 7.2^2 + 11^2 - 2(7.2)(11) \cos B$

$m\angle B \approx 98.4°$

Step 2 Find another angle measure.

$a^2 = b^2 + c^2 - 2bc \cos A$

$7.2^2 = 14^2 + 11^2 - 2(14)(11) \cos A$

$m\angle A \approx 30.6°$

Step 3 Find the third angle measure.

$m\angle C \approx 180° - 30.6° - 98.4° \approx 51.0°$

■ **A triangular tile has sides measuring 4 in., 5 in., and 8 in. What is the area of the tile to the nearest square inch?**

$s = \frac{1}{2}(4 + 5 + 8) = 8.5$ *Find the value of s.*

$A = \sqrt{s(s-a)(s-b)(s-c)}$ *Heron's Formula*

$A = \sqrt{8.5(8.5-4)(8.5-5)(8.5-8)} \approx 8.2 \text{ in}^2$

EXERCISES

Use the given measurements to solve △ABC. Round to the nearest tenth.

62. $m\angle C = 29°$, $a = 14$, $b = 30$

63. $m\angle A = 110°$, $b = 18$, $c = 12$

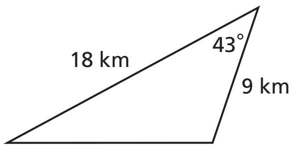
64. $a = 12$, $b = 3$, $c = 10$

65. $a = 7$, $b = 9$, $c = 11$

66. A bicycle race has a triangular course with the dimensions shown.

a. To the nearest tenth of a kilometer, how long is the race?

b. At an average speed of 28 km/h, how many hours will it take a rider to complete the race? Round to the nearest tenth.



67. A triangular wading pool has side lengths that measure 10 ft, 12 ft, and 16 ft. What is the area of the pool's surface to the nearest square foot?

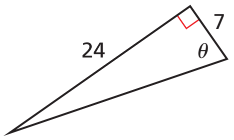
68. A triangular pennant has side lengths that measure 24 in., 24 in., and 8 in. What is the area of the pennant to the nearest square inch?

CHAPTER 13

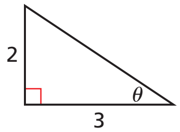
CHAPTER TEST

Find the values of the six trigonometric functions for θ.

1.



2.



3. Katrina is flying a kite on 150 ft of string. The string makes an angle of 62° with the horizontal. If Katrina holds the end of the string 5 ft above the ground, how high is the kite? Round to the nearest foot.

Draw an angle with the given measure in standard position.

4. 100°

5. $-210°$

***P* is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions of θ.**

6. $P(-32, 24)$

7. $P(-3, -7)$

Convert each measure from degrees to radians or from radians to degrees.

8. 310°

9. $-36°$

10. $\frac{2\pi}{9}$

11. $-\frac{5\pi}{6}$

Use the unit circle to find the exact value of each trigonometric function.

12. $\cos 210°$

13. $\tan \frac{11\pi}{6}$

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.

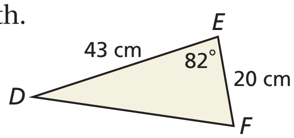
14. $\text{Cos}^{-1}\frac{\sqrt{2}}{2}$

15. $\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

16. A limestone cave is 6.2 mi south and 1.4 mi east of the entrance of a national park. To the nearest degree, in what direction should a group at the entrance head in order to reach the cave?

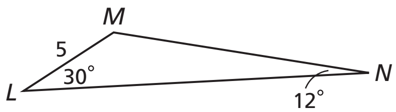
17. Find the area of $\triangle DEF$. Round to the nearest tenth.

18. Use the given measurements to solve $\triangle DEF$. Round to the nearest tenth.



19. An artist is designing a wallpaper pattern based on triangles. Determine the number of different triangles she can form using the measurements $a = 28$, $b = 13$, and $m\angle A = 102°$. Then solve the triangles. Round to the nearest tenth.

20. Solve $\triangle LMN$. Round to the nearest tenth.



21. A lawn next to an office building is shaped like a triangle with sides measuring 16 ft, 24 ft, and 30 ft. What is the area of the lawn to the nearest square foot?

College Entrance Exam Practice

CHAPTER 13

FOCUS ON SAT MATHEMATICS SUBJECT TESTS

Though both the SAT Mathematics Subject Tests Level 1 and Level 2 may include questions involving basic trigonometric functions, only the Level 2 test may include questions involving the Law of Sines, the Law of Cosines, and radian measure.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

HOT TIP! If you take the Level 1 test, make sure that your calculator is set to degree mode, because no questions will require you to use radians. If you take the Level 2 test, you will need to determine whether to use degree or radian mode as appropriate.

1. A right triangle has an angle measuring 22°. The shorter leg of the triangle has a length of 3 inches. What is the area of the triangle?

(A) 1.8 in^2

(B) 4.2 in^2

(C) 7.4 in^2

(D) 11.1 in^2

(E) 12.0 in^2

2. If $0 \leq \theta \leq \frac{\pi}{2}$ and $\cos^{-1}(\sin \theta) = \frac{\pi}{3}$, then what is the value of θ?

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{2}$

(C) $\frac{5\pi}{6}$

(D) $\frac{4\pi}{3}$

(E) $\frac{3\pi}{2}$

3. A triangle has side lengths of 7, 26, and 31. What is the measure of the smallest angle of the triangle?

(A) 8°

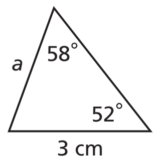
(B) 10°

(C) 26°

(D) 30°

(E) 40°

4. A manufacturer must produce a metal bar with a triangular cross section that meets the specifications shown. What is the length of side *a*?



Note: Figure not drawn to scale

(A) 2.4 cm

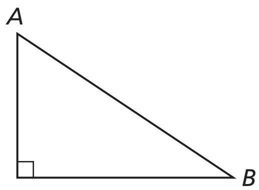
(B) 2.6 cm

(C) 2.8 cm

(D) 3.2 cm

(E) 3.6 cm

5. If $\sin B = \frac{5}{9}$ in the figure below, what is $\tan A$?



Note: Figure not drawn to scale

(A) 0.45

(B) 0.67

(C) 0.83

(D) 1.50

(E) 1.80

CHAPTER 13

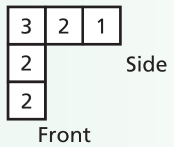
STRATEGIES FOR SUCCESS

Multiple Choice: Spatial-Reasoning Problems

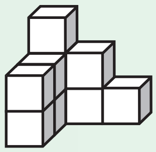
Some problems test your ability to use spatial reasoning. To solve these problems, you must be able to recognize different views of geometric figures. Orthographic drawings and nets are two common ways of representing three-dimensional objects. An *orthographic drawing* usually presents three views of a three-dimensional object: top, front, and side. A *net* is a diagram that can be folded to form a three-dimensional figure.

EXAMPLE 1

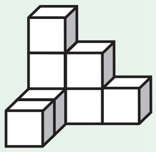
The drawing shows the top view of a structure made from cubes as well as the number of cubes in each column of the structure. Which three-dimensional view represents the same structure?



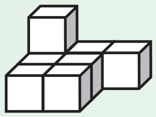
(A)



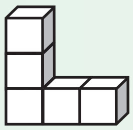
(C)



(B)

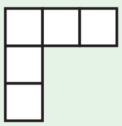


(D)

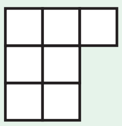


Start by sketching the top view of each answer choice.

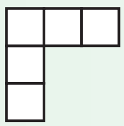
Choice A



Choice B



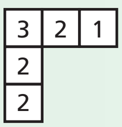
Choice C



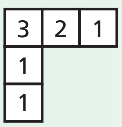
Choice D

Choices B and D can be eliminated because their top views do not match the one given in the problem. Next, count the number of cubes in each column of choices A and C.

Choice A



Choice C



The two front columns of choice C have only 1 cube each instead of 2. Therefore, choice C can be eliminated. Each column of choice A, however, has the correct number of cubes.

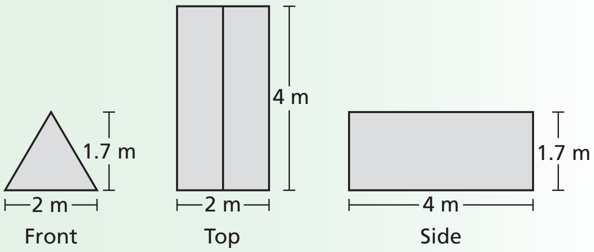
The correct answer is choice A.

HOT TIP! If you have trouble visualizing the geometric figures, make a quick sketch. Your sketch does not need to be exact, but it should show the sides or faces of the figures in the correct relationships to each other.

Read each test item and answer the questions that follow.

Item A

The front, top, and side views of a solid are shown below. What is the volume of the solid?



(A) $6.8\ m^3$

(B) $9.1\ m^3$

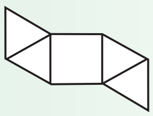
(C) $13.6\ m^3$

(D) $16.5\ m^3$

1. Make a sketch of the figure. What type of figure do the three views show?
2. How can you determine the volume of this figure?

Item B

What three-dimensional figure does this net represent?



(F) Square pyramid

(G) Triangular pyramid

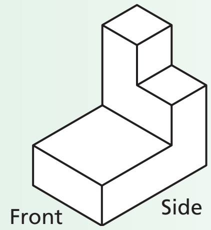
(H) Rectangular prism

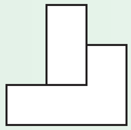
(J) Triangular prism

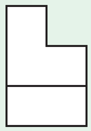
3. What type and number of faces does the figure have?
4. What type and number of faces does each of the answer choices have?

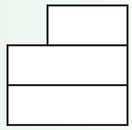
Item C

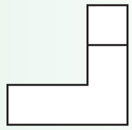
Which of the following is the front view of the figure shown?



(A)


(B)


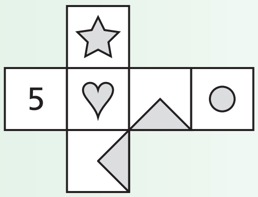
(C)


(D)


5. Make a sketch of the figure. Shade the faces on the front of the figure.
6. Describe the shapes of these faces.

Item D

Which is a true statement about the net of the cube shown?



(F) The face with the star and the face with the circle are parallel.

(G) The face with the heart and the face with the circle are perpendicular.

(H) The faces with the triangles are parallel.

(J) The face with the number 5 and the face with the star are perpendicular.

7. Which faces of the cube are opposite each other? Are opposite faces parallel or perpendicular to each other?
8. Can faces that share an edge be parallel to each other? Explain.

CHAPTER 9

STANDARDIZED TEST PREP

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CUMULATIVE ASSESSMENT, CHAPTERS 1–13

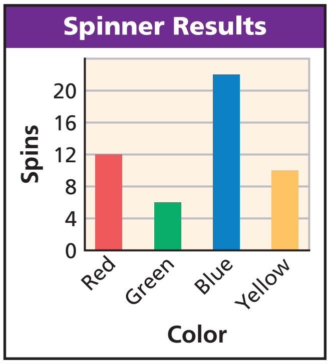
Multiple Choice

1. To the nearest tenth, what is the length of side a in $\triangle ABC$ if $m\angle A = 98°$, $b = 14.2$, and $c = 5.9$?

Ⓐ 8.4 Ⓒ 15.4

Ⓑ 12.9 Ⓓ 16.1

2. A spinner is divided into sections of different colors. The bar graph shows the results of spinning the pointer of the spinner 50 times.



What is the experimental probability that the pointer of the spinner will land on a blue section?

Ⓕ 0.05 Ⓗ 0.25

Ⓖ 0.22 Ⓙ 0.44

3. If $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, what value of θ makes the equation $\sin^{-1}\left(\frac{1}{2}\right) = \theta$ true?

Ⓐ $\frac{\pi}{6}$ Ⓒ $\frac{\pi}{3}$

Ⓑ $\frac{\pi}{4}$ Ⓓ $\frac{5\pi}{6}$

4. What is the inverse of the matrix $\begin{bmatrix} -3 & 4 \\ 0 & -1 \end{bmatrix}$?

Ⓕ $\begin{bmatrix} -\frac{1}{3} & -\frac{4}{3} \\ 0 & -1 \end{bmatrix}$ Ⓗ $\begin{bmatrix} -\frac{1}{3} & \frac{1}{4} \\ 0 & -1 \end{bmatrix}$

Ⓖ $\begin{bmatrix} 1 & 0 \\ -4 & 3 \end{bmatrix}$ Ⓙ $\begin{bmatrix} -1 & -4 \\ 0 & -3 \end{bmatrix}$

5. What is the equation of a hyperbola with center $(0, 0)$, a vertex at $(0, 6)$, and a focus at $(0, 10)$?

Ⓐ $\frac{x^2}{36} - \frac{y^2}{64} = 1$ Ⓒ $\frac{y^2}{36} + \frac{x^2}{64} = 1$

Ⓑ $\frac{y^2}{36} - \frac{x^2}{64} = 1$ Ⓓ $\frac{y^2}{64} - \frac{x^2}{36} = 1$

6. What is the 7th term of the following geometric sequence?

125, 25, 5, 1, ...

Ⓕ 0.2 Ⓗ 0.008

Ⓖ 0.04 Ⓙ 0.0016

7. What type of function best models the data in the table?

x	-2	-1	0	1	2
y	6.35	11.6	29.35	59.6	102.35

Ⓐ linear Ⓒ cubic

Ⓑ quadratic Ⓓ square root

8. If $f(x) = \frac{3}{2}x - 4$ and $g(x) = \frac{1}{2}f(x)$, what is the y-intercept of $g(x)$?

Ⓕ -4 Ⓗ $\frac{1}{2}$

Ⓖ -2 Ⓙ $\frac{8}{3}$

9. What is the common ratio of the exponential function represented by the table?

x	-2	-1	0	1	2
y	2.56	3.2	4	5	6.25

Ⓐ 0.16 Ⓒ 0.64

Ⓑ 0.25 Ⓓ 1.25

10. What effect does a translation 3 units right have on the graph of $f(x) = 2x - 5$?

Ⓕ The slope increases.

Ⓖ The slope decreases.

Ⓗ The value of the y-intercept increases.

Ⓙ The value of the y-intercept decreases.

HOT TIP! One way to check whether two expressions are equivalent is to substitute the same value for the variable in each expression. If the expressions simplify to different values, then they are *not* equivalent.

11. Which of the following is equivalent to $(x + 4)^3$?

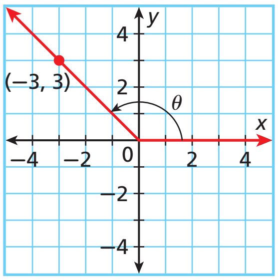
Ⓐ $x^3 + 64$

Ⓑ $x^3 + 8x + 16$

Ⓒ $4x^3 + 32x^2 + 64x$

Ⓓ $x^3 + 12x^2 + 48x + 64$

12. What is $\cos\theta$?



Ⓕ -1

Ⓖ $-\frac{\sqrt{2}}{2}$

Ⓗ $\frac{\sqrt{2}}{2}$

Ⓙ 1

13. What is the measure in degrees of an angle that measures $\frac{3\pi}{4}$ radians?

Ⓐ 45°

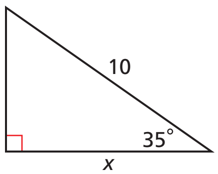
Ⓑ 90°

Ⓒ 135°

Ⓓ 180°

Gridded Response

14. To the nearest hundredth, what is the value of x in the triangle below?



15. What is the 15th term of an arithmetic sequence with $a_{11} = 425$ and $a_{17} = 515$?

16. How many different triangles can be formed with the following measurements?

$a = 6.38$, $b = 4.72$, $m\angle A = 132°$

17. The function $h(t) = -4.9t^2 + 4.9t + 1.75$ models the height in meters of a lacrosse ball, where t is the time in seconds since the ball was thrown. Based on the model, what is the maximum height in meters that the ball will reach?

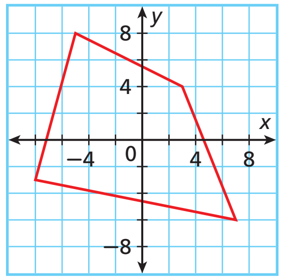
Short Response

18. Tickets for the water park cost $12 for children under 13, $20 for people ages 13 to 60, and $15 for people over 60 years of age.

a. Write a function to represent the cost y in dollars of a ticket to the water park given a person's age x in years.

b. Graph the function.

19. What are the constraints on the region bounded by the quadrilateral below?



20. The table shows the number of Atlantic hurricanes for the years 1997–2004.

Atlantic Hurricanes			
Year	**Number**	**Year**	**Number**
1997	3	2001	9
1998	10	2002	4
1999	8	2003	7
2000	8	2004	9

a. Make a box-and-whisker plot of the hurricane data.

b. What is the mean number of hurricanes per year for the time period shown in the table?

Extended Response

21. The choir director at a high school wants to rent an auditorium for an upcoming performance. To pay for the auditorium, $550 must be raised in ticket sales. The cost of the tickets will depend on the number of people who are expected to attend the performance.

a. Write a function to represent the number of dollars y a ticket should cost when x is the number of people who are expected to attend the performance.

b. What are the asymptotes of the function?

c. Graph the function.

d. What is a reasonable domain and range for the function? Explain.

e. How much should tickets cost if 250 people are expected to attend the performance?