Notes on Extrema

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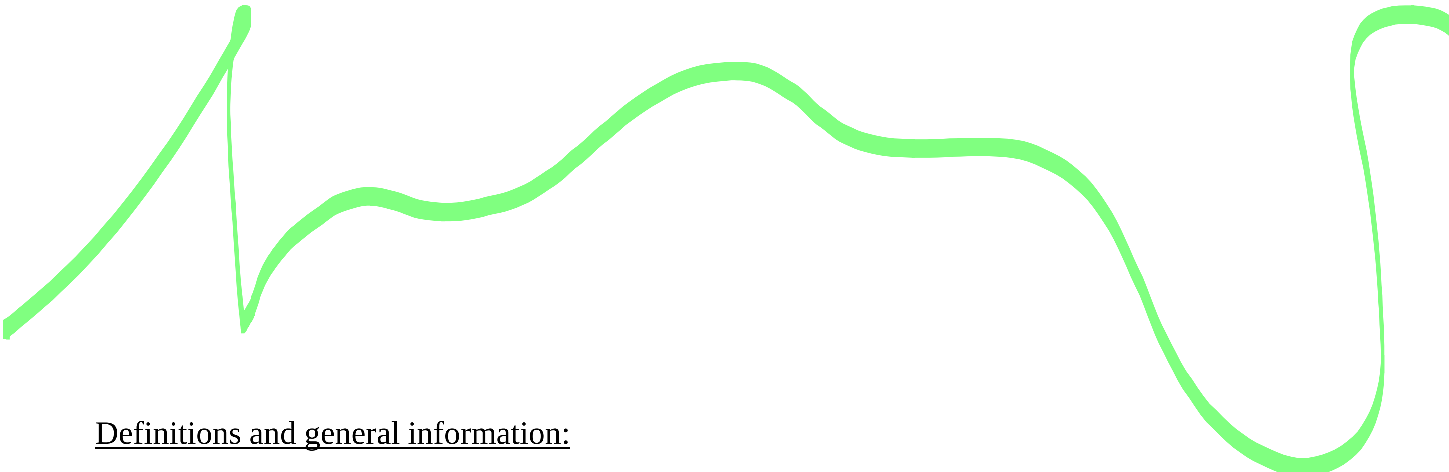
Notes on Extrema (Ch 3.1)



Definitions and general information:

Given some function f(x) on an Interval containing x = c,
f (c) is a maximum of the Interval if f (c)
$\geq$
f(x) for all x values in the Interval.

Given some function f(x) on an Interval containing x = c,
f (c) is a minimum of the Interval if f (c)
$\leq$
f(x) for all x values in the Interval.

On open intervals (no endpoints), functions do not need to have a minimum or maximum value, BUT functions on closed intervals (with endpoints) will have both a minimum and maximum value in the interval.

Extrema = absolute minimum and/or absolute maximum, could be none or one or one of each.
Relative extrema = minimums and/or maximums in a "neighborhood" and there could be none or one or several of each.

Minimums and maximums, whether local(relative) or absolute, must occur at some specific "name-able" point. You would be able to say, "the point (3, -2) is a local maximum" or "there is an absolute minimum at x = 5",

Critical Numbers

Critical numbers are our "candidates" for relative and absolute extrema.

A critical number is the point, c, where either

f'(c) = 0 (The graph of f(x) would show "c" at the top of a hill or bottom of a valley.) This means there would be a horizontal tangent at the point (c, f(c)).

OR

f'(c) is undefined but the point (c, f(c)) is defined.

Absolute extrema occur at critical numbers OR at endpoints. For absolute extrema, you do NOT need to test intervals around the critical point. You DO need to evaluate f(x) at the critical numbers and at the endpoints to find the highest and/or lowest function values.

(A side note: Relative extrema occur at critical numbers and DO need BOTH testing of the interval around the critical number and to evaluate the function value. This will include the First Derivative Test from Chapter 3.2)

Steps for finding Extrema (Absolute min and max.)

1. Find all critical numbers, c, where f' = 0 or f' is undefined.
2. Evaluate the function value(s), f(c), at all of the critical points in the interval.
3. Evaluate the function value, f(x), at the endpoints.
4. Compare the function values from the critical points (step 2) and the endpoints (step 3) and determine :

   The greatest f(x) is the Absolute Maximum.
   The least f(x) is the Absolute Minimum.

3.1: 1-12; 15-39 (3N)
41, 53-56.