Inverse Functions

Thursday, February 1, 2018 10:26 AM

Exp $\longleftrightarrow$ logarithms

Inverses:

old: $x + 3 = 7$

$y = x + 3$

get x alone

inverse operation

$y - 3 = x$

$f(x) = x + 3$

inverse of f. $f^{-1}(x) = x - 3$

Recipe for Finding an inverse function.

1. Replace $f(x)$ with y.
2. switch x with y.
3. solve for y
4. replace y with $f^{-1}(x)$

Ex:

$f(x) = 3x + 7$

1. $y = 3x + 7$

2. $x = 3y + 7$

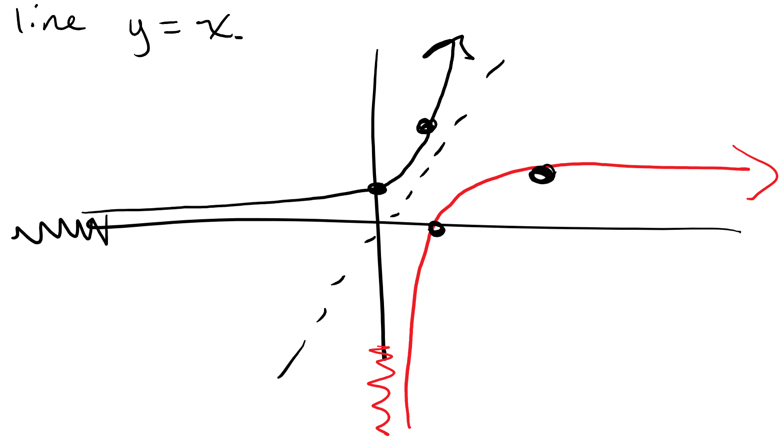
3. $x - 7 = 3y$

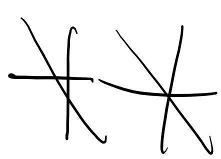
   $\frac{x-7}{3} = y$

4. $\boxed{f^{-1}(x) = \frac{x-7}{3}}$

Inverses graphically.

- inverses are reflections through line $y = x$.





- $(0,1) \rightarrow (1,0)$
- $(1,2) \rightarrow (2,1)$

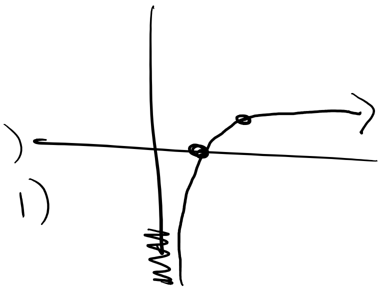
x & y change place

inverse

$y = 2^x$ $\qquad$ $x = 2^y$

$(0,1)$
$(1,2)$

tiny x axis asymptote



$(1,0)$
$(2,1)$

tiny y axis asymptote

Exponentials $\longleftrightarrow$ logarithms

$8 = 2^3 \longleftrightarrow 3 = \log_2 8$

3 is the exponent on 2 that yeilds answer 8.

$4^2 = 16$ $\qquad$ $2 = \log_4 16$

$5 = \log 32$

$= \log_2 32$

$2^5 = 32$

5 is exponent on 2

?

$\log_3 27 = ?$

$\log_3 27 = \boxed{3}$

What exponent on 3 yeilds answer of 27?

The answer to a log question is an exponent

$10{,}000(1.02)^{17} = \boxed{}$

$A = P(1 \pm r)^t = \$14{,}002.41$

Logs were invented to answer this question.

$$5{,}000 = 500(1.07)^t \text{ ?}$$

Logs help us find the missing exponent.

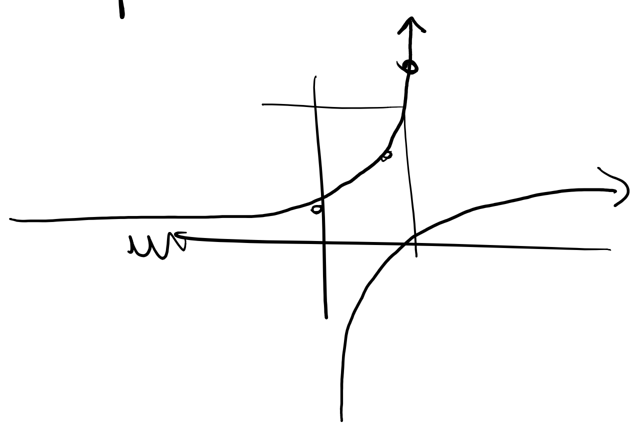
Solve:

$2^x = 16$

$x = 4$

$2^x = 47$

x is between 5 + 6



$\log_5 125 = \boxed{3}$

$2^0 = 1$ $\quad 3^0 \quad 4^0 \quad 5^0 \quad 6^0 \quad 7^0$

$2^0 = 1$ $\quad 3^0 \quad 4^0 \quad 5^0 \quad 6^0 \quad 7$

$2^1 = 2$

$2^2 = 4$

$\vdots$

$2^8 = 256$ $\quad 3^5 \quad 4^4 \quad 5^4 \quad 6^4 \quad 7^2$

Exponential $\qquad\qquad$ logarithm.

$b^x = a \quad \longleftrightarrow \quad \log_b a = x$

base, answer, exponent

$b > 0$

$b \neq 1$

Convert from exponential to logarithm.

$2^6 = 64 \implies \log_2 64 = 6$

$5^0 = 1 \implies \log_5 1 = 0$

$10^2 = 100 \implies \log_{10} 100 = 2$

Books

$\log 100 = 2$

understood to be 10.

Log $\longrightarrow$ Exponential

?

$\log_5 125 = \boxed{3} \longrightarrow 5^3 = 125$

$\log_2 8 = \boxed{3} \longrightarrow 2^3 = 8$

$\log_8 64 = \boxed{2} \longrightarrow 8^2 = 64$

$\log_7 7 = \boxed{1} \longrightarrow 7^1 = 7$

$\log_{216} 1 = \boxed{0} \longrightarrow 216^0 = 1$

$\log_8 \frac{1}{8} = \boxed{-1} \longrightarrow 8^{-1} = \frac{1}{8}$

Recall

$x^{1/2} = \sqrt{x}$

$x^{1/3} = \sqrt[3]{x}$

$\log_9 3 = \frac{1}{2} \longrightarrow 9^{1/2} = 3$

$\sqrt[3]{8} = 2$ $\quad \log_8 2 = \frac{1}{3} \longrightarrow 8^{1/3} = 2$

$\log_7 \frac{1}{49} = -2 \longrightarrow 7^{-2} = \frac{1}{49}$

$\sqrt[3]{2} = 2^{1/3}$ $\quad \log_2 \sqrt[3]{2} = \frac{1}{3} \longrightarrow 2^{1/3} = \sqrt[3]{2}$

$2^3 = 8$

$2 = \sqrt[3]{8}$

$2 = 8^{1/3}$

$7^2 = 49$

$7 = \sqrt{49}$

$7 = 49^{1/2}$

$\log_5 5 = 1$

$\log_5 1 = 0$

argument
can't be 0

$\log \square$

Domain:
$x > 0$

$$\boxed{\log_{\frac{1}{2}} 16 = x}$$

hint: $\frac{1}{2}$ is a power of 2.

$\left(\frac{1}{2}\right)^x = 16$

$\left(2^{-1}\right)^x = 16$

$2^{-x} = 16 = 2^4$

$-x = 4$

$x = -4$

$\left(\frac{1}{2}\right)^{-4} = 16$

$\left(\frac{1}{2}\right)^{-4} = 16$

$$\left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right)^{-4} = \left(2 \cdot 2 \cdot 2 \cdot 2\right)^{4}$$

7.2 20-28; 38

7.3 2-13; 17-28; 41, 42, 48-51