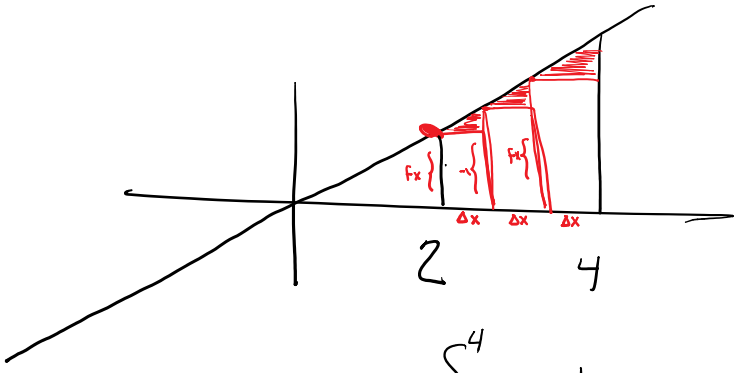
Summation

Thursday, January 11, 2018 8:17 AM

A. $\int x\,dx = \frac{1}{2}x^2$

Antiderivative

B. $\int x\,dx$ = Area under the curve $y = x$



Area btwn curve and x axis

$$\int_2^4 x\,dx$$

$$\int_2^4 f(x)\,dx = \underline{f(x_1)\cdot\Delta x} + \underline{f(x_2)\cdot\Delta x} + \underline{f(x_3)\Delta x}$$

$$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$$

$$\sum_{i=1}^{3} f(x_i)\,\Delta x$$

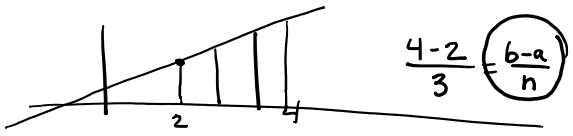
TRUE Area: $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$

$\Updownarrow$

$\int_2^4 f(x)\,dx$

aka $\int_a^b f(x)\,dx = \lim_{\Delta x\to 0} \sum_{i=1}^{n} f(x_i)\,\Delta x$

1. Summation w/ left endpoints



$\frac{4-2}{3} = \frac{b-a}{n}$

$\frac{2}{3} = \Delta x$

left endpoints

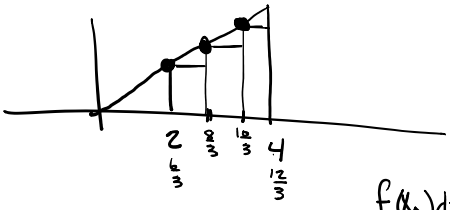
2	2
$2+\frac{2}{3}$	$\frac{8}{3}$
$2+\frac{2}{3}(2)$	$\frac{10}{3}$

$\Delta x = \frac{b-a}{n}$

Right: $x_i = x_0 + \Delta x \cdot i$

left $x_i = x_0 + \Delta x(i-1)$

Area using 3 Rectangles
left endpoints on $\int_2^4 f(x)dx$



$X_0\ X_1\ X_2\ X_3$

Left Sum

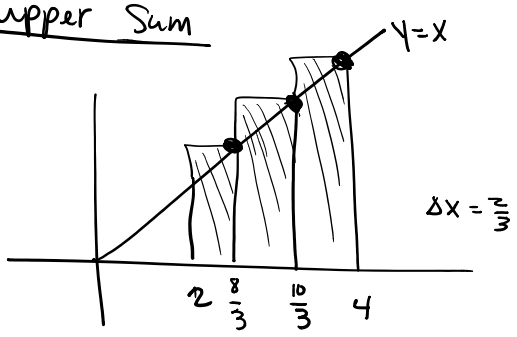
$f(x_0)dx + f(x_1)dx + f(x_2)dx$

$2\left(\frac{8}{3}\right) + \frac{8}{3}\left(\frac{2}{3}\right) + \frac{10}{3}\left(\frac{2}{3}\right)$

$\frac{4}{3} + \frac{16}{9} + \frac{20}{9}$

$\frac{48}{9} = \frac{16}{3}$ ✓

Upper Sum



$\frac{2}{3}\left(\frac{8}{3}\right) + \frac{2}{3}\left(\frac{10}{3}\right) + \frac{2}{3}\left(\frac{12}{3}\right)$

$\frac{16}{9} + \frac{20}{9} + \frac{24}{9}$

$\frac{60}{9} = \boxed{\frac{20}{3}}$

$\sum_{i=1}^{7} i$

$\sum_{i=1}^{16} i^2$ P 260

$\sum_{i=1}^{15} i^2 + i$

$\sum_{i=1}^{n} C = C \cdot n$ $\quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Ex: $\sum_{i=1}^{n} \frac{i+1}{n^2}$

$\frac{1}{n^2} \sum_{i=1}^{n} (i+1)$

$\frac{1}{n^2} \left[\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1 \right]$

$\frac{1}{n^2} \left[\frac{n(n+1)}{2} + n \right]$

$\frac{1}{n^2} \left[\frac{n^2+n}{2} + n \right]$

$\frac{1}{n^2} \left[\frac{n^2}{2} + \frac{n}{2} + n \right]$

$\frac{n^2}{2n^2} + \frac{n}{2n^2} + \frac{n}{n^2}$

$\frac{1}{2} + \frac{1}{2n} + \frac{1}{n^2}$

$\frac{n+3}{2n}$ *

4.2 11-29 odd